The volume of matter and dark energy dominated universe

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Abstract: The new mathematical model allows us to calculate the volume of infinite universe which varies with respect to time (t).It is shown that the mathematical equation for calculation of volume of infinite universe which varies with respect to time (t) accounts for scale factor of universe (a (t)),vacuum energy density, density parameter of present dark energy and matter dominated universe, density parameter of present dark energy dominated universe. The mathematical expressions was developed based on the cosmological concepts. Radius of Hubble sphere, vacuum energy density, critical density of universe and cosmological constant are brought togather in one frame of reference to explain the phenomenon of rate of expansion of universe. [Academia Arena, 2010;2(6):14-18] (ISSN 1553-992X).

Key words: Density parameter of universe, Scale factor of universe, vacuum energy density, critical density of universe, volume of universe

The Universe comprises everything we perceive to physically exist, the entirety of space and time, all forms of matter and energy. However, the term Universe may be used in slightly different contextual senses, denoting such concepts as the cosmos, the world, or Nature. Recent observations indicate that this expansion is accelerating because of dark energy, and that most of the matter in the Universe may be in a form which cannot be detected by present instruments, and so is not accounted for in the present models of the universe; this has been named dark matter. The universe is not expanding into anything, almost by definition; there is simply more space at later times than at earlier times. It may be that the size of the universe is infinite, which is easy to conceptualize.Even if the universe is finite, it is possible to make more space without having any "outside" space. It is believed the that the Universe has expanded from a primordial hot and dense initial condition at some finite time in the past .

Figure-1: Matter and dark energy dominated universe

Let us consider matter dominated universe.Mass density of matter dominated universe varies with respect to time (**t**)due to the cause of expansion of universe.If the universe is matter-dominated, then the mass density of the universe(**ρ)** can just be taken to include matter so

$$\rho_m = \rho_{m0} / a^3 \quad \text{.....(1)}$$

Here ρ_m =Mass density of universe which vary with respect to time(**t**) i.e $\rho_{m(t)}$, ρ_{m0} =Present mass density of universe ,**a**=Scale factor of universe(which is the function of time) i.e **a(t)**.

Density parameter of present matter dominated universe is given by

$$\Omega_m = \rho_{m0} / \rho_c \quad \text{.....(2)}$$

Here ρ_c=Critical density of present matter dominated universe
Thus (1) becomes

$$\rho_m = \Omega_m \rho_c / a^3 \quad \text{.....(3)}$$

The critical density is the watershed between an expanding and a contracting Universe**.**

Critical density of matter dominated universe is given by

$$\rho_c = 3H_o^2 / 8 \pi G \quad \text{.....(4)}$$

Here H_o= Present hubble parameter (it indicates rate of expansion of universe),**G**=Universal gravitational constant .

Thus (3) becomes

$$\rho_m = \Omega_m 3H_o^2 / 8 \pi G\, a^3 \quad \text{.....(5)}$$

By multiplying the equation (5) by C^2

Here **C**= Speed of light in vaccum**($3*10^8$m/s)**

We get

$$\rho_m C^2 = \Omega_m 3H_o^2\, C^2 / 8 \pi G\, a^3 \quad \text{.....(6)}$$

Vacuum energy is an underlying background energy that exists in space even when devoid of matter (known as free space). The vacuum energy is deduced from the concept of virtual particles, which are themselves derived from the energy-time uncertainty principle. Its effects can be observed in various phenomena (such as spontaneous emission, the Casimir effect, the van der Waals bonds, or the Lamb shift), and it is thought to have consequences for the behavior of the Universe on cosmological scales**.**

The vacuum energy density is constant and given by

$$\rho_{vaC} = \Lambda C^2 / 8 \pi G \quad \text{.....(7)}$$

Here Λ=Cosmological constant(dark energy).

$$\rho_{vaC} / \Lambda = C^2 / 8 \pi G$$

Recent observations indicate that the rate of expansion of universe is accelerating because of dark energy, and that most of the matter in the Universe may be in a form which cannot be detected by present instruments, and so is not accounted for in the present models of the universe; this has been named dark matter**. I**f the universe is both matter-dominated and dark energy-dominated. Let us now consider matter and dark energy dominated universe

Thus(6)becomes

$\rho_m C^2 = \Omega_m 3H_o^2 \rho_{vac} / a^3 \Lambda$.....(8)

Present cosmological (Dark energy)density parameter is given by

$\Omega_\Lambda = \Lambda C^2/3H_o^2$

$3H_o^2/ \Lambda = C^2/ \Omega_\Lambda$.....(9)

Thus (8) becomes

$\rho_m C^2 = \Omega_m \rho_{vac} C^2 / a^3 \Omega_\Lambda$.....(10)

$\rho_m = \Omega_m \rho_{vac} / a^3 \Omega_\Lambda$.....(11)

Present density parameter of matter and dark energy dominated universe is given by

$\Omega = \Omega_m + \Omega_\Lambda$.....(12)

Thus (11) becomes

$\rho_m a^3/ \rho_{vac} = (\Omega - \Omega_\Lambda) / \Omega_\Lambda$.....(13)

$\rho_m a^3/ \rho_{vac} = (\Omega / \Omega_\Lambda - 1)$.....(14)

Density of matter and dark energy dominated universe which vary with respect to time (t) is given by

$\rho = \rho_m + \rho_\Lambda$

Here ρ= Density of matter and dark energy dominated universe which vary with respect to time (t)
ρ_m = Density of matter dominated universe which vary with respect to time (t)
ρ_Λ= Density of dark energy dominated universe which vary with respect to time (t)

Thus (14) becomes

$\rho = \rho_{vac}/ a^3 (\Omega / \Omega_\Lambda - 1) + \rho_\Lambda$.....(15)

Density of matter and dark energy dominated universe which vary with respect to time (t) is given by

$\rho(t) = M(t) / V(t)$

Here **V(t) =** Volume of matter and dark energy dominated universe which vary with respect to time (**t**)

Thus (15) becomes

$V(t) = M(t) / [\rho_{vac}/ a^3 (\Omega / \Omega_\Lambda - 1) + \rho_\Lambda]$.....(16)

Here **M(t)=** Mass of matter and dark energy dominated universe which vary with respect to time(t).
a=Scale factor of universe.
ρ_{vac}= vaccum energy
Ω= Present density parameter of matter and dark energy dominated universe

Ω_Λ=Present density parameter of dark energy dominated universe.
ρ_Λ= Density of dark energydominated universe which vary with respect to time (t)

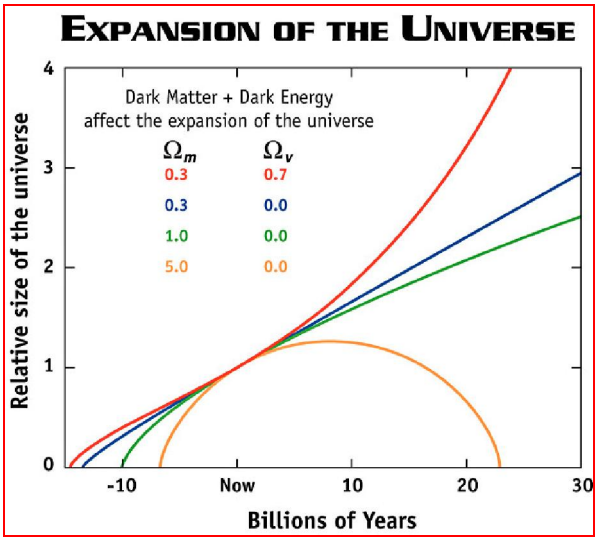

Figure-2:Expansion of matter and dark energy dominated universe.

Critical density of matter and dark energy dominated universe is given by

$\rho_c = 3H^2/8\pi G$.....(17)

Here **H**=Hubble parameter which is the function of time **H(t)**.

Divide (17) by C^2

$\rho_c / C^2 = 3H^2/8\pi G C^2$.....(19)

Radius of hubble sphere is given by

$r_{hs} = C/H$

Thus (17) becomes

$r_{hs}^2 = 3C^2/8\pi G \rho_c$.....(20)

From (7) we know

$\rho_{vaC} = \Lambda C^2/8\pi G$

Thus (20) becomes

$r_{hs}^2 = 3\rho_{vaC} / \Lambda \rho_c$.....(21)

Here r_{hs} = Radius of hubble sphere.
Λ = Cosmological constant

ρ_c=Critical density of universe
ρ_{vac}=Vacuum energy density

Result:
The volume of matter and dark energy dominated universe is given by the relation

$$V(t) = M(t) / [\rho_{vac} / a^3 (\Omega / \Omega_\Lambda - 1) + \rho_\Lambda].$$

[Here **M(t)=** Mass of matter and dark energy dominated universe which vary with respect to time(t),**a=**Scale factor of universe, ρ_{vac}= vaccum energy,**Ω=** Present density parameter of matter and dark energy dominated universe**,** Ω_Λ=Present density parameter of dark energy dominated universe,ρ_Λ= Density of dark energydominated universe which vary with respect to time (t)]

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Conclusion : The mass density of universe is not constant but varies with respect to time (t) due to the cause of expansion.The dark energy accelerates the rate of expansion of universe.The mathematical determination of the mass density of matter and dark energy dominated universe according to the formula(15)take into account vaccum energy, scale factor of universe and density parameter of universe respectively.The radius of hubble sphere can be calculated mathematically by knowing the value of ρ_{vac}, Λ ,ρ_c. The constants like **C,** Λ are included in the paper to lay a foundation for the mathematical equation to come into existence.

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