

Role of Convex Optimization with Nonlinear Programming: Problem Review

Ritu Sharma , Dr. Mayank Pawar, Dr. Sanjeev Rajan

¹Research Scholar, Hindu College, Moradabad²Teerthanker Mahaveer University, Moradabad³Hindu College, Moradabad

Abstract: In this paper we study the convex optimization problem with non linear programming. In this is a problem where all of the constraints are convex functions and the objective is a convex function if minimizing, or a concave function if maximizing.

[Ritu Sharma , Dr. Mayank Pawar, Dr. Sanjeev Rajan. *Role of Convex Optimization with Nonlinear Programming: Problem Review. Academia Arena 2013;5(2):1-2*] (ISSN 1553-992X). <http://www.sciencepub.net/academia.1>

Keywords: convex optimization, Concave Optimization, non linear programming, Convex function.

Introduction

There is in general no analytical formula for the solution of convex optimization problems, but (as with linear programming problems) there are very effective methods for solving them. Convex optimization methods work very well in practice, and in some cases can be proved to solve the problem to a specified accuracy with a number of operations that does not exceed a polynomial of the problem dimensions. [1]

We will see that interior-point methods can solve the problem in a number of steps or iterations that is almost always in the range between 10 and 100. Ignoring any structure in the problem (such as sparsity), each step requires on the order of

$$\max\{n^3, n^2m, F\}$$

operations, where F is the cost of evaluating the first and second derivatives of the objective and constraint functions f_0, \dots, f .

We cannot yet claim that solving general convex optimization problems is a mature technology, like solving least-squares or linear programming problems. Research on interior-point methods for general nonlinear convex optimization is still a very active research area, and no consensus has emerged yet as to what the best method or methods are. But it is reasonable to expect that solving general convex optimization problems will become a technology within a few years. And for some subclasses of convex optimization problems, for example second-order cone programming or geometric programming, it is fair to say that interior-point methods are approaching a technology. [1]

Role of convex optimization in non convex problems

In this paper we can focus primarily on convex optimization problems, and applications that can be reduced to convex optimization problems. But

convex optimization also plays an important role in problems that are not convex.[1]

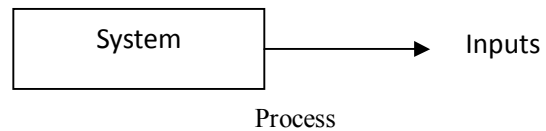


Figure (a): Optimization Problem

Convex heuristics algorithm for non convex optimization

Convex Optimization is a mathematically rigorous and well-studied field. In linear programming a whole host of tractable methods give your global optimums in lightning fast times. Quadratic programming is almost as easy, and there's a good deal of semi-definite, second-order cone and even integer programming methods that can do quite well on a lot of problems.

Non-convex optimization (and particularly weird formulations of certain integer programming and combinatorial optimization problems), however, are generally heuristics like "ant colony optimization". Essentially all generalizable non-convex optimization algorithms I've come across are some (often clever, but still) combination of gradient descent and genetic algorithms. [2]

Nonlinear optimization

Nonlinear optimization (or nonlinear programming) is the term used to describe an optimization problem when the objective or constraint functions are not linear, but not known to be convex. Sadly, there are no effective methods for solving the general nonlinear programming problem. Even simple looking problems with as few as ten variables can be extremely challenging, [3] while problems with a few hundreds of variables can be intractable. Methods for the general nonlinear programming problem therefore

take several different approaches, each of which involves some compromise. [6]

Consider the following non-linear programming problem

(P) Minimize $f(x)$
subject to

$$g_j(x) \leq 0, \quad j=1, 2, \dots, m \quad x \in X$$

where f and g_j , $j = 1, 2, \dots, m$ are real valued functions defined on $X \subseteq \mathbb{R}^n$. Let $X_0 = \{x \in X \mid g_j(x) \leq 0, j=1, 2, \dots, m\}$ denote the set of feasible solutions, which is also called the constraint set.

Since 1951, there has been tremendous growth in the field of non-linear programming problems. There is no single method which gives accurate solution of a non-linear programming problem. Some of the well known algorithms used in such situations are Golden section method, Fibonacci search method and conjugate gradient method discussed by Avriel [7]. Many of the non-linear programming problems have been successfully studied by linearizing the function and finding solutions to the corresponding linear programming problems.

LOCAL vs. GLOBAL OPTIMUM

The fastest optimization algorithms seek only a local solution, a point at which the objective function is smaller than at all other feasible points in its vicinity. They do not always find the best of all such minima, that is, the *global solution*. Global solutions are necessary (or at least highly desirable) in some applications, but they are usually difficult to identify and even more difficult to locate. An important special case is *convex programming*, in which all local solutions are also global solutions. Linear programming problems fall in the category of convex programming. However, general nonlinear problems, both constrained and unconstrained, may possess local solutions that are not global solutions. Geometrically, nonlinear programs can behave much differently from linear programs, even for problems with linear constraints. As shows, the optimal solution can occur:

- a) At an interior point of the feasible region;
- b) On the boundary of the feasible region, which is not an extreme point; or
- c) At an extreme point of the feasible region.

As a consequence, procedures, such as the simplex method, that search only extreme points may not determine an optimal solution.

Conclusion

In this paper we can study the role of convex programming is the non linear Optimization problem. And it can view the different type of problem in different method and here we can solve the convex optimization method. And in this paper we can see the non linear programming problem can solve with convex heuristic optimization method and solve the problems.

References

- [1]. Stephen Boyd and Lieven Vandenberghe, “Convex Optimization”, cambridge university press.
- [2]. <http://mathoverflow.net/questions/32533/is-all-non-convex-optimization-heuristic>
- [3]. M.P. Bendsøe, A. Ben-Tal and J. Zowe, “Optimization methods for truss geometry and topology design,” Structural Optimization, vol. 7, pp. 141–159, 1994.
- [4]. A.V. Fiacco and G.P. McCormick, “Nonlinear Programming: Sequential Unconstrained Minimization Techniques,” Research Analysis Corporation, McLean Virginia, 1968 (Republished in 1990 by SIAM, Philadelphia).
- [5]. J. M. Borwein and A. S. Lewis. Convex Analysis and Nonlinear Optimization. Springer, 2000.
- [6]. Goran Lesaja And Verlynda N. Slaughter, Interior-Point Algorithms For A Class Of Convex Optimization Problems, Yugoslav Journal Of Operations Research Volume 19 (2009) Number 2, 239-248.
- [7]. M. Avriel Non-linear Programming: Analysis and Methods, Prentice Hall, Inc, Englewood Cliffs, New Jersey 1976.

1/12/2013