Compton Theory Of The Scattering Of X-Rays By Light Elements

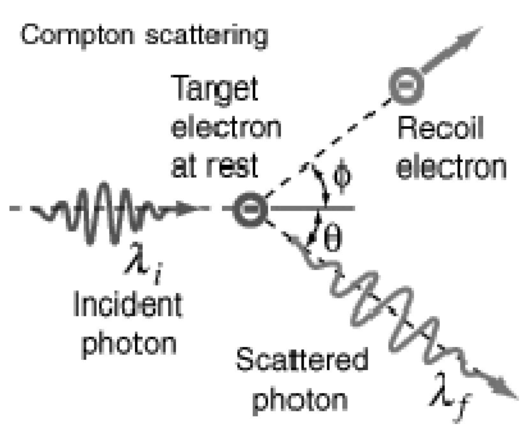
Manjunath. R

16/1, 8th Main Road, Shivanagar, Rajajinagar, Karnataka 560010, India
manjunath5496@gmail.com

Abstract: At early 1920 when the particle nature of photon suggested by the photoelectric effect was still being debated, the Compton Effect gave clear and independent evidence of particle-like behavior. The Compton hypothesis is suggested that when an X-ray photon is scattered it spends all of its energy and momentum upon some particular electron (which is treated as being at rest). This electron in turn scatters the ray in some definite direction. The change in momentum of the X-ray photon due to the change in its direction of propagation results in a recoil of the scattering electron. The energy in the scattered photon is thus less than the energy of the incident photon by the kinetic energy of recoil of the scattering electron. The corresponding increase in the wavelength of the scattered photon is $(\lambda_f - \lambda_i) = (h/m_0C) \times (1 - \cos\theta)$ and the time it takes for the incident photon to change its wavelength is $t = (h/m_0C^2) \times (1 - \cos\theta)$, where h is the Planck's constant, m_0 is the rest mass of the scattering electron, C is the speed of light in vacuum, and θ is the angle between the incident and the scattered photon. Experimental result is given which show that for graphite and the Mo- Kα radiation the value of t (for $\theta = 90^o$) has been found to be 7.333×10^{-21} s which being satisfactorily close to the computed value (8.089×10^{-21} s). In the case of $\theta = 135^o$, the value of t has been found to be 0.133×10^{-19} s which also being satisfactorily close to the computed value (0.138×10^{-19} s). However, the value of t has been found to vary with θ in agreement with the theory, increasing from 7.333×10^{-21} s ($\theta = 90^o$) to 0.133×10^{-19} s ($\theta = 135^o$). Velocities of recoil of the scattering electrons have not been experimentally determined. This is probably because the electrons which recoil in the process of the scattering of X-rays have not been observed. However, velocity of recoil of the scattering electron (when $\theta = 90^o$) has been calculated using: (1) Law of Conservation of Energy. (2) Law of Conservation of Momentum. The value of v computed using the Law of Conservation of Energy was found to be 0. And the value of v computed using the Law of Conservation of Momentum was found to be 0.04C. Using the conservation laws the law of variation of mass with velocity has been derived which in bad agreement with the Einstein's law of variation of mass with velocity. But according to which also when $v \rightarrow C$, $m_E \rightarrow \infty$.i.e., an electron travelling at the speed of light would have infinite mass and hence, no material particle can have a velocity equal to the speed of light in vacuum.

[Manjunath. R. **Compton Theory Of The Scattering Of X-Rays By Light Elements.** *Academ Arena* 2015;7(5):65-68]. (ISSN 1553-992X). http://www.sciencepub.net/academia. 8

Keywords: Photon; Electron; Compton; Scattering angle.



Above figure illustrates the scattering of an incident photon of energy $E = h\upsilon_0$ moving to the right with a momentum $p_0 = h/\lambda_i$ and interacting with an electron at rest with momentum = 0 and energy equal to its rest energy, m_0C^2. The symbols h, υ, and λ are the standard symbols used for Planck's constant, the photon's frequency, its wavelength, and m_0 is the rest mass of the electron. In the interaction, the X- ray photon is scattered in the direction at an angle θ with respect to the photon's incoming path with momentum $p = h/\lambda_f$ and energy $E = h\upsilon$. The electron is scattered in the direction at an angle φ with respect to the photon's incoming path with momentum $p = m_Ev$ and energy $E = m_EC^2$ where m_E is the relativistic mass of the electron after the interaction. The phenomenon of Compton scattering may be analyzed as an elastic collision of a photon with a free electron using relativistic mechanics. Since the energy of the photons (661. 6 *ke*V) is much greater than the binding energy of electrons (the most tightly bound electrons have a binding energy less than 1 *ke*V), the electrons which scatter the photons may be considered free electrons. Because energy and momentum must be conserved in an elastic collision, we can obtain the formula for the wavelength of the scattered photon, λ_f as a function of scattering angle θ: $\lambda_f = \{(h/m_0C) \times (1 - \cos\theta) + \lambda_i\}$

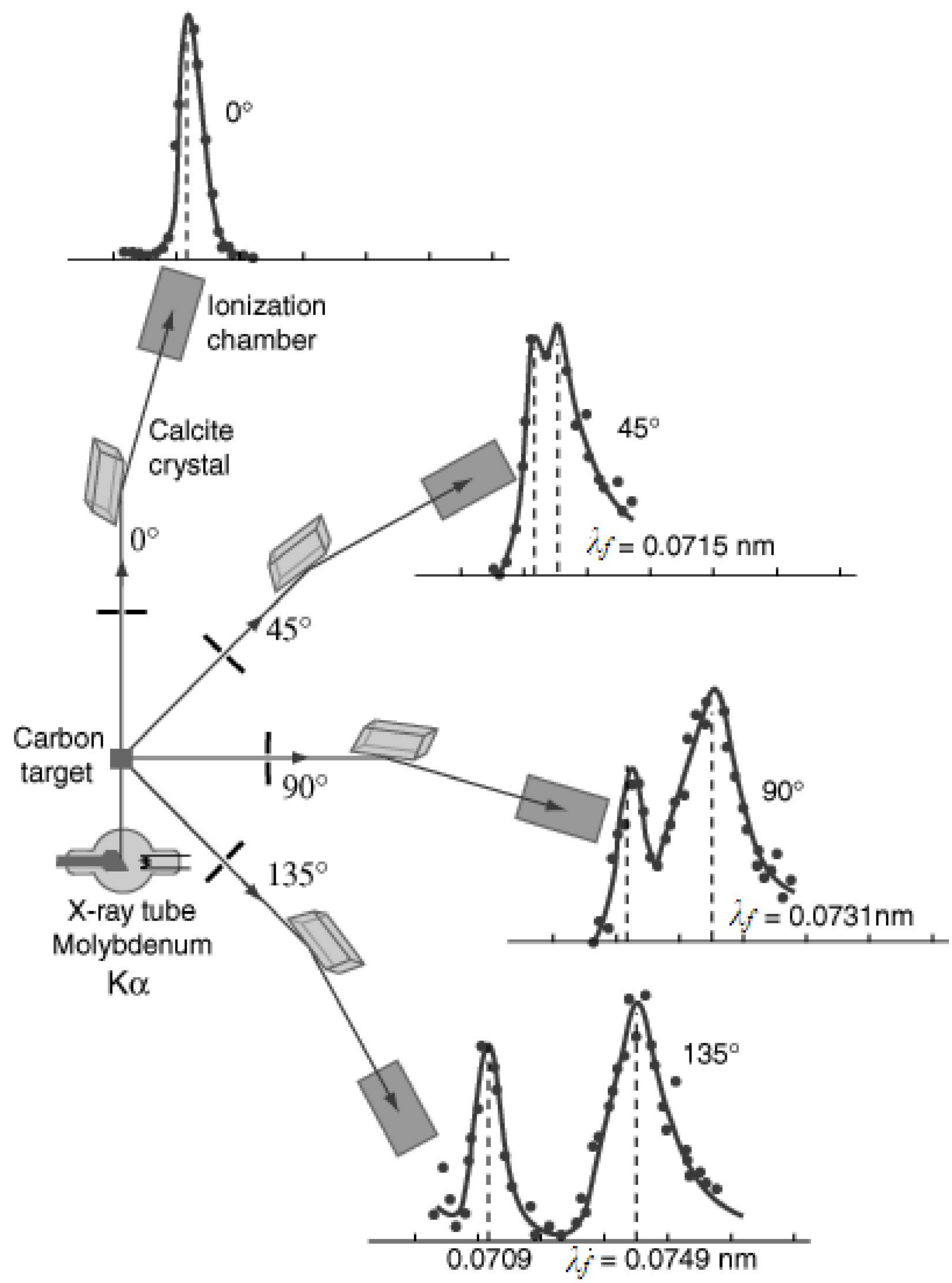
where λ_i is the wavelength of the incident photon and C is the speed of light in vacuum.

The photon power, $P = -(dE/dt)$, is given by the relation: $P = -dE/dt = h\upsilon^2$, where $E = h\upsilon$. But $\upsilon = C/\lambda$. Therefore:

$d\lambda = C \times dt$

Integrating over $d\lambda$ from λ_i (the wavelength of the incident photon) to λ_f (the wavelength of the scattered photon), and over dt from zero to t:

$(\lambda_f - \lambda_i) = C \times t$



It can be seen that that the wavelength of the scattered photon is unquestionably greater than that of the incident photon. The Kα line from molybdenum has a wavelength 0.0709 nm. The wavelength of this line in the scattered beam is found to be 0.0731nm.

$t = (\lambda_f - \lambda_i)/ C = 7.333 \times 10^{-21}$ s (experiment).

Substituting $(\lambda_f - \lambda_i) = C \times t$ in the equation $(\lambda_f - \lambda_i) = (h/m_0C) \times (1- \cos\theta)$, we get:

$t = (h/m_0C^2) \times (1- \cos\theta) = (8.089 \times 10^{-21}\ s) \times (1- \cos90^o) = 8.089 \times 10^{-21}$ s (theory), which is a very satisfactory agreement. In the case of $\theta = 135^o$, $(\lambda_f - \lambda_i) = (0.0749 - 0.0709)$ nm

$t = (\lambda_f - \lambda_i)/ C = 0.133 \times 10^{-19}$ s (experiment)

and $t = (h/m_0C^2) \times (1- \cos\theta) = (h/m_0C^2) \times (1- \cos135^o) = 0.138 \times 10^{-19}$ s (theory), which is also a very satisfactory agreement. However, the value of t has been found to vary with θ in agreement with the theory, increasing from 7.333×10^{-21} s ($\theta = 90^o$) to 0.133×10^{-19} s ($\theta = 135^o$). Velocities of recoil of the scattering electrons have not been experimentally determined. This is probably because the electrons which recoil in the process of the scattering of X-rays have not been observed. However, velocity of recoil of the scattering electrons can be calculated using the

- Law of Conservation of Energy.
- Law of Conservation of Momentum.

Case 1: Calculating the velocity of recoil of the scattering electron (when $\theta = 90^o$) using the Law of Conservation of Energy.

From the law of conservation of energy, the energy of the incident X-ray, $h\upsilon_0$, and the rest energy of the electron, m_0C^2, before scattering is equal to the energy of the scattered X-ray, $h\upsilon$, and the total energy of the electron, m_EC^2, after scattering

$h\upsilon_0 + m_0C^2 = h\upsilon + m_EC^2$ which on rearranging: $(h\upsilon_0 - h\upsilon) = m_EC^2 - m_0C^2$

But according to law of variation of mass with velocity

$m_EC^2 = m_0C^2 / (1- v^2/C^2)^{½}$

Therefore:

$(h\upsilon_0 - h\upsilon) = m_0C^2 \{1/ (1- v^2/C^2)^{½} - 1\}$

Solving $h\upsilon_0 = hC/\lambda_i = 28.072 \times 10^{-36}$ J, $h\upsilon = hC/\lambda_f = 27.226 \times 10^{-36}$ J and $m_0C^2 = 81.9 \times 10^{-15}$ J, we get:

$(28.072 - 27.226) \times 10^{-36} = 81.9 \times 10^{-15} \times \{1/ (1- v^2/C^2)^{½} - 1\}$

$(0.846 \times 10^{-36} / 81.9 \times 10^{-15}) + 1 = 1/ (1- v^2/C^2)^{½}$

$[1.0329 \times 10^{-23} + 1] = 1/ (1- v^2/C^2)^{½}$

Since: $1.0329 \times 10^{-23} <<<< 1$. Therefore: $[1.0329 \times 10^{-23} + 1] \approx 1$

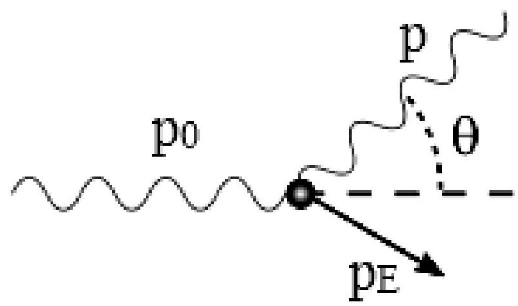
$1 = 1/ (1- v^2/C^2)^{½}$

From this it follows that

$v = 0$, which is meaningless. There can be no bigger limitation than this.

Case 2: Calculating the velocity of recoil of the scattering electron (when $\theta = 90^o$) using the Law of Conservation of Momentum.

Imagine, as in figure below,



that an X-ray photon of frequency υ_0 is scattered by an electron of mass m_0. The momentum of the incident photon will be $p_0 = h / \lambda_i$, where h is the Planck's constant, and that of the scattered photon is $p = h / \lambda_f$ at angle θ with the initial momentum. The principle of the conservation of momentum accordingly demands that the momentum of recoil of the scattering electron shall equal the vector difference between the momenta of these photons. The momentum of the electron, $p_E = m_0Cv/ (C^2 - v^2)^{½}$, is thus given by the relation

$m_0^2 C^2v^2/ (C^2 - v^2) = p_0^2 + p^2 - 2p_0p\cos\theta$

Solving $p_0^2 = (h / \lambda_i) = 87.553 \times 10^{-48}\ J^2s^2/m^2$, $p^2 = (h / \lambda_f)^2 = 82.355 \times 10^{-48}\ J^2s^2/m^2$ and $\theta = 90^o$, we get:

$m_0^2 C^2v^2/ (C^2 - v^2) = (p_0^2 + p^2) = (87.553 + 82.355) \times 10^{-48}$

$m_0^2 C^2v^2/ (C^2 - v^2) = 169.908 \times 10^{-48}\ J^2s^2/m^2$

But $m_0^2C^2 = 745.29 \times 10^{-46}\ J^2$. Therefore:

$v^2/ (C^2 - v^2) = (169.908 \times 10^{-48} / 745.29 \times 10^{-46}) = 2.279 \times 10^{-3}$

$v^2 = 2.279 \times 10^{-3}C^2 - 2.279 \times 10^{-3}v^2$

$v^2 (1 + 2.279 \times 10^{-3}) = 2.279 \times 10^{-3}C^2$

From this it follows that

$v = 0.04C$

Derivation of law of variation of mass with velocity

From the law of conservation of mass, the mass of the incident X-ray, m_i, and the rest mass of the electron, m_0, before scattering is equal to the mass of the scattered X-ray, m_f, and the total mass of the electron, m_E, after scattering

$m_i + m_0 = m_f + m_E$ which on rearranging: $m_E = m_0 + (m_i - m_f)$

The photon masses are related to the frequencies by

$m_i = h\upsilon_0 /C^2$

$m_f = h\upsilon /C^2$

Therefore:

$m_E = m_0 + (h\upsilon_0 - h\upsilon) / C^2$

But $(h\upsilon_0 - h\upsilon) = m_E C^2 - m_0 C^2$ = kinetic energy of the electron (KE). Therefore:

$m_E = m_0 + (KE / C^2)$

In physics, we define the kinetic energy of the electron to be equal to the work done by a photon impulse to increase velocity of the electron from zero to some value v. That is,

$KE = J \times v$

Impulse applied to an object produces an equivalent change in its linear momentum. The photon impulse may be expressed in a simpler form:

$J = \Delta p_E = p_2 - p_1$

where:

p_2 = momentum of the electron after scattering = $m_E v$

p_1 = momentum of the electron before scattering = 0 (since the electron was initially at rest).

$J = m_E v$

$KE = m_E v^2$

Solving for KE we get:

$m_E = m_0 + (m_E v^2 / C^2)$

$m_E = m_0 / (1 - v^2/C^2)$ which in bad agreement with the Einstein's law of variation of mass with velocity. But according to which also when $v \rightarrow C$, $m_E \rightarrow \infty$.i.e., an electron travelling at the speed of light would have infinite mass and hence, no material particle can have a velocity equal to the speed of light in vacuum. However, this is in conflict with the body of scientific knowledge. Cerenkov radiation is emitted by a material particle that is traveling faster than the light in a medium.

References:

1. A Quantum Theory of the Scattering of X-rays by Light Elements by Arthur. H Compton (1923).
2. ABOUT THE DETERMINATION OF THE PHOTON POWER by Dr Sergej Reissig (2003), EFBR (Research & Development Office Reissig).
3. The Compton Effect-- Compton Scattering and Gamma Ray Spectroscopy by Dr. James E. Parks (2015).
1. Derive the mass to velocity relation by William J. Harrison (the general science journal).
2. Physics I For Dummies Paperback- June 17, 2011 by Steven Holzner.
3. Physics II For Dummies Paperback- July 13, 2010 by Steven Holzner.

5/19/2015