BLACK HOLE

Manjunath.R

16/1, 8th Main Road, Shivanagar, Rajajinagar, Karnataka 560010, India
manjunath5496@gmail.com

Abstract: Most people think of a black hole as a voracious whirlpool in space, sucking down everything around it. But that's not really true! A black hole is a place where gravity has gotten so strong that even light cannot escape out of its influence. In this paper, we are going to look in more detail at some of the aspects of the black hole. [Manjunath R. **BLACK HOLE.** *Academ Arena* 2015; 7(5):106-107]. (ISSN 1553-992X). http://www.sciencepub.net/academia.11

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A photon generated at the center of the star makes its way to the surface. It may take up to several million years to get to the surface, and the gravitational potential energy of the photon at the surface of the star is given by: $PE = -\,GMm/R$, where $G = 6.673 \times 10^{-11}\ Nm^2/kg^2$ is Gravitational constant, m is the photon mass, M and R denote the mass and radius of the star. If the photon wants to detach from the star surface, it should obey the condition: $F_G = F_P$, where $F_G = GMm/R^2$ is the force of gravitation experienced by the photon and $F_P = mC^2/\lambda$ is the force which moves the photon.

$$F_G = F_P \text{ or } GMm/R^2 = mC^2/\lambda$$

From this it follows that: $R^2 = GM\lambda/C^2$. If $F_P<F_G$, then photon cannot detach from the star surface. And if the condition $F_G = F_P$ is obeyed and the photon detaches the star surface, its energy shifts from $h\upsilon$ to $h\upsilon_0$. The change in photon energy is given by the relation: $(h\upsilon - h\upsilon_0) = -\,GMm/R$ or $(h\upsilon - h\upsilon_0)/h\upsilon = -\,GM/RC^2$. Since the gravitational binding energy of a star is given by $U = -3GM^2/4R$. Therefore: $(h\upsilon - h\upsilon_0)/h\upsilon = 4U/3MC^2$. If a star collapses to a black hole a photon will be red shifted to zero frequency i.e., $h\upsilon_0 = 0$.

$$(h\upsilon - 0)/h\upsilon = 4U/3MC^2 \text{ or } U = \tfrac{3}{4}\, MC^2$$

This means that the gravitational binding energy of a black hole frames ¾ of its total energy. For a black hole of one solar mass ($M = 2 \times 10^{30}$ kg), we get a gravitational binding energy of 13.5×10^{46} joules – much higher than its entropic energy. The rate of loss of photon energy, $P = -\,(dE/dt)$, is related to the photon frequency υ by: $P = -\,dE/dt = h\upsilon^2$, where $E = h\upsilon$. But $\upsilon = C/\lambda$. Therefore: $d\lambda = C \times dt$. Integrating over $d\lambda$ from λ (the wavelength of the photon before detaching from the star surface) to λ_0 (the wavelength of the detached photon), and over dt from zero to t: $(\lambda_0 - \lambda) = C \times t$. From this it follows that:

$$(\upsilon - \upsilon_0)/\upsilon\upsilon_0 = t \text{ or } h(\upsilon - \upsilon_0)/h\upsilon\upsilon_0 = t$$

Since $(h\upsilon - h\upsilon_0)/h\upsilon = -\,GM/RC^2$. Therefore:

$$t = -\,GM/R\upsilon_0C^2 \text{ or } t = -\,GM\lambda_0/RC^3$$

The negative sign indicates that more the time the photon takes to detach from the star surface the smaller the wavelength of the detached photon. Looking at the unusual nature of Hawking radiation; it may be natural to question if such radiation exists in nature or to suggest that it is merely a theoretical solution to the hidden world of quantum gravity. The attempt to understand the Hawking radiation has had a profound impact upon the understanding of the black hole thermodynamics, leading to the description of what the black hole entropic energy is. Black hole entropic energy = Black hole temperature × Black hole entropy.

$$E_s = T_{BH} \times S_{BH}$$

$$E_s = \tfrac{1}{2}\, MC^2$$

This means that the entropic energy makes up half of the total energy of the black hole. For a black hole of one solar mass ($M = 2 \times 10^{30}$ kg), we get an entropic energy of 9×10^{46} joules – much higher than the thermal entropic energy of the sun. The gravitational binding energy of a black hole is related to its entropic energy by: $U = ¾ MC^2$

$$U = ¾ (2 E_s) = 1.5 E_s$$

This means that the gravitational binding energy of a black hole is 1.5 times its entropic energy. Given that power emitted in Hawking radiation is the rate of energy loss of the black hole:

$$P = -C^2 (dM / dt) \text{ or } P = 2 \times (-dEs / dt)$$

The more power a black hole radiates per second, the more entropic energy being lost in Hawking radiation. However, the total entropic energy of the black hole of one solar mass is about 9×10^{46} joules of which only 4.502×10^{-29} joules per second is lost in Hawking radiation. The image we often see of photons as a tiny bit of light circling a black hole in well-defined orbit of radius. The image we often see of photons as a tiny bit of light circling a black hole in well-defined circular orbit of radius $r = 3GM/C^2$ (where G = Newton's universal constant of gravitation, C= speed of light in vacuum and M = mass of the black hole) is actually quite interesting. The angular velocity of the photon orbiting the black hole is given by: $\omega = C/r$.

For circular motion the angular velocity is the same as the angular frequency. Thus

$$\omega = C/r = 2\pi C/\lambda$$

From this it follows that: $\lambda = 2\pi r$

The De Broglie wavelength λ associated with the photon of mass m orbiting the black hole is given by: $\lambda = h/mC$. Therefore: $r = \hbar/mC$, where $\hbar$ is the reduced Planck constant. The photon must satisfy the condition $r = \hbar/mC$ much like an electron moving in a circular orbit. Since this condition forces the photon to orbit the hole in a circular orbit.

$$r = 3GM/C^2 = \hbar/mC \text{ or } 3GM/C^2 = \hbar/mC$$

$$\text{Or } 3mM = (\text{Planck mass})^2$$

Because of this condition the photons orbiting the small black hole carry more mass than those orbiting the big black hole. For a black hole of one Planck mass (M = Planck mass),

$$m = 1/3 \times \text{Planck mass}$$

The average energy of the emitted Hawking radiation photon is given by: $L = 2.821\, k_B T$ (where k_B = Boltzmann constant and T = black hole temperature).

$L = 2.821\, k_B T = (\hbar C^3 / 8\pi GM)$ which on rearranging:

$$GM / C^2 = 2.821 (\hbar C / 8\pi L)$$

Since $3GM/C^2 = \hbar/mC$. Therefore:

$$\hbar / 3mC = 2.821 (\hbar C / 8\pi L)$$

From this it follows that

$$mC^2 = 2.968L \text{ or } mC^2 > L$$

If a photon with energy mC^2 orbiting the black hole can't slip out of its influence, and so how can a Hawking radiation photon with energy $L < mC^2$ is emitted from the event horizon of the black hole? So it may be natural to question if such radiation exists in nature or to suggest that it is merely a theoretical solution to the hidden world of quantum gravity.

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