Algebraic Structures of Soft subsets and Partitions of a soft set

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Abstract: This research paper describes certain monoids, semirings and lattices of subsets and partitions of a soft set.

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1. Introduction

Certain algebraic structures consisting of subsets and partitions of a set have been developed which are shown to be useful in applications in the areas of computer arithmetic, sequential machines, formal languages and syntactic analysis (see [5, 13], for details).In the recent years, following the formulation of soft set theory in [7], various operations on soft sets and their properties have been studied (see [2, 3, 6, 9, 10], for example). In the sequel, a number of underlying algebraic structures associated with these operations got developed (see [3, 4,8, and 12] for example). Recently, monoids of multiset partitions have been studied in [11]. In this paper, certain monoids, semirings and lattices of soft subsets and partitions of a soft set are described.

2. Preliminaries

In this section, some basic notions relevant to this paper are presented.

Definition 2.1 [9]

Let U be an initial universe set and E a set of parameters or attributes with respect to U. Let $P(U)$ denote the power set of U and $A \subseteq E$. A pair (F, A) is called a **soft set** over U, where F is a mapping given by $F: A \to P(U)$.

In other words, a soft set (F, A) over U is a parameterized family of subsets of U. For $e \in A$, $F(e)$ may be considered as the set of e-elements or e-approximate elements of the soft set (F, A). Thus (F,A) is defined as

$$(F, A) = \{F(e) \in P(U) : e \in E,\ F(e) = \varnothing \text{ if } e \notin A\}$$

Characteristically, F is a *set-valued* function of a set.

Definition 2.2 [6]

Let (F, A) and (G, B) be two soft sets over a common universe U, we say that

(a) (F,A) is a **soft subset** of (G,B), denoted $(F, A) \tilde{\subseteq} (G, B)$, if

(i) $A \subseteq B$, and

(ii) $\forall\ e \in A, F(e) \tilde{\subseteq} G(e)$.

(b) (F, A) is **soft equal** to (G, B), denoted $(F, A) = (G, B)$, if $(F, A) \tilde{\subseteq} (G, B)$ and $(G, B) \tilde{\subseteq} (F, A)$.

Definitions 2.3 [2]

Let U *be* a universe, E a set of parameters and $A \subseteq E$.

a) *(F, A)* is called a **relative null soft set** with respect to *A*, denoted $\tilde{\Phi}_A$, if $F(e) = \emptyset$, $\forall\ e \in A$.

b) *(F , A)* is called a **relative whole soft set** or *A*-universal with respect to *A*, denoted $\tilde{U}_A$, if $F(e) = U$, $\forall\ e \in A$.

c) The relative whole soft set with respect to *E*, denoted $\tilde{U}_E$, is called the **absolute soft set** over *U*.

d) The relative null soft set with respect to E, denoted $\tilde{\Phi}_E$, is called the **null soft set** over U.

e) The unique soft set (F.A) is called the **empty soft set** over *U*, denoted $\tilde{\Phi}_\phi$, with empty parameter set.

Remark 2.1: $\tilde{\Phi}_\phi \tilde{\subseteq} \tilde{\Phi}_A \tilde{\subseteq} \tilde{\Phi}_E \tilde{\subseteq} (F, A) \tilde{\subseteq} \tilde{U}_A \tilde{\subseteq} \tilde{U}_E$.

Definitions 2.4 [6,9]

Let (F,A) and (G,B) be two soft sets over a common universe U.

(i) The **union** of (*F*,*A*) and (*G*,*B*), denoted $(F,A)\tilde{\cup}(G,B)$, is a soft set (*H*,*C*) where $C = A\cup B$ and $\forall\ e\in C$,

$$H(e)=\begin{cases}F(e), & e\in A-B\\ G(e), & e\in B-A\\ F(e)\cup G(e), & e\in A\cap B.\end{cases}$$

The **extended intersection** of (*F*,*A*) and (*G*,*B*), denoted $(F,A)\tilde{\cap}(G,B)$, is a soft set (*H*,*C*) where $C = A\cup B$ and $\forall\ e\in C$,

$$H(e)=\begin{cases}F(e), \text{ if } e\in A-B\\ G(e), \text{ if } e\in B-A\\ F(e)\cap G(e), \text{ if } e\in A\cap B.\end{cases}$$

(ii) The **restricted intersection** of (*F*,*A*) and (*G*,*B*), denoted (*F*,*A*) $\cap_R$ (*G*,*B*), is a soft set (*H*,*C*) where $C = A\cap B$ and $\forall\ e\in C,\ H(e)=F(e)\cap G(e)$. If $A\cap B$ =ϕ then (*F*,*A*) $\cap_R$ (*G*,*B*)= $\tilde{\Phi}_\phi$.

(iii) The **restricted union** of (*F*,*A*) and (*G*,*B*), denoted (*F*,*A*) $\cup_R$ (*G*,*B*), is a soft set (*H*,*C*) where $C = A\cap B$ and $\forall\ e\in C$, $H(e)=F(e)\cup G(e)$. If $A\cap B$ =ϕ then (*F*,*A*) $\cup_R$ (*G*,*B*) = $\tilde{\Phi}_\phi$.

In the following Propositions, some results, abstracted from [2,3,6,9,10], which will be useful for later discussion, are presented.

Propositions 2.1

Let (F, A), (G, B) and (H, C) be soft sets over a common universe.

1. **Idempotent properties**

(i) $(F,A)\tilde{\cup}(F,A)=(F,A)=(F,A)\cup_R(F,A)$

(ii) $(F,A)\tilde{\cap}(F,A)=(F,A)=(F,A)\cap_R(F,A)$

2. **Identity Properties**

(i) $(F,A)\tilde{\cup}\tilde{\Phi}=(F,A)=(F,A)\cup_R\tilde{\Phi}$

(ii) $(F,A)\tilde{\cap}\tilde{U}=(F,A)=(F,A)\cap_R\tilde{U}$

3. **Commutative Properties**

(i) $(F,A)\tilde{\cup}(G,B)=(G,B)\tilde{\cup}(F,A)$

(ii) $(F,A)\cup_R(G,B)=(G,B)\cup_R(F,A)$

(iii) $(F,A)\tilde{\cap}(G,B)=(G,B)\tilde{\cap}(F,A)$

(iv) $(F,A)\cap_R(G,B)=(G,B)\cap_R(F,A)$

4. **Associative Properties**

(i) $(F,A)\tilde{\cup}\big((G,B)\tilde{\cup}(H,C)\big)=\big((F,A)\tilde{\cup}(G,B)\big)\tilde{\cup}(H,C)$

(ii) $(F,A)\cup_R\big((G,B)\cup_R(H,C)\big)=\big((F,A)\cup_R(G,B)\big)\cup_R(H,C)$

(iii) $(F,A)\tilde{\cap}\big((G,B)\tilde{\cap}(H,C)\big)=\big((F,A)\tilde{\cap}(G,B)\big)\tilde{\cap}(H,C)$

(iv) $(F,A)\cap_R\big((G,B)\cap_R(H,C)\big)=\big((F,A)\cap_R(G,B)\big)\cap_R(H,C)$

5. **Distributive properties**

(i)$(F,A)\tilde{\cup}\big((G,B)\cap_R(H,C)\big)=\big((F,A)\tilde{\cup}(G,B)\big)\cap_R\big((F,A)\tilde{\cup}(H,C)\big)$

(ii)$(F,A)\cap_R\big((G,B)\tilde{\cup}(H,C)\big)=\big((F,A)\cap_R(G,B)\big)\tilde{\cup}\big((F,A)\cap_R(H,C)\big)$

(iii)$(F,A)\cup_R\big((G,B)\tilde{\cap}(H,C)\big)=\big((F,A)\cup_R(G,B)\big)\tilde{\cap}\big((F,A)\cup_R(H,C)\big)$

(iv)$(F,A)\tilde{\cap}\big((G,B)\cup_R(H,C)\big)=\big((F,A)\tilde{\cap}(G,B)\big)\cup_R\big((F,A)\tilde{\cap}(H,C)\big)$

(v)$(F,A)\cup_R\big((G,B)\cap_R(H,C)\big)=\big((F,A)\cup_R(G,B)\big)\cap_R\big((F,A)\cup_R(H,C)\big)$

(vi)$(F,A)\cap_R\big((G,B)\cup_R(H,C)\big)=\big((F,A)\cap_R(G,B)\big)\cup_R\big((F,A)\cap_R(H,C)\big)$

(vii)$(F,A)\tilde{\cup}\big((G,B)\tilde{\cap}(H,C)\big)\neq\big((F,A)\tilde{\cup}(G,B)\big)\tilde{\cap}\big((F,A)\tilde{\cup}(H,C)\big)$

(viii)$(F,A)\tilde{\cap}\big((G,B)\tilde{\cup}(H,C)\big)\neq\big((F,A)\tilde{\cap}(G,B)\big)\tilde{\cup}\big((F,A)\tilde{\cap}(H,C)\big)$

(ix)$(F,A)\cup_R\big((G,B)\tilde{\cup}(H,C)\big)=\big((F,A)\cup_R(G,B)\big)\tilde{\cup}\big((F,A)\cup_R(H,C)\big)$

(x)$(F,A)\cap_R\big((G,B)\tilde{\cap}(H,C)\big)=\big((F,A)\cap_R(G,B)\big)\tilde{\cap}\big((F,A)\cap_R(H,C)\big)$

$$\text{(xi)}(F,A)\tilde{\cup}\left((G,B)\cup_R(H,C)\right)\neq\left((F,A)\tilde{\cup}(G,B)\right)\cup_R\left((F,A)\tilde{\cup}(H,C)\right)$$
$$\text{(xii)}(F,A)\tilde{\cap}\left((G,B)\cap_R(H,C)\right)\neq\left((F,A)\tilde{\cap}(G,B)\right)\cap_R\left((F,A)\tilde{\cap}(H,C)\right).$$

6. **Absorption Properties**

i. $(F,A)\tilde{\cup}\left((F,A)\cap_R(G,B)\right)=(F,A)$

ii. $(F,A)\cap_R\left((F,A)\tilde{\cup}(G,B)\right)=(F,A)$

iii. $(F,A)\cup_R\left((F,A)\tilde{\cap}(G,B)\right)=(F,A)$

iv. $(F,A)\tilde{\cap}\left((F,A)\cup_R(G,B)\right)=(F,A).$

7. **Absorption inclusion properties**

(i) $(F,A)\cup_R\left((F,A)\cap_R(G,B)\right)\tilde{\subseteq}(F,A)$

(ii) $(F,A)\cap_R\left((F,A)\cup_R(G,B)\right)\tilde{\subseteq}(F,A)$

(iii) $(F,A)\tilde{\cup}\left((F,A)\tilde{\cap}(G,B)\right)\tilde{\supseteq}(F,A)$

(iv) $(F,A)\tilde{\cap}\left((F,A)\tilde{\cup}(G,B)\right)\tilde{\supseteq}(F,A).$

Definition 2.5

Let (*F*,*A*) and (*G*,*B*) be two soft sets over a common universe *U*. (*F*,*A*), and (*G*,*B*) are said to be **disjoint** if A and B are disjoint or $(F,A)\tilde{\cap}(G,B)=\tilde{\Phi}_C$ where $C=A\cap B.$

Definition 2.6

A collection $\tilde{P}$ of non-empty pair wise disjoint soft subsets of a soft set (F, A) whose union

is (F,A), is called a **partition** of (F,A). In other words, a collection $\tilde{P}$ = {(F_i, A_i), i ∈ I} is called a partition of (F,A) if

(i) (F, A) = $\bigcup_{i\in I}$ (F_i, A_i), and
(ii) $A_i \cap A_j = \emptyset$ for $i \neq j$.

The elements of $\tilde{P}$ are called the *blocks* or *cells* of $\tilde{P}$.

3. Algebraic Structures Associated with Soft Sets

In this section, algebraic structures associated with collections of soft subsets and that of partitions of a soft set are developed.

Let us recall the following definitions of algebraic structures associated with sets.

An algebraic structure (S, ∗) consisting of a non-empty set S with an associative binary operation ∗ is called a **semigroup**. If there exists an element e in (S, ∗) such that e∗x = x∗e = x for all x ∈ S, then S is called a **monoid** with e as the identity element. If for every element x ∈ S, x∗x = x, then S is called **idempotent**.

An algebraic structure (R, +,.) consisting of a non-empty set R together with two binary operations, usually called addition (+) and multiplication (.), is called a **semiring** if the following conditions are satisfied:

(i) (R, +) is a commutative monoid,
(ii) (R, .) is a monoid, and
(iii) Multiplication distributes over addition from both sides i.e.,
a(b + c) = ab + ac and (b + c)a = ba + ca.

A **lattice** (L,∨,∧) is a non-empty set L with two binary operations ∨ and ∧ such that

(i) (L ,∨) is a commutative, idempotent semigroup,
(ii) (L ,∧) is a commutative, idempotent semigroup, and
(iii) Absorption properties hold.

If a lattice has identity elements with respect to both the operations, then it is called **bounded**.

If distributive properties hold in a lattice, then it is called a **distributive lattice**.

3.1 Commutative, Idempotent Monoids and Semirings of Soft Subsets of a Soft Set

Let U be an initial universe, A a parameter set with respect to U, and (F, A) a soft set over U.

Let SS(F, A) denote the collection of all soft subsets of a soft set (F, A).

It is immediate to see from Proposition 2.1:(1), (3) and (4) that (SS(F, A), ∗) are commutative, idempotent semigroups for ∗ ∈ {$\tilde{\cup}$, $\cup_R$, $\tilde{\cap}$, $\cap_R$}.

A. Monoids

1. (SS(F,A), ∗ ∈ {$\tilde{\cup}$, $\tilde{\cap}$}) are commutative, idempotent monoids with $\tilde{\Phi}_\emptyset$ as the identity element.

2. (SS(F,A), $\cup_R$) is a commutative, idempotent monoid with $\tilde{\Phi}_A$ as the identity element.

3. (SS(F,A), $\cap_R$) is a commutative, idempotent monoid with(F,A) as the identity element.

B. Semirings

1. (SS(F,A), $\tilde{\cup}$, $\cup_R$) is a commutative, idempotent semiring with identity element $\tilde{\Phi}_\emptyset$.

2. (SS(F,A), $\cap_R$, $\tilde{\cup}$) is a commutative, idempotent semiring with identity element (F,A).

3. (SS(F,A), $\cup_R$, $\tilde{\cap}$) is a commutative, idempotent semiring with identity element $\tilde{\Phi}_A$.

4. (SS(F,A), $\tilde{\cap}$, $\cup_R$) is a commutative, idempotent semiring with identity element $\tilde{\Phi}_\emptyset$.

5. (SS(F,A), $\cup_R$, $\cap_R$) is a commutative, idempotent semiring with identity element $\tilde{\Phi}_A$.

6. (SS(F,A), $\cap_R$, $\cup_R$) is a commutative, idempotent semiring with identity element (F,A).

7. (SS(F,A), $\tilde{\cap}$, $\cap_R$) is a commutative, idempotent semiring with identity element $\tilde{\Phi}_\emptyset$.

8. (SS(F,A), $\tilde{\cup}$, $\cap_R$) is a s, idempotent semiring with identity element $\tilde{\Phi}_\emptyset$.

[Proofs of the results stated above in (A) and (B) follow from definitions and properties of the respective operations given in section 2] .

Remark 3.1

1. (SS(F,A), $\tilde{\cap}$, $\tilde{\cup}$) and (SS(F,A), $\tilde{\cup}$, $\tilde{\cap}$) are not semirings since $\tilde{\cap}$ and $\tilde{\cup}$ do not distribute over each other(see Proposition 2.1: 5(vii and viii))

2. (SS(F,A), $\cup_R$, $\tilde{\cup}$) and (SS(F,A), $\cap_R$, $\tilde{\cap}$) are not semirings since $\tilde{\cap}$ and $\tilde{\cup}$ do not distribute over $\cap_R$ and $\cup_R$, respectively (see Proposition 2.1: 5(xi and xii)).

3.2 Bounded, Distributive Lattices of Soft Subsets of a Soft Set

Let SS(F, A) be as defined in 3.1 above.

Proposition 3.1

(i) (SS(F,A), $\tilde{\cap}$, $\cup_R$) and (SS(F,A), $\cup_R$, $\tilde{\cap}$), and

(ii) (SS(F,A), $\cap_R$, $\tilde{\cup}$) and (SS(F,A), $\tilde{\cup}$, $\cap_R$)

are bounded, distributive lattices.

Proof

(i) (SS(F,A), $\tilde{\cap}$) and (SS(F,A), $\cup_R$) are commutative, idempotent semi groups with $\tilde{\Phi}_\emptyset$ and $\tilde{\Phi}_A$ as the identity elements, respectively(see 3.1 above). Moreover $\tilde{\cap}$ and $\cup_R$ distribute and absorb over each other [see Proposition 2.1:5 (iii, iv) and 6(iii, iv), respectively]. Hence (i) holds.

(ii)Similarly, (SS(F,A), $\tilde{\cup}$) and (SS(F,A), $\cap_R$) are commutative, idempotent semi groups with $\tilde{\Phi}_\emptyset$ and (F,A) as the identity elements, respectively. Also $\tilde{\cup}$ and $\cap_R$ distribute and absorb over each other [see Proposition 2.1:5 (i, ii) and 6(i, ii), respectively]. Hence (ii) holds.

Remark 3.1

1. (SS(F,A), $\tilde{\cap}$, $\tilde{\cup}$) and (SS(F,A), $\tilde{\cup}$, $\tilde{\cap}$) are not lattices since $\tilde{\cap}$ and $\tilde{\cup}$ do not distribute and absorb over each other[see Proposition 2.1: 5(vii , viii) and 7(iii, iv)].

2. (SS(F,A), $\cup_R$, $\cap_R$) and (SS(F,A), $\cap_R$, $\cup_R$) are not lattices since $\cup_R$,and $\cap_R$ do not absorb over each other [see Proposition 2.1: 7(i ,ii)].

3.3 Commmutative, Idempotent Monoids and Semirings of Partitions of a Soft Set

Let $\tilde{P}$ = {(F_i, A_i) i ∈ I} be a partition of a soft set (F, A) and let $\bar{P}_k$ denote the block of a partition.

Let ℘ (F, A) denote the collection of all partitions of (F, A).

We define a binary operation ⊓ on ℘ (F, A) as follows:

Given $\tilde{P}_1$, $\tilde{P}_2$ ∈ ℘(F, A), let $\tilde{P}_1 \sqcap \tilde{P}_2$ be the soft set consisting of non-empty restricted intersections

of every block of $\tilde{P}_1$ with every block of $\tilde{P}_2$.

Clearly, the operation ⊓ on ℘(F, A) is both associative and commutative, since the operation $\cap_R$ on (F, A) is both associative and commutative and also every element $\tilde{P}_k$ ∈ ℘ (F, A) is idempotent with respect to ⊓ i.e. $\tilde{P}_k \sqcap \tilde{P}_k = \tilde{P}_k$.

Moreover, the identity element with respect to ⊓ is the partition with the single block.

Thus (℘(F, A), ⊓) is a commutative ,idempotent monoid.

Similarly, if we define a binary operation ⊔ on ℘(F, A) such that every resulting soft set consists of restricted union of every block of $\tilde{P}_1$ with every block of $\tilde{P}_2$, for all $\tilde{P}_1$, $\tilde{P}_2$ ∈ ℘(F, A), then (℘(F, A) ⊔) also gives rise to a commutative, idempotent monoid with the partition consisting of singleton blocks as the identity element.

Furthermore the following collections give rise to commutative, idempotent semirings:

1. ($\wp$(F, A), ⊔, ⊓) with the partition consisting of the single block as the identity element.

2. ($\wp$(F, A), ⊓,⊔) with the partition consisting of the singleton blocks as the identity element.

Example 3.1

Let $U=\{u_1,u_2,u_3,u_4\}$ and A = {a_1, a_2, a_3}. Let (F, A) be a soft set over U given by

(F, A) = { F(a_1) = $\{u_1,u_2\}$, F(a_2) = $\{u_3\}$, F(a_3) = $\{u_3,u_4\}$ }; Now let $A_1=\{a_1\}$, $A_2=\{a_2\}$, $A_3=\{a_3\}$;

$(F_1,A_1)=\{F_1(a_1)\}$, $(F_2,A_2)=\{F_2(a_2)\}$, $(F_3,A_3)=\{F_3(a_3)\}$, where F_i is the restriction of F to A_i , i=1,2,3.

It is easy to verify that $\left\{\overline{(F_1,A_1)}, \overline{(F_2,A_2)}, \overline{(F_3,A_3)}\right\}$, is a partition of (F,A). In other words, $\left\{\overline{F(a_1)}, \overline{F(a_2)}, \overline{F(a_3)}\right\}$ is a partition of (F,A).

Similarly, let B_1= {a_1,a_2} ,B_2={a_3};

$(F_1,B_1)=\{F_1(a_1), F_1(a_2)\}$, $(F_2,B_2) = \{F_2(a_3)\}$ where F_i is the restriction of F to B_i , i=1,2.

Then $\left\{\overline{F(a_1), F(a_2)}, \overline{F(a_3)}\right\}$ is another partition of (F,A).

Thus the collection $\wp$(F, A) of all partitions of (F, A) can be given as

$\wp$ (F, A) = $\{\tilde{P}_1,\tilde{P}_2,\tilde{P}_3,\tilde{P}_4,\tilde{P}_5\}$, where

$$\tilde{P}_1=\left\{\overline{F(a_1)}, \overline{F(a_2)}, \overline{F(a_3)}\right\},$$

$$\tilde{P}_2=\left\{\overline{F(a_1), F(a_2)}, \overline{F(a_3)}\right\},$$

$$\tilde{P}_3=\left\{\overline{F(a_1)}, \overline{F(a_2),F(a_3)}\right\},$$

$$\tilde{P}_4=\left\{\overline{F(a_1)\ F(a_3)}, \overline{F(a_2)}\right\}, \text{ and}$$

$$\tilde{P}_5=\left\{\overline{F(a_1), F(a_2), F(a_3)}\right\}.$$

It is not hard to see that $\tilde{P}_1 \sqcap \tilde{P}_2 = \left\{\overline{F(a_1)}, \overline{F(a_2)}, \overline{F(a_3)}\right\}$, $\tilde{P}_2 \sqcap \tilde{P}_2 = \left\{\overline{F(a_1), F(a_2)}, \overline{F(a_3)}\right\} = \tilde{P}_2$,and $\tilde{P}_1 \sqcap \tilde{P}_2 = \tilde{P}_2 \sqcap \tilde{P}_1$. As an illustration, let us consider Example 3.1:

Let $\tilde{P}_1=\left\{\overline{(F_1,A_1)},\overline{(F_2,A_2)},\overline{(F_3,A_3)}\right\} =\left\{\overline{(F,A_1)},\overline{(F,A_2)},\overline{(F,A_3)}\right\}$, and $\tilde{P}_2=\left\{\overline{(F_1,B_1)},\overline{(F_2,B_2)}\right\}=\left\{\overline{(F,B_1)},\overline{(F,B_2)}\right\}$.

Then, $\tilde{P}_1 \sqcap \tilde{P}_2$ =

$$\{(F,A_1)\cap_R(F,B_1),(F,A_1)\cap_R(F,B_2),(F,A_2)\cap_R(F,B_1),$$
$$(F,A_2)\cap_R(F,B_2),(F,A_3)\cap_R(F,B_1),(F,A_3)\cap_R(F,B_2)\}.$$

$$=\left\{\overline{F(a_1)}, \overline{F(a_2)}, \overline{F(a_3)}\right\}.$$

Similarly, $\tilde{P}_2 \sqcap \tilde{P}_1$ = $\left\{\overline{F(a_1)}, \overline{F(a_2)}, \overline{F(a_3)}\right\}$.

Also, $\tilde{P}_2 \sqcap \tilde{P}_2$ =

$$\{(F,B_1)\cap_R(F,B_1),(F,B_1)\cap_R(F,B_2),$$
$$(F,B_2)\cap_R(F,B_1),(F,B_2)\cap_R(F,B_2)\}.$$

$$=\left\{\overline{F(a_1), F(a_2)}, \overline{F(a_3)}\right\} = \tilde{P}_2.$$

Similar results hold for all other combinations.

Thus ($\wp$ (F, A), ⊓) is a commutative, idempotent monoid with $\left\{\overline{F(a_1), F(a_2), F(a_3)}\right\}$ as the identity element. Similarly, the other structures described in this paper can be illustrated.

3.4 Bounded, Distributive Lattices of Partitions of a Soft Set

Let $\wp$ (F, A) be as defined in 3.3 above.

We define a binary operation $\sqcap_R$ on $\wp$ (F, A) as follows:

Given $\tilde{P}_1$, $\tilde{P}_2$ ∈ $\wp$(F, A), let $\tilde{P}_1 \sqcap_R \tilde{P}_2$ be the soft set consisting of non-empty restricted intersections of every block of $\tilde{P}_1$ with every block of $\tilde{P}_2$.

Similarly, let the operation $\sqcup_E$ on $\wp$(F, A) be defined as: $\tilde{P}_1 \sqcup_E \tilde{P}_2$ be the soft set consisting of union of every block of $\tilde{P}_1$ with every block of $\tilde{P}_2$, for all $\tilde{P}_1$, $\tilde{P}_2$ ∈ $\wp$(F, A).

Proposition 3.2

(i) ($\wp$(F, A), $\sqcup_E$, $\sqcap_R$), (ii) ($\wp$(F, A), $\sqcap_R$, $\sqcup_E$), (iii) ($\wp$(F, A), ⊔, ⊓) and (iv) ($\wp$(F, A), ⊓, ⊔) are bounded , distributive lattices.

Proof

Clearly, the operations $\sqcap_R$ and $\sqcup_E$ on $\wp$(F, A) are both associative and commutative, since the operations $\cap_R$ and $\tilde{\cup}$ on (F, A) are both associative and commutative [see Proposition 2.1:3(i ,iv) and 4(ii,iv), respectively]. Also every element $\tilde{P}_k$ $\in \wp$ (F, A) is idempotent with respect to $\sqcap_R$ and $\sqcup_E$ i.e., $\tilde{P}_k \sqcap \tilde{P}_k = \tilde{P}_k$, since both $\cap_R$ and $\tilde{\cup}$ are idempotent [see Proposition 2.1:s 1(i ii)].Hence ($\wp$(F, A), $\sqcap_R$) and($\wp$(F, A), $\sqcup_E$) are commutative, idempotent semigroups. Also $\sqcap_R$ and $\sqcup_E$ absorb over each other [see Proposition 2.1: 6(i ii)]. Thus ($\wp$(F, A), $\sqcap_R$) and ($\wp$(F, A), $\sqcup_E$) give rise to lattices. Moreover, the identity elements with respect to $\sqcap_R$ and $\sqcup_E$ are the partition consisting of the single block and the partition consisting of the singleton blocks, respectively. Furthermore $\sqcap_R$ and $\sqcup_E$ distribute over each other [see Proposition 2.1: 5(i , ii)].

Hence (i) and (ii) hold.

Similarly, (iii) and (iv) can be proved.

Concluding Remarks

Since the algebraic structures developed in this paper are relatively new, it seems promising to introduce some other operations which would not only be alternatives to rather extensions of $\sqcap$ (see [11], for some related details).

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