# There are infinitely many prime triplets $P, 3P-2, 3P+2$

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**Abstract:** Using Jiang's function we prove that there are infinitely many primes $P$ such that $3P-2$ and $3P+2$ are primes. In studying Williams numbers Echi conjectures that there are infiniitely many prime triplets $P, 3P-2, 3P+2$ [1]. In this paper using Jiang's function we prove this conjecture and find the best asymptotic formula for the number of primes $P$.

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**Theorem 1. The prime equations are**

$$P_1 = 3P-2 \text{ and } P_2 = 3P+2 \tag{1}$$

There are infinitely many primes $P$ such that $P_1$ and $P_2$ are primes.

**Proof**. Jiang's function is

$$J_2(\omega) = \prod_P (P-1-x(P)) \tag{2}$$

where $\omega = \prod_P P$, $x(P)$ is the number of solutions of the following congruence

$$(3q-2)(3q+2) = 0 (\bmod P) \tag{3}$$

where $q = 1, 2, \cdots P-1$

Table 1 (From (3) we obtain)

| $q$ | $3q-2$ | $3q+2$ | |
|---|---|---|---|
| 1 | 1 | 5 | $x(2)=0, J_2(2)=1$ |
| 2 | 4 | 8 | $x(3)=0, J_2(3)=2$ |
| 3 | 7 | 11 | |
| 4 | $2\times 5$ | $2\times 7$ | $x(5)=2, J_2(5)=2$ |
| 5 | 13 | 17 | |
| 6 | 16 | 20 | $x(7)=2, J_2(7)=4$ |
| 7 | 19 | 23 | |
| 8 | $2\times 11$ | $2\times 13$ | |
| 9 | 25 | 29 | |
| 10 | 28 | 32 | $x(11)=2, J_2(11)=8$ |
| … | … | … | … |
| $P-1$ | $3P-5$ | $3P-1$ | $x(P)=2, J_2(P)=P-3$ |

In order to understand the Jiang's function we explain the table 1.

Let $\omega=2$, Euler function $\phi(2)=1$. There is the prime equation

$$2n+1 \quad (4)$$

where $n=1,2,\cdots$.

$J_2(2)=1$, there is the prime equation

$$P=2n+1 \quad (5)$$

Substituting (5) into (1) we obtain

$$P_1=6n+1 \text{ and } P_2=6n+5 \quad (6)$$

There are infinitely many integers *n* such that $P, P_1$ and $P_2$ are primes.

Let $\omega=6$, $\phi(6)=2$. There are the prime equations

$$6n+h \quad (7)$$

where $n=0,1,2,\cdots$ $h=1,5$.

$J_2(6)=2$, there is the prime equation

$$P=6n+h \quad (8)$$

Substituting (8) into (1) we obtain

$$P_1=18n+3h-2 \text{ and } P_2=18n+3h+2. \quad (9)$$

There are infinitely many integers $n$ such that $P, P_1$ and $P_2$ are primes.

Let $\omega=30$, $\phi(30)=8$. There are the prime equations

$$30n+h \quad (10)$$

where $h=1,3,11,13,17,19,23,29$, $n=0,1,...$

$J_2(30)=4$, there is the prime equation

$$P=30n+u, \quad (11)$$

where $u=7,13,17,23$.

Substituting (11) into (1)

$$P_1=90n+3u-2 \text{ and } P_2=90n+3u+2 \quad (12)$$

There are infinitely many integers $n$ such that $P, P_1$ and $P_2$ are primes.

Let $\omega=210$, $\phi(210)=48$. There are the prime equations

$$210n+u \quad (13)$$

where $u=$1, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101, 103, 107, 109, 113, 121, 127, 131, 137, 139, 143, 149, 151, 157, 163, 167, 169, 173, 179, 181, 187, 191, 193, 197, 199, 209, $n=0,1,2...$

$J_2(210)=16$, there is the prime equations

$$P=210n+u \quad (14)$$

where *u*=13, 23, 37, 43, 47, 83, 97, 103, 107, 113, 127, 163, 167, 173, 187, 197.

Substituting (14) into (1) we obtain

$$P_1=630n+3u-2 \text{ and } P_2=630n+3u+2 \quad (15)$$

There are infinitely many integers *n* such that $P, P_1$ and $P_2$ are primes.

Let $\omega = 2310$, $\phi(2310) = 480$. There are the prime equations

$$2310n + h, \quad (16)$$

where $n = 0,1,\cdots, h = 13,17,\ldots,\ldots,2309$.

$J_2(2310) = 128$, there is the prime equation

$$P = 2310n + u, \quad (17)$$

where $u$=13, 17,… 2287, 2297.

Substituting (17) into (1)

$$P_1 = 6930n + 3u - 2 \text{ and } P_2 = 6930n + 3u + 2. \quad (18)$$

There are infinitely many integers $n$ such that $P, P_1$ and $P_3$ are primes.

$J_2(\omega) \to \infty$ as $\omega \to \infty$, there infinitely many prime equations such that $P, P_1$ and $P_2$ are primes.

We prove the theorem 1

From (2) we obtain

$$J_2(\omega) = 2\prod_{5 \le P}(P - 3) \quad (19)$$

We obtain the best asymptotic formula for the number of primes $P$ [2]

$$\pi_3(N,2) = \left|\{P \le N, 3P - 2 = prime, 3P + 2 = prime\}\right| \sim \frac{J_2(\omega)\omega^2}{\phi^3(\omega)}\frac{N}{\log^3 N}$$

$$= 9\prod_{5 \le P}\left(1 - \frac{3P - 1}{(P - 1)^3}\right)\frac{N}{\log^3 N} \sim 5.77\frac{N}{\log^3 N} \quad (20)$$

where $\phi(\omega) = \prod_P (P - 1)$.

Table 2（From (20) we obtain）

| | $10^2$ | $10^3$ | $10^4$ | $10^5$ | $10^6$ | $10^7$ | $10^8$ |
|---|---|---|---|---|---|---|---|
| $\pi_3(N,2)$ (20) | 6 | 18 | 74 | 378 | 2188 | 13780 | 92312 |
| $\pi_3(N,2)$ (exact) | 7 | 21 | 89 | 445 | 2420 | 14828 | 98220 |

From table 2 we consider that prime distribution is order rather than random [2].

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