

Direct Boundary Element Method for Calculation of Oseen Flow Past a Circular Cylinder in Case of Linear Variation

Ghulam Muhammad¹, Nawazish Ali Shah², and Muhammad Mushtaq²

¹Department of Mathematics, GCS, Lahore, Pakistan

²Department of Mathematics, University of Engineering & Technology, Lahore – 54890, Pakistan

* Corresponding Author: g_muhammad123@hotmail.com

Abstract: In present paper, direct boundary element method (DBEM) has been applied to calculate Oseen flow past a circular cylinder in case of linear variation. The boundary of the circular cylinder is discretized into constant boundary elements over which the velocity distribution is calculated. The calculated results are also compared with exact results.

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Keywords: Direct boundary element method, Oseen flow, Circular cylinder, Linear variation.

Introduction

Boundary element method is a numerical technique used to solve the different types of problems today facing in science and technology. The well-known computational methods such as finite difference method (FDM) and finite element method (FEM) are very costly and time-consuming because in these methods the whole domain under study is discretised into a number of block-type elements, whereas in boundary element method the process of discretisation takes place on the surface of body. Which considerably reduces the size of system of equations resulting the reduction in data and that is requisite to run a computer program efficiently. In other words, boundary element methods are superior in several aspects to other computational methods because of their surface modeling approach. That is why, the complicated structures can be more easily modeled by these methods and are therefore preferred by engineers. The results of boundary element methods are more accurate and reliable than those of classical methods. which establishes the fact that these methods (BEMs) are time-saving, accurate, efficient and economical techniques as compared to other numerical techniques (Mushtaq, M; 2008, 2009, 2011). These salient features of BEMs make them popular in communities of engineering and science. Such methods are essentially the methods for solving the partial differential equations arising in wide range of fields, e.g., fluid mechanics, solid and fracture mechanics, heat transfer and electromagnetic theory, potential theory, elasticity, elastostatics and elastodynamics, etc. as detailed in Brebbia and Walker, (1980). Furthermore, the area of their applications is increasing day to day. Boundary element methods have been classified into direct and

indirect methods. The direct method is in the form of a statement which gives the values of unknown variables at the field point under discussion in terms of a complete set of the entire boundary data. whereas the indirect method is based on the distribution of sources or doublets over the boundary of the body and calculates such distribution in terms of the solution of an integral equation. The first work on flow field calculations around three-dimensional bodies was probably done by Hess and Smith, (1962 & 1967). The direct boundary element method (DBEM) for potential flow calculations around objects was first applied in past by Morino et al (1975). In recent past, the direct boundary element methods have been applied by the author himself for flow field calculations around two- and three-dimensional objects (Muhammad, G; 2008, 2010).

Calculation of Oseen Flow Past a Circular Cylinder

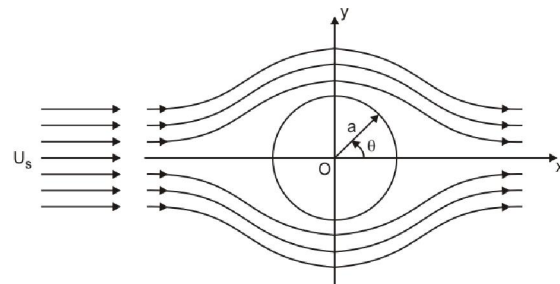


Figure 1

Boundary element methods are applied for both problems of exterior and interior flows in two dimensional space.. In this section, direct boundary element method is used to calculate the Oseen's flow

around a circular cylinder. A circular cylinder of radius ‘a’ is held fixed in a uniform stream of incompressible viscous fluid flowing steadily around it and let the centre of a cylinder be taken as origin and U_s be the velocity of uniform stream in the positive.

x – direction as given in figure 1 (Shah, 2008).

The hydrodynamical equations are (Lamb, 1932).

$$\left. \begin{aligned} U_s \frac{\partial u}{\partial x} &= -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \nabla^2 u \\ U_s \frac{\partial v}{\partial x} &= -\frac{1}{\rho} \frac{\partial p}{\partial y} + \nu \nabla^2 v \end{aligned} \right\} \quad (1)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (2)$$

$$\text{Also } \nabla^2 p = 0 \quad (3)$$

$$\text{and } p = \rho U_s \frac{\partial \phi}{\partial x} \quad (4')$$

The relations for the velocity components are (Milne Thomson, 1968) and (Lamb, 1932).

$$\left. \begin{aligned} u &= \frac{\partial \phi}{\partial x} + \frac{1}{2k} \frac{\partial \chi}{\partial x} - \chi \\ v &= \frac{\partial \phi}{\partial y} + \frac{1}{2k} \frac{\partial \chi}{\partial y} \end{aligned} \right\} \quad (5)$$

where k is an inertia coefficient.

Also, we know that

$$\nabla^2 \phi = 0 \quad (6)$$

$$\text{Now } \frac{\partial u}{\partial x} = \frac{\partial^2 \phi}{\partial x^2} + \frac{1}{2k} \frac{\partial^2 \chi}{\partial x^2} - \frac{\partial \chi}{\partial x}$$

$$\frac{\partial u}{\partial y} = \frac{\partial^2 \phi}{\partial y^2} + \frac{1}{2k} \frac{\partial^2 \chi}{\partial y^2}$$

Using these relations in equation (2), we get

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{1}{2k} \frac{\partial^2 \chi}{\partial x^2} - \frac{\partial \chi}{\partial x} - \frac{\partial^2 \phi}{\partial y^2} + \frac{1}{2k} \frac{\partial^2 \chi}{\partial y^2} = 0$$

$$\left(\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} \right) + \frac{1}{2k} \left(\frac{\partial^2 \chi}{\partial x^2} + \frac{\partial^2 \chi}{\partial y^2} \right) - \frac{\partial \chi}{\partial x} = 0$$

$$\nabla^2 \phi + \frac{1}{2k} \left(\nabla^2 \chi - \frac{\partial \chi}{\partial x} \right) = 0$$

Since $\nabla^2 \phi = 0$

$$\frac{1}{2k} \left(\nabla^2 \chi - 2k \frac{\partial \chi}{\partial x} \right) = 0$$

$$\text{or } \nabla^2 \chi - 2k \frac{\partial \chi}{\partial x} = 0$$

$$\left(\nabla^2 - 2k \frac{\partial}{\partial x} \right) \chi = 0 \quad (7)$$

Let the appropriate solution of equation (7) for small values of kr be

$$\chi = -C(1+kx) \left(\gamma + \ln \frac{1}{2}kr \right)$$

where γ is Euler constant.

Let us assume

$$\phi = -U_s x + A_0 \ln r + A_1 \frac{\partial}{\partial x} \ln r + \dots$$

so that the boundary conditions $u = 0, v = 0$ are satisfied on the boundary of a circular cylinder.

$$\frac{\partial \chi}{\partial x} = -Ck \left(\gamma + \ln \frac{1}{2}kr \right) - C(1+kx) \left(\frac{x}{r^2} \right)$$

$$\frac{1}{2k} \frac{\partial \chi}{\partial x} = -\frac{C}{2k} \left\{ k \left(\gamma + \ln \frac{1}{2}kr \right) + \frac{x}{r^2} + \frac{kx^2}{r^2} \right\}$$

$$\frac{1}{2k} \frac{\partial \chi}{\partial x} - \chi = -\frac{C}{2k} \left\{ k \left(\gamma + \ln \frac{1}{2}kr \right) + \frac{x}{r^2} + \frac{kx^2}{r^2} \right\}$$

$$-C(1+kx) \left(\gamma + \ln \frac{1}{2}kr \right) \quad (8)$$

Also

$$\frac{\partial \phi}{\partial x} = -U_s + A_0 \frac{\partial}{\partial x} (\ln r) + A_1 \frac{\partial^2}{\partial x^2} (\ln r) + \dots \quad (9)$$

Similarly,

$$\frac{\partial \chi}{\partial y} = -C(1+kx) \left(\frac{y}{r^2} \right)$$

$$\frac{1}{2k} \frac{\partial \chi}{\partial y} = -\frac{C}{2k} \left\{ \frac{y}{r^2} + k \frac{xy}{r^2} \right\}$$

$$= -\frac{C}{2k} \left\{ \frac{\partial}{\partial y} (\ln r) - \frac{1}{2}kr^2 \frac{\partial^2}{\partial x \partial y} (\ln r) + \dots \right\} \quad (10)$$

and

$$\frac{\partial \phi}{\partial y} = A_0 \frac{\partial}{\partial y} (\ln r) + A_1 \frac{\partial^2}{\partial x \partial y} (\ln r) + \dots \quad (11)$$

Using the equations (8), (9), (10) and (11) in equation (5), we have

$$\begin{aligned} u &= -U_s - C \left(\frac{1}{2} - \gamma - \ln \frac{1}{2}kr \right) + \left(A_0 - \frac{C}{2k} \right) \frac{\partial}{\partial x} (\ln r) \\ &+ \left(A_1 + \frac{C}{4}r^2 \right) \frac{\partial^2}{\partial x^2} (\ln r) + \dots \quad (12) \end{aligned}$$

$$\begin{aligned} v &= A_0 \frac{\partial}{\partial y} \ln r + A_1 \frac{\partial^2}{\partial x \partial y} \ln r + \dots \\ &- \frac{C}{2k} \frac{\partial}{\partial y} \ln r + \frac{C}{4}r^2 \frac{\partial^2}{\partial x \partial y} \ln r + \dots \end{aligned}$$

or

$$v = \left(A_0 - \frac{C}{2k} \right) \frac{\partial}{\partial y} \ln r + \left(A_1 + \frac{C}{4}r^2 \right) \frac{\partial^2}{\partial x \partial y} \ln r + \dots \quad (13)$$

Boundary conditions are

$$u = 0, v = 0, \text{ for } r = a$$

Using the above boundary conditions in the equations (12) and (13), we obtain

$$0 = -U_s - C \left(\frac{1}{2} - \gamma - \ln \frac{1}{2} k a \right) + \left(A_0 - \frac{C}{2k} \right) \frac{\partial}{\partial x} (\ln a) + \left(A_1 + \frac{C}{4} a^2 \right) \frac{\partial^2}{\partial x^2} (\ln r) + \dots$$

and

$$0 =$$

which give on comparison

$$-U_s - C \left(\frac{1}{2} - \gamma - \ln \frac{1}{2} k a \right) = 0,$$

$$A_0 - \frac{C}{2k} = 0$$

$$\text{and } A_1 + \frac{C}{4} a^2 = 0$$

$$\text{or } C = -\frac{2U_s}{\frac{1}{2} - \gamma - \ln \frac{1}{2} k a}$$

$$A_0 = \frac{C}{2k}$$

$$\text{and } A_1 = -\frac{C}{4} a^2$$

Using these constants in equations (12) and (13), we obtain

$$u = -U_s - C \left(\frac{1}{2} - \gamma - \ln \frac{1}{2} k r \right) + \left(-\frac{C}{2k} + \frac{C}{2k} \right) \frac{\partial}{\partial x} (\ln r) + \left(\frac{C}{4} a^2 - \frac{C}{4} r^2 \right) \frac{\partial^2}{\partial x^2} (\ln r) = -U_s \tag{14}$$

$$v = \frac{C}{4} (r^2 - a^2) \frac{\partial^2}{\partial x \partial y} (\ln r) = \frac{U_s}{2(\gamma + \ln \frac{1}{2} k a - \frac{1}{2})} (r^2 - a^2) \frac{\partial^2}{\partial x \partial y} (\ln r) \tag{15}$$

The magnitude of velocity is given by the relation

$$V = \sqrt{u^2 + v^2} \tag{16}$$

Now to approximate the surface of a circular cylinder, the coordinates of extreme points on the boundary elements are generated in a computer program as under.

The boundary of the cylinder can be divided into elements by the formula.

$$\theta_k = \frac{[(m+2) - 2k] \pi}{m}, \quad k = 1, 2, \dots, m \tag{17}$$

Then the coordinates of the extreme points of these m elements are calculated from

$$\left. \begin{aligned} x_k &= a \cos \theta_k \\ y_k &= a \sin \theta_k \end{aligned} \right\} k = 1, 2, \dots, m \tag{18}$$

Take m = 8 and a = 1.

Linear Variation

Next the case is considered in which the boundary of the circular cylinder is divided into linear elements. In this case the nodes where the boundary conditions are specified are at the intersection of the elements (Muhammad, G 2008).

The figure 2 shows the discretization of the circular cylinder into 8 linear boundary elements.

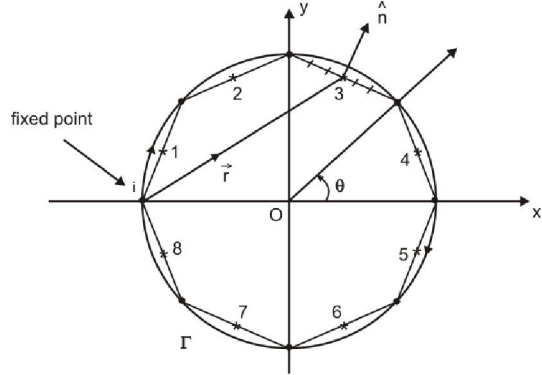


Figure 2

The discretization of the surface of a circular cylinder is given in figure 2.

The mid-node coordinates over every element are defined by the formula.

$$\left. \begin{aligned} x_m &= \frac{x_k + x_{k+1}}{2} \\ y_m &= \frac{y_k + y_{k+1}}{2} \end{aligned} \right\} k, m = 1, 2, \dots, 8 \tag{19}$$

The coordinates of the extreme points of the elements and the coordinates of the mid-point of each element where the velocity will be calculated can be found from the equations (18) and (19) respectively. The reason for distributing the elements in this way is such that the velocities are calculated at the same values of θ in both figure 2 so that computed results could be easily compared.

Equation for the direct boundary element method is given by

$$c_i \phi_i = \phi_\infty - \frac{1}{2\pi} \int_\Gamma \ln \left(\frac{1}{r} \right) \frac{\partial \phi}{\partial n} d\Gamma + \frac{1}{2\pi} \int_{\Gamma-i} \phi \frac{\partial}{\partial n} \left(\ln \frac{1}{r} \right) d\Gamma \tag{20}$$

Which can be in the discretised form as

$$-c_i \phi_i + j \int_{\Gamma_j-i} \phi \frac{\partial}{\partial n} \left(\frac{1}{2\pi} \ln \frac{1}{r} \right) d\Gamma + \phi_\infty = j \int_{\Gamma_j} \left(\frac{1}{2\pi} \ln \frac{1}{r} \right) d\Gamma \tag{21}$$

Since ϕ and $\frac{\partial \phi}{\partial n}$ vary linearly over the element their values at any point on the element can be defined in terms of their nodal values and the interpolation functions N_1 and N_2 as

$$\phi = [N_1 N_2] \begin{Bmatrix} \phi_1 \\ \phi_2 \end{Bmatrix}$$

$$\frac{\partial \phi}{\partial n} = [N_1 N_2] \begin{Bmatrix} \frac{\partial \phi_1}{\partial n} \\ \frac{\partial \phi_2}{\partial n} \end{Bmatrix} \quad (22)$$

The integrals along an element ‘j’ on the L.H.S. of equation (21) can now written as

$$\int_{\Gamma_{j-i}} \phi \frac{\partial}{\partial n} \left(\frac{1}{2\pi} \ln \frac{1}{r} \right) d\Gamma$$

$$= \int_{\Gamma_{j-i}} [N_1 N_2] \frac{\partial}{\partial n} \left(\frac{1}{2\pi} \ln \frac{1}{r} \right) d\Gamma \begin{Bmatrix} \phi_1 \\ \phi_2 \end{Bmatrix}$$

$$= [h_{ij}^1 \ h_{ij}^2] \begin{Bmatrix} \phi_1 \\ \phi_2 \end{Bmatrix}$$

Where

$$h_{ij}^k = \int_{\Gamma_{j-i}} N_k \frac{\partial}{\partial n} \left(\frac{1}{2\pi} \ln \frac{1}{r} \right) d\Gamma,$$

$$k = 1, 2, \dots \dots \dots \quad (23)$$

The integrals on the R.H.S. of equation (21) can be written as

$$\int_{\Gamma_j} \frac{\partial}{\partial n} \left(\frac{1}{2\pi} \ln \frac{1}{r} \right) d\Gamma$$

$$= \int_{\Gamma_j} [N_1 N_2] \left(\frac{1}{2\pi} \ln \frac{1}{r} \right) d\Gamma \begin{Bmatrix} \frac{\partial \phi_1}{\partial n} \\ \frac{\partial \phi_2}{\partial n} \end{Bmatrix}$$

$$= [g_{ij}^1 \ g_{ij}^2] \begin{Bmatrix} \frac{\partial \phi_1}{\partial n} \\ \frac{\partial \phi_2}{\partial n} \end{Bmatrix}$$

where $g_{ij}^k = \int_{\Gamma_j} N_k \left(\frac{1}{2\pi} \ln \frac{1}{r} \right) d\Gamma,$

$$k = 1, 2, \dots \dots \dots \quad (24)$$

Again, the integrals in equations (23) and (24) are evaluated numerically as before except for the element on which the fixed point ‘i’ is lying. For this element, the integrals are evaluated analytically. The integrals h_{ii}^1 and h_{ii}^2 are zero because \hat{r} and \hat{n} are orthogonal to each other over the element. The integrals g_{ii}^1 and g_{ii}^2 can be calculated as follows:

$$g_{ii}^1 = \int_{\Gamma_i} N_1 \left(\frac{1}{2\pi} \ln \frac{1}{r} \right) d\Gamma \quad (25)$$

Since N_1 and r are functions of ξ , and the integral is with respect to ‘ Γ ’, so the calculation of this integral requires the use of a Jacobian $|J|$ of the transformation. As in Brebbia and Walker, 1980. such Jacobian is given by

$$|J| = \sqrt{\left(\frac{dx}{d\xi} \right)^2 + \left(\frac{dy}{d\xi} \right)^2} \quad (26)$$

Thus $d\Gamma = |J| d\xi$

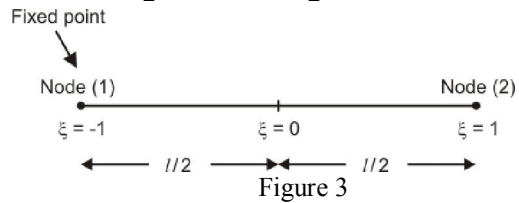
Now to find $|J|$,

$$x = N_1 x_1 + N_2 x_2$$

$$= \frac{1}{2} (1 - \xi) x_1 + \frac{1}{2} (1 + \xi) x_2$$

and $y = N_1 y_1 + N_2 y_2$

$$= \frac{1}{2} (1 - \xi) y_1 + \frac{1}{2} (1 + \xi) y_2$$



where x_i, y_i are the coordinates of the nodes referred to the global system.

Therefore $\frac{dx}{d\xi} = \frac{1}{2} (x_2 - x_1),$

$$\frac{dy}{d\xi} = \frac{1}{2} (y_2 - y_1)$$

and thus from the equation (6.3.29),

$$|J| = \sqrt{\left(\frac{x_2 - x_1}{2} \right)^2 + \left(\frac{y_2 - y_1}{2} \right)^2} = \frac{l}{2} \quad (27)$$

where l is the length of the element shown in figure 3. For the linear element shown this is obvious, but the same procedure yields the Jacobian for more complicated elements.

Therefore, the integral in equation (25) becomes

$$g_{ii}^1 = \frac{1}{2\pi} \int_{-1}^1 \frac{1}{2} (1 - \xi) \ln \left(\frac{1}{r} \right) \frac{l}{2} d\xi \quad (28)$$

If the node (1) is taken as the fixed point ‘i’, then r is measured from node (1).

$$\text{Thus } r = \frac{l}{2} (1 + \xi) \quad (29)$$

From the equations (28) and (29), we get

$$g_{ii}^1 = \frac{l}{8\pi} \int_{-1}^1 (1 - \xi) \ln \frac{1}{\frac{l}{2} (1 + \xi)} d\xi$$

$$= \frac{l}{8\pi} \left[\int_{-1}^1 (1 - \xi) \ln \frac{1}{1 + \xi} d\xi \right]$$

Integrating by parts and simplifying, then

$$g_{ii}^1 = \frac{l}{8\pi} [3 - 2 \ln l] \quad (30)$$

Also,

$$\begin{aligned} g_{ii}^2 &= \int_{\Gamma_i} N_2 \left(\frac{1}{2\pi} \ln \frac{1}{r} \right) d\Gamma \\ &= \frac{1}{2\pi} \int_{-1}^1 \frac{1}{2} (1 + \xi) \ln \frac{1}{r} |J| d\xi \quad (31) \end{aligned}$$

From the equations (27), (29) and (31), we have

$$\begin{aligned} g_{ii}^2 &= \frac{l}{8\pi} \int_{-1}^1 (1 + \xi) \ln \frac{1}{\frac{l}{2}(1 + \xi)} d\xi \\ &= \\ \frac{l}{8\pi} \lim_{\epsilon \rightarrow 0+} \int_{-1+\epsilon}^1 (1 + \xi) \ln \frac{1}{\frac{l}{2}(1 + \xi)} d\xi \end{aligned}$$

Again integrating by parts and simplifying, we obtain

$$g_{ii}^2 = \frac{l}{8\pi} [1 - 2 \ln l] \quad (32)$$

Since $\frac{\partial \phi}{\partial n}$ is specified at each node of the element, the values of the perturbation velocity potential ϕ can be found at each node on the boundary. The total potential Φ is then found, which

will then be used to calculate the velocity on the circular cylinder.

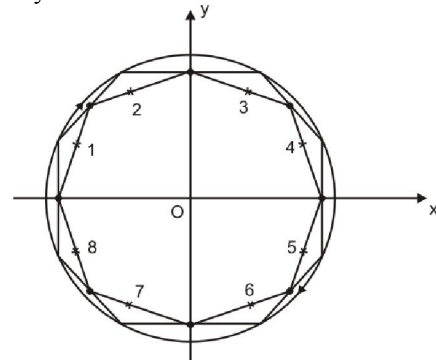


Figure 4

The velocity midway between two nodes on the boundary can then be approximated by using the formula

$$\text{Velocity } \otimes = \frac{\Phi_{k+1} - \Phi_k}{\text{Length from node } k \text{ to } k+1} \quad (33)$$

The method has been implemented using FORTRAN programming with 16, 32 and 64 constant boundary elements.

TABLE (1)

ELEMENT	XM	YM	VELOCITY	EXACT VELOCITY
1	-.96	.19	.19509E+00	.43862E+00
2	-.82	.54	.55557E+00	.66601E+00
3	-.54	.82	.83147E+00	.89375E+00
4	-.19	.96	.98079E+00	.10247E+01
5	.19	.96	.98079E+00	.10247E+01
6	.54	.82	.83147E+00	.89375E+00
7	.82	.54	.55557E+00	.66601E+00
8	.96	.19	.19509E+00	.43862E+00
9	.96	-.19	.19509E+00	.43862E+00
10	.82	-.54	.55557E+00	.66601E+00
11	.54	-.82	.83147E+00	.89375E+00
12	.19	-.96	.98079E+00	.10247E+01
13	-.19	-.96	.98079E+00	.10247E+01
14	-.54	-.82	.83147E+00	.89375E+00
15	-.82	-.54	.55557E+00	.66601E+00
16	-.96	-.19	.19509E+00	.43862E+00

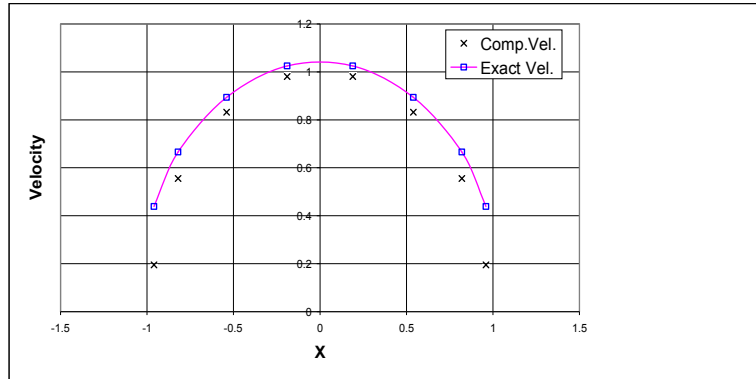


Figure 5: Comparison of exact and computed values over the boundary of a circular cylinder for 16 linear boundary elements.

TABLE (2)

ELEMENT	XM	YM	VELOCITY	EXACT VELOCITY
1	-.99	.10	.98018E-01	.38677E+00
2	-.95	.29	.29029E+00	.46360E+00
3	-.88	.47	.47140E+00	.57934E+00
4	-.77	.63	.63439E+00	.70239E+00
5	-.63	.77	.77301E+00	.81490E+00
6	-.47	.88	.88192E+00	.90652E+00
7	-.29	.95	.95694E+00	.97082E+00
8	-.10	.99	.99518E+00	.10039E+01
9	.10	.99	.99518E+00	.10039E+01
10	.29	.95	.95694E+00	.97082E+00
11	.47	.88	.88192E+00	.90652E+00
12	.63	.77	.77301E+00	.81490E+00
13	.77	.63	.63439E+00	.70239E+00
14	.88	.47	.47140E+00	.57934E+00
15	.95	.29	.29028E+00	.46359E+00
16	.99	.10	.98017E-01	.38677E+00
17	.99	-.10	.98017E-01	.38677E+00
18	.95	-.29	.29028E+00	.46359E+00
19	.88	-.47	.47140E+00	.57934E+00
20	.77	-.63	.63439E+00	.70239E+00
21	.63	-.77	.77301E+00	.81490E+00
22	.47	-.88	.88192E+00	.90652E+00
23	.29	-.95	.95694E+00	.97082E+00
24	.10	-.99	.99518E+00	.10039E+01
25	-.10	-.99	.99518E+00	.10039E+01
26	-.29	-.95	.95694E+00	.97082E+00
27	-.47	-.88	.88192E+00	.90652E+00
28	-.63	-.77	.77301E+00	.81490E+00
29	-.77	-.63	.63439E+00	.70239E+00
30	-.88	-.47	.47140E+00	.57934E+00
31	-.95	-.29	.29029E+00	.46360E+00
32	-.99	-.10	.98017E-01	.38677E+00

TABLE (3)

ELEMENT	XM	YM	VELOCITY	EXACT VELOCITY
1	-1.00	.05	.49068E-01	.37355E+00
2	-.99	.15	.14673E+00	.39500E+00
3	-.97	.24	.24298E+00	.43401E+00
4	-.94	.34	.33689E+00	.48510E+00
5	-.90	.43	.42756E+00	.54321E+00
6	-.86	.51	.51410E+00	.60444E+00
7	-.80	.59	.59570E+00	.66590E+00
8	-.74	.67	.67156E+00	.72547E+00
9	-.67	.74	.74095E+00	.78155E+00
10	-.59	.80	.80321E+00	.83289E+00
11	-.51	.86	.85773E+00	.87852E+00
12	-.43	.90	.90399E+00	.91764E+00
13	-.34	.94	.94154E+00	.94964E+00
14	-.24	.97	.97003E+00	.97405E+00
15	-.15	.99	.98918E+00	.99051E+00
16	-.05	1.00	.99880E+00	.99880E+00
17	.05	1.00	.99880E+00	.99880E+00
18	.15	.99	.98918E+00	.99051E+00
19	.24	.97	.97003E+00	.97405E+00
20	.34	.94	.94154E+00	.94964E+00
21	.43	.90	.90399E+00	.91764E+00
22	.51	.86	.85773E+00	.87852E+00
23	.59	.80	.80321E+00	.83289E+00
24	.67	.74	.74095E+00	.78155E+00
25	.74	.67	.67156E+00	.72547E+00
26	.80	.59	.59570E+00	.66590E+00
27	.86	.51	.51410E+00	.60444E+00
28	.90	.43	.42756E+00	.54321E+00
29	.94	.34	.33689E+00	.48509E+00
30	.97	.24	.24298E+00	.43401E+00
31	.99	.15	.14673E+00	.39500E+00
32	1.00	.05	.49067E-01	.37355E+00
33	1.00	-.05	.49067E-01	.37355E+00
34	.99	-.15	.14673E+00	.39500E+00
35	.97	-.24	.24298E+00	.43401E+00
36	.94	-.34	.33689E+00	.48509E+00
37	.90	-.43	.42756E+00	.54321E+00
38	.86	-.51	.51410E+00	.60444E+00
39	.80	-.59	.59570E+00	.66590E+00
40	.74	-.67	.67156E+00	.72547E+00
41	.67	-.74	.74095E+00	.78155E+00
42	.59	-.80	.80321E+00	.83289E+00
43	.51	-.86	.85773E+00	.87852E+00
44	.43	-.90	.90399E+00	.91764E+00
45	.34	-.94	.94154E+00	.94964E+00
46	.24	-.97	.97003E+00	.97405E+00
47	.15	-.99	.98918E+00	.99051E+00
48	.05	-1.00	.99880E+00	.99880E+00
49	-.05	-1.00	.99880E+00	.99880E+00
50	-.15	-.99	.98918E+00	.99051E+00
51	-.24	-.97	.97003E+00	.97405E+00
52	-.34	-.94	.94154E+00	.94964E+00
53	-.43	-.90	.90399E+00	.91764E+00
54	-.51	-.86	.85773E+00	.87852E+00
55	-.59	-.80	.80321E+00	.83289E+00
56	-.67	-.74	.74095E+00	.78155E+00
57	-.74	-.67	.67156E+00	.72547E+00
58	-.80	-.59	.59570E+00	.66590E+00
59	-.86	-.51	.51410E+00	.60444E+00
60	-.90	-.43	.42756E+00	.54321E+00
61	-.94	-.34	.33689E+00	.48510E+00
62	-.97	-.24	.24298E+00	.43401E+00
63	-.99	-.15	.14673E+00	.39500E+00
64	-1.00	-.05	.49068E-01	.37355E+00

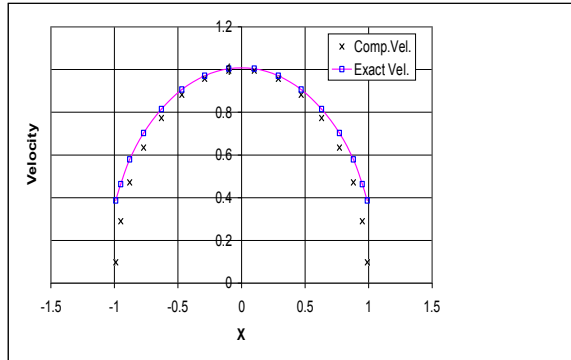


Figure 6: Comparison of exact and computed values over the boundary of a circular cylinder for 32 linear boundary elements.

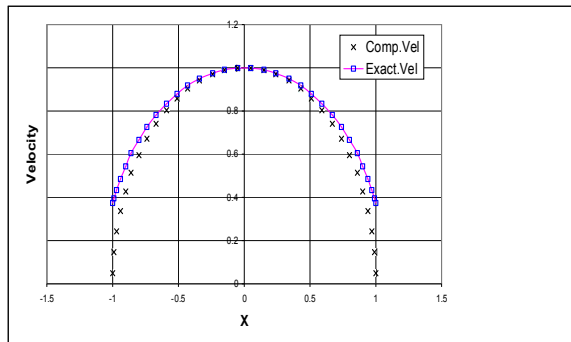


Figure 7: Comparison of exact and computed values over the boundary of a circular cylinder for 64 linear boundary elements.

Conclusion

Direct boundary element method has been applied to calculate Oseen's flow past a circular cylinder in case of linear variation. The improvement in results gained by taking 32 and 64 constant elements can be seen from the tables (2), (3) and figures 6, 7 and such improvement increases with increase in number of boundary elements. Moreover, at the top of figure 7, the computed results are convergent with exact results and as we come down, such results are slightly divergent from the exact ones due to increase of viscous effects.

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