There are finite Fermat primes

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Abstract: Using Jiang function we prove the finite Fermat primes.

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Theorem. Suppose the prime equation

$$P_1 = (P-1)^{2^n} + 1$$

（1）

There exist infinitely many primes P such that P_1 is a prime.

Proof. We have Jiang function[1]

$$J_2(\omega) = \prod_P [P - 1 - \chi(P)],$$

（2）

where $\omega = \prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$(q-1)^{2^n} + 1 \equiv 0 \pmod{P}, \qquad q = 1, \cdots, P-1.$$

（3）

From (3) we have $\chi(P) = 2^n$ if $P \equiv 1 \pmod{2^{n+1}}$, $\chi(P) = 0$ otherwise.

Since $J_2(\omega) \neq 0$, there exist infinitely many primes P such that P_1 is a prime.

We have the asymptotic formula [1]

$$\pi_2(N, 2) = \left| \left\{ P \le N : (P-1)^{2^n} + 1 = prime \right\} \right| \sim \frac{J_2(\omega)}{2^n \phi^2(\omega)} \frac{N}{\log^2 N}.$$

（4）

When $P = 3$. From (1) we have the equation of Fermat number [2]

$$P_1 = 2^{2^n} + 1$$

（5）

From (4) we have

$$\pi_2(3, 2) = \left| \left\{ 3 \le N : 2^{2^n} + 1 = prime \right\} \right| \sim \frac{J_2(\omega)}{2^n \phi^2(\omega)} \frac{3}{\log^2 3} \to 0 \quad \text{as } n \to \infty$$

（4）

From (4) we prove the finite Fermat primes.

In the same way we are able to prove that $4^{2^n} + 1$ and $6^{2^n} + 1$ have finite prime solutions [2]

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