

**The New Prime theorem (11)**

$$2 \times a^2 \pm 1$$

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**Abstract:** Using Jiang function we prove that  $2 \times a^2 \pm 1$  has infinitely many prime solutions.

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**Theorem. We define the prime equation**

$$P_1 = 2 \times (P-1)^2 - 1 \quad (1)$$

There exist infinitely many primes  $P$  such that  $P_2$  is a prime.

**Proof.** We have Jiang function[1]

$$J_2(\omega) = \prod_P [P-1 - \chi(P)] \quad (2)$$

where  $\omega = \prod_P P$ ,  $\chi(P)$  is the number of solutions of congruence

$$2 \times (q-1)^2 - 1 \equiv 0 \pmod{P}, \quad q = 1, \dots, P-1 \quad (3)$$

From (3) we have  $\left(\frac{2}{P}\right) = (-1)^{\frac{P^2-1}{8}}$ , if  $\left(\frac{2}{P}\right) = 1$  then  $\chi(P) = 2$ , if  $\left(\frac{2}{P}\right) = -1$  then  $\chi(P) = 0$ .

Substituting it into (2) we have

$$J_2(\omega) = \prod_{3 \leq P} \left[ P - 2 - (-1)^{\frac{P^2-1}{8}} \right] \neq 0 \quad (4)$$

We prove there exist infinitely many primes  $P$  such that  $P_2$  is a prime.

We have asymptotic formula [1]:

$$\pi_2(N, 2) = \left| \left\{ P \leq N : 2 \times (P-1)^2 - 1 = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega}{2\phi^2(\omega)} \frac{N}{\log^2 N} \quad (5)$$

where  $\phi(\omega) = \prod_P (P-1)$ .

In the same way we are able to prove that  $2 \times a^2 + 1$  has infinitely many prime solutions.

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