

Gaps Among Products of m Primes

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摘要 (Abstract): Theorem 1.

$$d(x) = d(x+1) = d(x+2) = 2 \text{ infinitely-often.} \quad (1)$$

where $d(x)$ represents the number of distinct prime factors of x , $d(x) = \sum_{P|x} 1$, $d(3) = 1$, $d(15) = 2$, $d(105) = 3$.

Proof (see[1] p.146 theorem 3.1.154). Prime equations are

$$p_2 = 10p_1 + 1, \quad p_3 = 15p_1 + 2, \quad p_4 = 6p_1 + 1 \quad (2)$$

We have Jiang function

$$J_2(\omega) = 3 \prod_{7 < P} (P-4) \neq 0, \quad (3)$$

where $\omega = \prod_{2 \leq P} P$

We prove that $J_2(\omega) \neq 0$ there exist infinitely many primes P_1 such that P_2, P_3, P_4 are primes.
We have asymptotic formula

$$\pi_4(N, 2) = |\{P_1 \leq N : 10P_1 + 1, 15P_1 + 2, 6P_1 + 1\}| \sim \frac{J_2(\omega)\omega}{\phi^4(\omega)} \frac{N}{\log^4 N}, \quad (4)$$

where $\phi(\omega) = \prod_{2 \leq P} (P-1)$

From (2) we have $3p_2 + 1 = 30p_1 + 4 = 2p_3$, $3p_2 + 2 = 30p_1 + 5 = 5p_4$. We prove that there exist infinitely many triples of consecutive integers, each being the products of two distinct primes.

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关键词 (Keywords): Theorem; number; factor; Prime; equation; function

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From (2) we have $3p_2 + 1 = 30p_1 + 4 = 2p_3$, $3p_2 + 2 = 30p_1 + 5 = 5p_4$. We prove that there exist infinitely many triples of consecutive integers, each being the products of two distinct primes.

Theorem 2.

$$d(x) = d(x+1) = d(x+2) = m > 1 \text{ infinitely-often} \quad (5)$$

Proof (see [1] p.148, theorem 3.1.158). Suppose that $u, u+1$ and $u+2$ are three consecutive integers, each being the products of $m-1$ distinct primes. Let $M = u(u+1)(u+2)$. We define the three prime equations

$$P_2 = \frac{2M}{u}P_1 + 1, \quad P_3 = \frac{2M}{u+1}P_1 + 1, \quad P_4 = \frac{2M}{u+2}P_1 + 1 \quad (6)$$

Using Jiang function $J_2(\omega)$ we prove that there exist infinitely many primes P_1 such that P_2, P_3 and P_4 are primes.

From (6) we have

$$uP_2 = 2MP_1 + u, \quad uP_2 + 1 = 2MP_1 + u + 1 = (u+1)\left(\frac{2M}{u+1}P_1 + 1\right) = (u+1)P_3$$

$$uP_2 + 2 = 2MP_1 + u + 2 = (u+2)\left(\frac{2M}{u+2}P_1 + 1\right) = (u+2)P_4$$

We prove

$$d(x) = d(x+1) = d(x+2) = m > 1 \text{ infinitely-often.} \quad (7)$$

Theorem 3.

$$d(x) = d(x+2) = d(x+4) = 2 \text{ infinitely-often} \quad (8)$$

Proof [1,2,3]. Prime equations are

$$P_2 = 70P_1 + 1, \quad P_3 = 42P_1 + 1, \quad P_4 = 30P_1 + 1 \quad (9)$$

Using Jiang function $J_2(\omega)$ we prove that there exist infinitely many primes P_1 such that P_2, P_3 and P_4 are primes.

From (9) we have

$$3P_2 = 210P_1 + 3, \quad 3P_2 + 2 = 210P_1 + 5 = 5(42P_1 + 1) = 5P_3$$

$$3P_2 + 4 = 210P_1 + 7 = 7(30P_1 + 1) = 7P_4 \quad (10)$$

We prove

$$d(3P_2) = d(3P_2 + 2) = d(3P_2 + 4) = 2 \text{ infinitely-often.} \quad (11)$$

Theorem 4.

$$d(x) = d(x+2) = d(x+4) = m > 1 \text{ infinitely-often.} \quad (12)$$

Proof [1, 2, 3]. Suppose that $u, u+2$ and $u+4$ are three odd integers, each being the products of $m-1$ distinct primes. Let $M = u(u+2)(u+4)$

We define three prime equations

$$P_2 = \frac{2M}{u}P_1 + 1, \quad P_3 = \frac{2M}{u+2}P_1 + 1, \quad P_4 = \frac{2M}{u+4}P_1 + 1 \quad (13)$$

Using Jiang function $J_2(\omega)$ we prove that there exist infinitely many primes P_1 such that P_2 , P_3 and P_4 are primes.

From (13) we have $uP_2 = 2MP_1 + u$,

$$\begin{aligned} uP_2 + 2 &= 2MP_1 + u + 2 = (u+2) \left(\frac{2M}{u+2} P_1 + 1 \right) = (u+2)P_3, \\ uP_2 + 4 &= MP_1 + u + 4 = (u+4) \left(\frac{2M}{u+4} P_1 + 1 \right) = (u+4)P_4. \end{aligned} \quad (14)$$

We prove

$$d(x) = d(x+2) = d(x+4) = m > 1 \text{ infinitely-often.} \quad (15)$$

Theorem 5.

$$d(x) = d(x+1) = d(x+2) = d(x+4) = m > 1 \text{ infinitely-often.} \quad (16)$$

Proof. From (9) we have prime equations

$$P_2 = 70P_1 + 1, \quad P_3 = 105P_1 + 2, \quad P_4 = 42P_1 + 1, \quad P_5 = 30P_1 + 1 \quad (17)$$

Using Jiang function we prove there exist infinitely many primes P_1 such that P_2 , P_3 , P_4 and P_5 are primes. From (17) we have

$$\begin{aligned} 3P_2 &= 210P_1 + 3 \\ 3P_2 + 1 &= 210P_1 + 4 = 2(105P_1 + 2) = 2P_3 \\ 3P_2 + 2 &= 210P_1 + 5 = 5(42P_1 + 1) = 5P_4 \\ 3P_2 + 4 &= 210P_1 + 7 = 7(30P_1 + 1) = 7P_5. \end{aligned} \quad (18)$$

Using Jiang function we prove

$$d(x) = d(x+1) = d(x+2) = d(x+4) = m > 1 \text{ infinitely-often.} \quad (19)$$

$$d(x) = d(x+1) = d(x+2) = d(x+4) = d(x+8) = d(x+10) = m > 1 \text{ infinitely-often.} \quad (20)$$

Using Jiang function $J_2(\omega)$ we are able to prove

$$d(x) = d(x+n) = m > 1 \text{ infinitely-often.} \quad (21)$$

$$d(x) = d(x+5-3) = d(x+7-3) = \dots = d(x+P-3) = m > 1 \text{ infinitely-often.} \quad (22)$$

Goldston *et. al* prove only $d(x) = d(x+n \leq 6) = 2$ infinitely-often [4].

我们发现素数分布新的规律，这个问题比哥德巴赫猜想要难一万倍。这是人类最伟大数学发现。欧拉高斯没接触这个问题，20世纪最伟大数学家埃尔德什开始关注这个问题，但也没得出有用结果。最近国际顶尖数学家 Goldston 等正在研究这个问题。得到国际数学界广泛的支持和关注，但文章都发表在著名杂志上，但没有得出任何实质性进展，蒋春暄在 2002 年[1]就彻底证明了它，但国内外数学家都读了它，都不说话，看到文献[4]后，我们决定写本文，如不用 Jiang 函数，再过两百年也不一定能证明它，国内更无人研究它，这才是研究方向！2009 年 1 月 10 日蒋春暄为休息去参加宋正海讲座在公共汽车上发现公式(22),1 月 11 日发现定理五。这样算一篇完整论文。

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