

**Jiang Chun-Xuan is one of the greatest mathematicians In the world beyond Gauss and Euler  
-Jiang Proof Of Fermat Last Theorem-  
-蒋春暄证明费马大定理-**

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**Abstract:** 蒋春暄证明费马大定理, 他们正在等待这么一天。中国蒋春暄 1991 年证明了费马大定理, 怀尔斯 1994 年证明费马大定理已被否定。这将会引起数学界翻天覆地变化。

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**Keywords:** 世界; 数学; 蒋春暄; 费马大定理; 证明

在两千年前古希腊毕达哥拉斯提出毕达哥拉斯定理

$$a^2+b^2=c^2 \quad (1)$$

有无限多整数解,例如  $3^2+4^2=5^2, 9+16=25$ , 中国称为勾股定理.一般中学生都知勾股定理.1637 年法国业余数学家费马推广毕达哥拉斯定理提出今天称为费马大定理

$$a^n+b^n=c^n \quad (2)$$

$n>2$  时,(2)无整数解.一般中学生都知费马大定理.费马大定理是 354 年以来没有证明数学难题。国际对费马大定理证明评论:“它的证明是 20 世纪最大成就, 是人类的智力最高峰, 它相当若干个普通诺贝尔奖, 它可同人类登月球相提并论成就.它可同人类发现 DNA 和原子分裂相提并论的成就”.1991 年蒋春暄用他发明数学非常简单证明了费马大定理.中国数学家刘培杰写一本宣传怀尔斯书,书名为“从毕达哥拉斯到怀尔斯”中 365 页指出:只用 4 页不需要任何数论知识蒋春暄就证明了费马大定理,只找到了有限的赞同者,但却从未收到过任何公开的来自学术上的反驳.这才是真正费马大定理证明,中学生都能理解这个证明.蒋春暄因首先证明费马大定理获特勒肖-伽利略科学院 2009 年度金奖.有人说:“应该先把费马大定理的问题澄清,因为这是舆论的焦点, 现在谈别的都毫无意义”。所以本文就谈费马大定理。

#### 本文介绍

(1) 蒋春暄 1991 年只用 4 页证明费马大定理,In 1991 Jiang proved Fermat last theorem;

(2) 怀尔斯 1994 年用 120 页证明费马大定理,In 1994 Wiles proved Fermat last theorem;

#### Top 10 Greatest Mathematicians

蒋春暄是大数学家

[M. R. Sexton](#) December 7, 2010

Often called the language of the universe, mathematics is fundamental to our understanding of the world and, as such, is vitally important in a modern society such as ours. Everywhere you look it is likely mathematics has made an impact, from the faucet in your kitchen to the satellite that beams your television programs to your home. As such, great mathematicians are undoubtedly going to rise above the rest and have their name embedded within history. This list documents some such people. I have rated them based on contributions and how they effected mathematics at the time, as well as their lasting effect. I also suggest one looks deeper into the lives of these men, as they are truly fascinating people and their discoveries are astonishing – too much to include here. As always, such lists are highly subjective, and as such please include your own additions in the comments!

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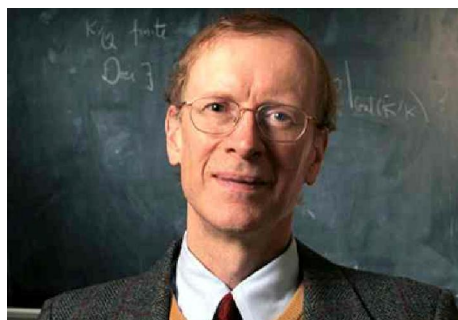
Pythagoras of Samos



Greek Mathematician Pythagoras is considered by some to be one of the first great mathematicians. Living around 570 to 495 BC, in modern day Greece, he is known to have founded the Pythagorean cult, who were noted by Aristotle to be one of the first groups to actively study and advance mathematics. He is also commonly credited with the Pythagorean Theorem within trigonometry. However, some sources doubt that it was him who constructed the proof (Some attribute it to his students, or Baudhayana, who lived some 300 years earlier in India). Nonetheless, the effect of such, as with large portions of fundamental mathematics, is commonly felt today, with the theorem playing a large part in modern measurements and technological equipment, as well as being the base of a large portion of other areas and theorems in mathematics. But, unlike most ancient theories, it played a bearing on the development of geometry, as well as opening the door to the study of mathematics as a worthwhile endeavor. Thus, he could be called the founding father of modern mathematics.

9

Andrew Wiles



The only currently living mathematician on this list, Andrew Wiles is most well known for his proof of Fermat's Last Theorem: That no positive integers,  $a$ ,  $b$  and  $c$  can satisfy the equation  $a^n + b^n = c^n$  For  $n$  greater than 2. (If  $n=2$  it is the Pythagoras Formula). Although the contributions to math are not, perhaps, as grand as other on this list, he did 'invent' large portions of new mathematics for his proof of the theorem. Besides, his dedication is often admired by most, as he quite literally shut himself away for 7 years to formulate a solution. When it was found that the solution contained an error, he returned to solitude for a further year before the solution was accepted. To put in perspective how ground breaking and new the math was, it had been said that you could count the number of mathematicians in the world on one hand who, at the time, could understand and validate his proof. Nonetheless, the effects of such are likely to only increase as time passes (and more and more people can understand it).

8

Isaac Newton and Wilhelm Leibniz



I have placed these two together as they are both often given the honor of being the 'inventor' of modern infinitesimal calculus, and as such have both made monolithic contributions to the field. To start, Leibniz is often given the credit for introducing modern standard notation, notably the integral sign. He made large contributions to the field of Topology. Whereas all round genius Isaac Newton has, because of the grand scientific epic Principia, generally become the primary man hailed by most to be the actual inventor of calculus. Nonetheless, what can be said is that both men made considerable vast contributions in their own manner.

7

Leonardo Pisano Bigollo



Bigollo, also known as Leonardo Fibonacci, is perhaps one of the middle ages greatest mathematicians. Living from 1170 to 1250, he is best known for introducing the infamous Fibonacci Series to the western world. Although known to Indian mathematicians since approximately 200 BC, it was, nonetheless, a truly insightful sequence, appearing in biological systems frequently. In addition, from this

Fibonacci also contributed greatly to the introduction of the Arabic numbering system. Something he is often forgotten for.

Having spent a large portion of his childhood within North Africa he learned the Arabic numbering system, and upon realizing it was far simpler and more efficient than the bulky Roman numerals, decided to travel the Arab world learning from the leading mathematicians of the day. Upon returning to Italy in 1202, he published his *Liber Abaci*, whereupon the Arabic numbers were introduced and applied to many world situations to further advocate their use. As a result of his work the system was gradually adopted and today he is considered a major player in the development of modern mathematics.

6

Alan Turing



Computer Scientist and Cryptanalyst Alan Turing is regarded by many, if not most, to be one of the greatest minds of the 20th Century. Having worked in the Government Code and Cypher School in Britain during the second world war, he made significant discoveries and created ground breaking methods of code breaking that would eventually aid in cracking the German Enigma Encryptions. Undoubtedly affecting the outcome of the war, or at least the time-scale.

After the end of the war he invested his time in computing. Having come up with idea of a computing style machine before the war, he is considered one of the first true computer scientists. Furthermore, he wrote a range of brilliant papers on the subject of computing that are still relevant today, notably on Artificial Intelligence, on which he developed the Turing test which is still used to evaluate a computer's 'intelligence'. Remarkably, he began in 1948 working

with D. G. Champernowne, an undergraduate acquaintance on a computer chess program for a machine not yet in existence. He would play the 'part' of the machine in testing such programs.

5

René Descartes



French Philosopher, Physicist and Mathematician René Descartes is best known for his 'Cogito Ergo Sum' philosophy. Despite this, the Frenchman, who lived 1596 to 1650, made ground breaking contributions to mathematics. Alongside Newton and Leibniz, Descartes helped provide the foundations of modern calculus (which Newton and Leibniz later built upon), which in itself had great bearing on the modern day field. Alongside this, and perhaps more familiar to the reader, is his development of Cartesian Geometry, known to most as the standard graph (Square grid lines, x and y axis, etc.) and its use of algebra to describe the various locations on such. Before this most geometers used plain paper (or another material or surface) to perform their art. Previously, such distances had to be measured literally, or scaled. With the introduction of Cartesian Geometry this changed dramatically, points could now be expressed as points on a graph, and as such, graphs could be drawn to any scale, also these points did not necessarily have to be numbers. The final contribution to the field was his introduction of superscripts within algebra to express powers. And thus, like many others in this list, contributed to the development of modern mathematical notation.



4  
Euclid

Living around 300BC, he is considered the Father of Geometry and his magnum opus: Elements, is one the greatest mathematical works in history, with its being in use in education up until the 20th century. Unfortunately, very little is known about his life, and what exists was written long after his presumed death. Nonetheless, Euclid is credited with the instruction of the rigorous, logical proof for theorems and conjectures. Such a framework is still used to this day, and thus, arguably, he has had the greatest influence of all mathematicians on this list. Alongside his Elements were five other surviving works, thought to have been written by him, all generally on the topic of Geometry or Number theory. There are also another five works that have, sadly, been lost throughout history.

3  
G. F. Bernhard Riemann

Bernhard Riemann, born to a poor family in 1826, would rise to become one of the worlds prominent

mathematicians in the 19th Century. The list of contributions to geometry are large, and he has a wide range of theorems bearing his name. To name just a few: Riemannian Geometry, Riemannian Surfaces and the Riemann Integral. However, he is perhaps most famous (or infamous) for his legendarily difficult Riemann Hypothesis; an extremely complex problem on the matter of the distributions of prime numbers. Largely ignored for the first 50 years following its appearance, due to few other mathematicians actually understanding his work at the time, it has quickly risen to become one of the greatest open questions in modern science, baffling and confounding even the greatest mathematicians. Although progress has been made, its has been incredibly slow. However, a prize of \$1 million has been offered from the Clay Maths Institute for a proof, and one would almost undoubtedly receive a Fields medal if under 40 (The Nobel prize of mathematics). The fallout from such a proof is hypothesized to be large: Major encryption systems are thought to be breakable with such a proof, and all that rely on them would collapse. As well as this, a proof of the hypothesis is expected to use 'new mathematics'. It would seem that, even in death, Riemann's work may still pave the way for new contributions to the field, just as he did in life.

2  
Carl Friedrich Gauss 高斯不敢证明费马大定理

Child prodigy Gauss, the 'Prince of Mathematics', made his first major discovery whilst still a teenager, and wrote the incredible Disquisitiones Arithmeticae, his magnum opus, by the time he was 21. Many know Gauss for his outstanding mental ability – quoted to have added the numbers 1 to 100 within seconds whilst attending primary school (with the aid of a clever trick). The local Duke, recognizing his talent, sent him to Collegium Carolinum before he left

for Gottingen (at the time it was the most prestigious mathematical university in the world, with many of the best attending). After graduating in 1798 (at the age of 22), he began to make several important contributions in major areas of mathematics, most notably number theory (especially on Prime numbers). He went on to prove the fundamental theorem of algebra, and introduced the Gaussian gravitational constant in physics, as well as much more – all this before he was 24! Needless to say, he continued his work up until his death at the age of 77, and had made major advances in the field which have echoed down through time.

我是一名在哈佛数学系的博士生, 我看到蒋老师的成就, 真的非常振奋. 您是我们中国的骄傲, 可以说是数学史上最伟大天才(以前我以为是高斯). 2006-12-17 来信.

1

Leonhard Euler



欧拉只证明费马大定理指数 3, 蒋春暄用他的方法证明了只要证明指数 3 就证明了费马大定理. 只有欧拉和蒋春暄证明是正确的, 直观的, 任何人都可理解的. 其它证明都是猜想使人难以理解的。

If Gauss is the Prince, Euler is the King. Living from 1707 to 1783, he is regarded as the greatest mathematician to have ever walked this planet. It is said that all mathematical formulas are named after the next person after Euler to discover them. In his day he was ground breaking and on par with Einstein in genius. His primary (if that's possible) contribution to the field is with the introduction of mathematical notation including the concept of a function (and how it is written as  $f(x)$ ), shorthand trigonometric functions, the 'e' for the base of the natural logarithm (The Euler Constant), the Greek letter Sigma for summation and the letter 'i' for imaginary units, as well as the symbol pi for the ratio of a circles circumference to its diameter. All of which play a huge bearing on modern mathematics, from the everyday to the incredibly complex.

As well as this, he also solved the Seven Bridges of Koenigsberg problem in graph theory, found the Euler Characteristic for connecting the number of vertices, edges and faces of an object, and (dis)proved many well known theories, too many to list. Furthermore, he continued to develop calculus, topology, number theory, analysis and graph theory as well as much, much more – and ultimately he paved the way for modern mathematics and all its revelations. It is probably no coincidence that industry and technological developments rapidly increased around this time.

## Automorphic Functions And Fermat's Last Theorem(1)

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**Abstract:** In 1637 Fermat wrote: "It is impossible to separate a cube into two cubes, or a biquadrate into two biquadrates, or in general any power higher than the second into powers of like degree: I have discovered a truly marvelous proof, which this margin is too small to contain."

This means:  $x^n + y^n = z^n (n > 2)$  has no integer solutions, all different from 0 (i.e., it has only the trivial solution, where one of the integers is equal to 0). It has been called Fermat's last theorem (FLT). It suffices to prove FLT for exponent 4. and every prime exponent  $P$ . Fermat proved FLT for exponent 4. Euler proved FLT for exponent 3.

In this paper using automorphic functions we prove FLT for exponents  $3P$  and  $P$ , where  $P$  is an odd prime. The proof of FLT must be direct. But indirect proof of FLT is disbelieving.

In 1974 Jiang found out Euler formula of the cyclotomic real numbers in the cyclotomic fields

$$\exp\left(\sum_{i=1}^{n-1} t_i J^i\right) = \sum_{i=1}^n S_i J^{i-1} \quad (1)$$

where  $J$  denotes a  $n$ th root of unity,  $J^n = 1$ ,  $n$  is an odd number,  $t_i$  are the real numbers.

$S_i$  is called the automorphic functions (complex hyperbolic functions) of order  $n$  with  $n-1$  variables [1-7].

$$S_i = \frac{1}{n} \left[ e^A + 2 \sum_{j=1}^{\frac{n-1}{2}} (-1)^{(i-1)j} e^{B_j} \cos\left(\theta_j + (-1)^j \frac{(i-1)j\pi}{n}\right) \right] \quad (2)$$

where  $i=1, 2, \dots, n$ ;

$$A = \sum_{\alpha=1}^{n-1} t_\alpha, \quad B_j = \sum_{\alpha=1}^{n-1} t_\alpha (-1)^{\alpha j} \cos \frac{\alpha j \pi}{n},$$

$$\theta_j = (-1)^{j+1} \sum_{\alpha=1}^{n-1} t_\alpha (-1)^{\alpha j} \sin \frac{\alpha j \pi}{n}, \quad A + 2 \sum_{j=1}^{\frac{n-1}{2}} B_j = 0 \quad (3)$$

(2) may be written in the matrix form

$$\begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ \dots \\ S_n \end{bmatrix} = \frac{1}{n} \begin{bmatrix} 1 & 1 & 0 & \dots & 0 \\ 1 & -\cos \frac{\pi}{n} & -\sin \frac{\pi}{n} & \dots & -\sin \frac{(n-1)\pi}{2n} \\ 1 & \cos \frac{2\pi}{n} & \sin \frac{2\pi}{n} & \dots & -\sin \frac{(n-1)\pi}{n} \\ \dots & \dots & \dots & \dots & \dots \\ 1 & \cos \frac{(n-1)\pi}{n} & \sin \frac{(n-1)\pi}{n} & \dots & -\sin \frac{(n-1)^2 \pi}{2n} \end{bmatrix} \begin{bmatrix} e^A \\ 2e^{B_1} \cos \theta_1 \\ 2e^{B_1} \sin \theta_1 \\ \dots \\ 2 \exp B_{\frac{n-1}{2}} \sin \theta_{\frac{n-1}{2}} \end{bmatrix} \quad (4)$$

where  $(n-1)/2$  is an even number.

From (4) we have its inverse transformation

$$\begin{bmatrix} e^A \\ e^{B_1} \cos \theta_1 \\ e^{B_1} \sin \theta_1 \\ \dots \\ \exp(B_{\frac{n-1}{2}}) \sin(\theta_{\frac{n-1}{2}}) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ 1 & -\cos \frac{\pi}{n} & \cos \frac{2\pi}{n} & \dots & \cos \frac{(n-1)\pi}{n} \\ 0 & -\sin \frac{\pi}{n} & \sin \frac{2\pi}{n} & \dots & \sin \frac{(n-1)\pi}{n} \\ \dots & \dots & \dots & \dots & \dots \\ 0 & -\sin \frac{(n-1)\pi}{2n} & -\sin \frac{(n-1)\pi}{n} & \dots & -\sin \frac{(n-1)^2\pi}{2n} \end{bmatrix} \begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ \dots \\ S_n \end{bmatrix} \quad (5)$$

From (5) we have

$$\begin{aligned} e^A &= \sum_{i=1}^n S_i, & e^{B_j} \cos \theta_j &= S_1 + \sum_{i=1}^{n-1} S_{1+i} (-1)^{ij} \cos \frac{ij\pi}{n} \\ e^{B_j} \sin \theta_j &= (-1)^{j+1} \sum_{i=1}^{n-1} S_{1+i} (-1)^{ij} \sin \frac{ij\pi}{n} \end{aligned} \quad (6)$$

In (3) and (6)  $t_i$  and  $S_i$  have the same formulas. (4) and (5) are the most critical formulas of proofs for FLT. Using (4) and (5) in 1991 Jiang invented that every factor of exponent  $n$  has the Fermat equation and proved FLT [1-7] Substituting (4) into (5) we prove (5).

$$\begin{bmatrix} e^A \\ e^{B_1} \cos \theta_1 \\ e^{B_1} \sin \theta_1 \\ \dots \\ \exp(B_{\frac{n-1}{2}}) \sin(\theta_{\frac{n-1}{2}}) \end{bmatrix} = \frac{1}{n} \begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ 1 & -\cos \frac{\pi}{n} & \cos \frac{2\pi}{n} & \dots & \cos \frac{(n-1)\pi}{n} \\ 0 & -\sin \frac{\pi}{n} & \sin \frac{2\pi}{n} & \dots & \sin \frac{(n-1)\pi}{n} \\ \dots & \dots & \dots & \dots & \dots \\ 0 & -\sin \frac{(n-1)\pi}{2n} & -\sin \frac{(n-1)\pi}{n} & \dots & -\sin \frac{(n-1)^2\pi}{2n} \end{bmatrix} \times$$

$$\begin{bmatrix} 1 & 1 & 0 & \dots & 0 \\ 1 & -\cos \frac{\pi}{n} & -\sin \frac{\pi}{n} & \dots & -\sin \frac{(n-1)\pi}{2n} \\ 1 & \cos \frac{2\pi}{n} & \sin \frac{2\pi}{n} & \dots & -\sin \frac{(n-1)\pi}{n} \\ \dots & \dots & \dots & \dots & \dots \\ 1 & \cos \frac{(n-1)\pi}{n} & \sin \frac{(n-1)\pi}{n} & \dots & -\sin \frac{(n-1)^2\pi}{2n} \end{bmatrix} \begin{bmatrix} e^A \\ 2e^{B_1} \cos \theta_1 \\ 2e^{B_1} \sin \theta_1 \\ \dots \\ 2 \exp(B_{\frac{n-1}{2}}) \sin(\theta_{\frac{n-1}{2}}) \end{bmatrix}$$

$$\begin{aligned}
 &= \frac{1}{n} \begin{bmatrix} n & 0 & 0 & \dots & 0 \\ 0 & \frac{n}{2} & 0 & \dots & 0 \\ 0 & 0 & \frac{n}{2} & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & \frac{n}{2} \end{bmatrix} \begin{bmatrix} e^A \\ 2e^{B_1} \cos \theta_1 \\ 2e^{B_1} \sin \theta_1 \\ \dots \\ 2 \exp(B_{\frac{n-1}{2}}) \sin(\theta_{\frac{n-1}{2}}) \end{bmatrix} \\
 &= \begin{bmatrix} e^A \\ e^{B_1} \cos \theta_1 \\ e^{B_1} \sin \theta_1 \\ \dots \\ \exp(B_{\frac{n-1}{2}}) \sin(\theta_{\frac{n-1}{2}}) \end{bmatrix}, \tag{7}
 \end{aligned}$$

where  $1 + \sum_{j=1}^{n-1} (\cos \frac{j\pi}{n})^2 = \frac{n}{2}$ ,  $\sum_{j=1}^{n-1} (\sin \frac{j\pi}{n})^2 = \frac{n}{2}$ .  
 From (3) we have

$$\exp(A + 2 \sum_{j=1}^{\frac{n-1}{2}} B_j) = 1 \tag{8}$$

From (6) we have

$$\exp(A + 2 \sum_{j=1}^{\frac{n-1}{2}} B_j) = \begin{vmatrix} S_1 & S_n & \dots & S_2 \\ S_2 & S_1 & \dots & S_3 \\ \dots & \dots & \dots & \dots \\ S_n & S_{n-1} & \dots & S_1 \end{vmatrix} = \begin{vmatrix} S_1 & (S_1)_1 & \dots & (S_1)_{n-1} \\ S_2 & (S_2)_1 & \dots & (S_2)_{n-1} \\ \dots & \dots & \dots & \dots \\ S_n & (S_n)_1 & \dots & (S_n)_{n-1} \end{vmatrix}, \tag{9}$$

where  $(S_i)_j = \frac{\partial S_i}{\partial t_j}$  [7].

From (8) and (9) we have the circulant determinant

$$\exp(A + 2 \sum_{j=1}^{\frac{n-1}{2}} B_j) = \begin{vmatrix} S_1 & S_n & \dots & S_2 \\ S_2 & S_1 & \dots & S_3 \\ \dots & \dots & \dots & \vdots \\ S_n & S_{n-1} & \dots & S_1 \end{vmatrix} = 1 \tag{10}$$

If  $S_i \neq 0$ , where  $i = 1, 2, \dots, n$ , then (10) has infinitely many rational solutions.

Assume  $S_1 \neq 0$ ,  $S_2 \neq 0$ ,  $S_i = 0$  where  $i = 3, 4, \dots, n$ .  $S_i = 0$  are  $n-2$  indeterminate equations with  $n-1$  variables. From (6) we have



$$e^A = S_1 + S_2, \quad e^{2B_j} = S_1^2 + S_2^2 + 2S_1S_2(-1)^j \cos \frac{j\pi}{n} \quad (11)$$

From (10) and (11) we have the Fermat equation

$$\exp\left(A + 2 \sum_{j=1}^{\frac{n-1}{2}} B_j\right) = (S_1 + S_2) \prod_{j=1}^{\frac{n-1}{2}} (S_1^2 + S_2^2 + 2S_1S_2(-1)^j \cos \frac{j\pi}{n}) = S_1^n + S_2^n = 1 \quad (12)$$

**Example[1].** Let  $n = 15$ . From (3) we have

$$\begin{aligned} A &= (t_1 + t_{14}) + (t_2 + t_{13}) + (t_3 + t_{12}) + (t_4 + t_{11}) + (t_5 + t_{10}) + (t_6 + t_9) + (t_7 + t_8) \\ B_1 &= -(t_1 + t_{14}) \cos \frac{\pi}{15} + (t_2 + t_{13}) \cos \frac{2\pi}{15} - (t_3 + t_{12}) \cos \frac{3\pi}{15} + (t_4 + t_{11}) \cos \frac{4\pi}{15} \\ &\quad - (t_5 + t_{10}) \cos \frac{5\pi}{15} + (t_6 + t_9) \cos \frac{6\pi}{15} - (t_7 + t_8) \cos \frac{7\pi}{15}, \\ B_2 &= (t_1 + t_{14}) \cos \frac{2\pi}{15} + (t_2 + t_{13}) \cos \frac{4\pi}{15} + (t_3 + t_{12}) \cos \frac{6\pi}{15} + (t_4 + t_{11}) \cos \frac{8\pi}{15} \\ &\quad + (t_5 + t_{10}) \cos \frac{10\pi}{15} + (t_6 + t_9) \cos \frac{12\pi}{15} + (t_7 + t_8) \cos \frac{14\pi}{15}, \\ B_3 &= -(t_1 + t_{14}) \cos \frac{3\pi}{15} + (t_2 + t_{13}) \cos \frac{6\pi}{15} - (t_3 + t_{12}) \cos \frac{9\pi}{15} + (t_4 + t_{11}) \cos \frac{12\pi}{15} \\ &\quad - (t_5 + t_{10}) \cos \frac{15\pi}{15} + (t_6 + t_9) \cos \frac{18\pi}{15} - (t_7 + t_8) \cos \frac{21\pi}{15}, \\ B_4 &= (t_1 + t_{14}) \cos \frac{4\pi}{15} + (t_2 + t_{13}) \cos \frac{8\pi}{15} + (t_3 + t_{12}) \cos \frac{12\pi}{15} + (t_4 + t_{11}) \cos \frac{16\pi}{15} \\ &\quad + (t_5 + t_{10}) \cos \frac{20\pi}{15} + (t_6 + t_9) \cos \frac{24\pi}{15} + (t_7 + t_8) \cos \frac{28\pi}{15}, \\ B_5 &= -(t_1 + t_{14}) \cos \frac{5\pi}{15} + (t_2 + t_{13}) \cos \frac{10\pi}{15} - (t_3 + t_{12}) \cos \frac{15\pi}{15} + (t_4 + t_{11}) \cos \frac{20\pi}{15} \\ &\quad - (t_5 + t_{10}) \cos \frac{25\pi}{15} + (t_6 + t_9) \cos \frac{30\pi}{15} - (t_7 + t_8) \cos \frac{35\pi}{15}, \\ B_6 &= (t_1 + t_{14}) \cos \frac{6\pi}{15} + (t_2 + t_{13}) \cos \frac{12\pi}{15} + (t_3 + t_{12}) \cos \frac{18\pi}{15} + (t_4 + t_{11}) \cos \frac{24\pi}{15} \\ &\quad + (t_5 + t_{10}) \cos \frac{30\pi}{15} + (t_6 + t_9) \cos \frac{36\pi}{15} + (t_7 + t_8) \cos \frac{42\pi}{15}, \\ B_7 &= -(t_1 + t_{14}) \cos \frac{7\pi}{15} + (t_2 + t_{13}) \cos \frac{14\pi}{15} - (t_3 + t_{12}) \cos \frac{21\pi}{15} + (t_4 + t_{11}) \cos \frac{28\pi}{15} \\ &\quad - (t_5 + t_{10}) \cos \frac{35\pi}{15} + (t_6 + t_9) \cos \frac{42\pi}{15} - (t_7 + t_8) \cos \frac{49\pi}{15}, \\ A + 2 \sum_{j=1}^7 B_j &= 0, \quad A + 2B_3 + 2B_6 = 5(t_5 + t_{10}) \end{aligned} \quad (13)$$

Form (12) we have the Fermat equation

$$\exp(A + 2 \sum_{j=1}^7 B_j) = S_1^{15} + S_2^{15} = (S_1^5)^3 + (S_2^5)^3 = 1 \quad (14)$$

From (13) we have

$$\exp(A + 2B_3 + 2B_6) = [\exp(t_5 + t_{10})]^5 \quad (15)$$

From (11) we have

$$\exp(A + 2B_3 + 2B_6) = S_1^5 + S_2^5 \quad (16)$$

From (15) and (16) we have the Fermat equation

$$\exp(A + 2B_3 + 2B_6) = S_1^5 + S_2^5 = [\exp(t_5 + t_{10})]^5 \quad (17)$$

Euler proved that (14) has no rational solutions for exponent 3[8]. Therefore we prove that (17) has no rational solutions for exponent 5[1].

**Theorem 1.** [1-7]. Let  $n = 3P$ , where  $P > 3$  is odd prime. From (12) we have the Fermat's equation

$$\exp(A + 2 \sum_{j=1}^{\frac{3P-1}{2}} B_j) = S_1^{3P} + S_2^{3P} = (S_1^P)^3 + (S_2^P)^3 = 1 \quad (18)$$

From (3) we have

$$\exp(A + 2 \sum_{j=1}^{\frac{P-1}{2}} B_{3j}) = [\exp(t_P + t_{2P})]^P \quad (19)$$

From (11) we have

$$\exp(A + 2 \sum_{j=1}^{\frac{P-1}{2}} B_{3j}) = S_1^P + S_2^P \quad (20)$$

From (19) and (20) we have the Fermat equation

$$\exp(A + 2 \sum_{j=1}^{\frac{P-1}{2}} B_{3j}) = S_1^P + S_2^P = [\exp(t_P + t_{2P})]^P \quad (21)$$

Euler proved that (18) has no rational solutions for exponent 3[8]. Therefore we prove that (21) has no rational solutions for  $P > 3$  [1, 3-7].

**Theorem 2.** In 1847 Kummer write the Fermat's equation

$$x^P + y^P = z^P \quad (22)$$

in the form

$$(x + y)(x + ry)(x + r^2y) \cdots (x + r^{P-1}y) = z^P \quad (23)$$

where  $P$  is odd prime,  $r = \cos \frac{2\pi}{P} + i \sin \frac{2\pi}{P}$

Kummer assume the divisor of each factor is a  $P$  th power. Kummer proved FLT for prime exponent  $p < 100$  [8]. We consider the Fermat's equation

$$x^{3P} + y^{3P} = z^{3P} \quad (24)$$

we rewrite (24)

$$(x^P)^3 + (y^P)^3 = (z^P)^3 \quad (25)$$

From (24) we have

$$(x^P + y^P)(x^P + ry^P)(x^P + r^2y^P) = z^{3P} \quad (26)$$

$$r = \cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3}$$

where

We assume the divisor of each factor is a  $P$  th power.

Let  $S_1 = \frac{x}{z}$ ,  $S_2 = \frac{y}{z}$ . From (20) and (26) we have the Fermat's equation

$$x^P + y^P = [z \times \exp(t_p + t_{2p})]^P \quad (27)$$

Euler proved that (25) has no integer solutions for exponent 3[8]. Therefore we prove that (27) has no integer solutions for prime exponent  $P$ .

**Fermat Theorem.** It suffices to prove FLT for exponent 4. We rewrite (24)

$$(x^3)^P + (y^3)^P = (z^3)^P \quad (28)$$

Euler proved that (25) has no integer solutions for exponent 3 [8]. Therefore we prove that (28) has no integer solutions for all prime exponent  $P$  [1-7].

We consider Fermat equation

$$x^{4P} + y^{4P} = z^{4P} \quad (29)$$

We rewrite (29)

$$(x^P)^4 + (y^P)^4 = (z^P)^4 \quad (30)$$

$$(x^4)^P + (y^4)^P = (z^4)^P \quad (31)$$

Fermat proved that (30) has no integer solutions for exponent 4 [8]. Therefore we prove that (31) has no integer solutions for all prime exponent  $P$  [2,5,7]. This is the proof that Fermat thought to have had.

**Remark.** It suffices to prove FLT for exponent 4. Let  $n = 4P$ , where  $P$  is an odd prime. We have the Fermat's equation for exponent  $4P$  and the Fermat's equation for exponent  $P$  [2,5,7]. This is the proof that Fermat thought to have had. In complex hyperbolic functions let exponent  $n$  be  $n = \Pi P$ ,  $n = 2\Pi P$  and  $n = 4\Pi P$ . Every factor of exponent  $n$  has the Fermat's equation [1-7]. In complex trigonometric functions let exponent  $n$  be  $n = \Pi P$ ,  $n = 2\Pi P$  and  $n = 4\Pi P$ . Every factor of exponent  $n$  has Fermat's equation [1-7]. Using modular elliptic curves Wiles and Taylor prove FLT[9,10]. This is not the proof that Fermat thought to have had. The classical theory of automorphic functions, created by Klein and Poincare, was concerned with the study of analytic functions in the unit circle that are invariant under a discrete group of transformations. Automorphic functions are generalization of the trigonometric, hyperbolic, elliptic, and certain other functions of elementary analysis. The complex trigonometric functions and complex hyperbolic functions have a wide application in mathematics and physics.

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### Automorphic Functions And Fermat's Last Theorem (3) (Fermat's Proof of FLT)

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**Abstract:** In 1637 Fermat wrote: "It is impossible to separate a cube into two cubes, or a biquadrate into two biquadrates, or in general any power higher than the second into powers of like degree: I have discovered a truly marvelous proof, which this margin is too small to contain."

This means:  $x^n + y^n = z^n$  ( $n > 2$ ) has no integer solutions, all different from 0 (i.e., it has only the trivial solution, where one of the integers is equal to 0). It has been called Fermat's last theorem (FLT). It suffices to prove FLT for exponent 4 and every prime exponent  $P$ . Fermat proved FLT for exponent 4. Euler proved FLT for exponent 3.

In this paper using automorphic functions we prove FLT for exponents  $4P$  and  $P$ , where  $P$  is an odd prime. We rediscover the Fermat proof. The proof of FLT must be direct. But indirect proof of FLT is disbelieving.

In 1974 Jiang found out Euler formula of the cyclotomic real numbers in the cyclotomic fields

$$\exp\left(\sum_{i=1}^{4m-1} t_i J^i\right) = \sum_{i=1}^{4m} S_i J^{i-1}, \quad (1)$$

where  $J$  denotes a  $4m$  th root of unity,  $J^{4m} = 1$ ,  $m=1,2,3,\dots$ ,  $t_i$  are the real numbers.

$S_i$  is called the automorphic functions (complex hyperbolic functions) of order  $4m$  with  $4m-1$  variables [2,5,7].

$$S_i = \frac{1}{4m} \left[ e^{A_i} + 2e^H \cos\left(\beta + \frac{(i-1)\pi}{2}\right) + 2\sum_{j=1}^{m-1} e^{B_j} \cos\left(\theta_j + \frac{(i-1)j\pi}{2m}\right) \right] \\ + \frac{(-1)^{(i-1)}}{4m} \left[ e^{A_2} + 2\sum_{j=1}^{m-1} e^{D_j} \cos\left(\phi_j - \frac{(i-1)j\pi}{2m}\right) \right] \quad (2)$$

where  $i = 1, \dots, 4m$ ;

$$A_1 = \sum_{\alpha=1}^{4m-1} t_\alpha, \quad A_2 = \sum_{\alpha=1}^{4m-1} t_\alpha (-1)^\alpha, \quad H = \sum_{\alpha=1}^{2m-1} t_{2\alpha} (-1)^\alpha, \quad \beta = \sum_{\alpha=1}^{2m} t_{2\alpha-1} (-1)^\alpha, \\ B_j = \sum_{\alpha=1}^{4m-1} t_\alpha \cos \frac{\alpha j \pi}{2m}, \quad \theta_j = -\sum_{\alpha=1}^{4m-1} t_\alpha \sin \frac{\alpha j \pi}{2m}, \\ D_j = \sum_{\alpha=1}^{4m-1} t_\alpha (-1)^\alpha \cos \frac{\alpha j \pi}{2m}, \quad \phi_j = \sum_{\alpha=1}^{4m-1} t_\alpha (-1)^\alpha \sin \frac{\alpha j \pi}{2m}, \\ A_1 + A_2 + 2H + 2\sum_{j=1}^{m-1} (B_j + D_j) = 0 \quad (3)$$

From (2) we have its inverse transformation[5,7]

$$e^{A_i} = \sum_{i=1}^{4m} S_i, \quad e^{A_2} = \sum_{i=1}^{4m} S_i (-1)^{1+i}$$



$$\begin{aligned}
 e^H \cos \beta &= \sum_{i=1}^{2m} S_{2i-1}(-1)^{1+i}, & e^H \sin \beta &= \sum_{i=1}^{2m} S_{2i}(-1)^i \\
 e^{B_j} \cos \theta_j &= S_1 + \sum_{i=1}^{4m-1} S_{1+i} \cos \frac{ij\pi}{2m}, & e^{B_j} \sin \theta_j &= -\sum_{i=1}^{4m-1} S_{1+i} \sin \frac{ij\pi}{2m}, \\
 e^{D_j} \cos \phi_j &= S_1 + \sum_{i=1}^{4m-1} S_{1+i}(-1)^i \cos \frac{ij\pi}{2m}, & e^{D_j} \sin \phi_j &= \sum_{i=1}^{4m-1} S_{1+i}(-1)^i \sin \frac{ij\pi}{2m}.
 \end{aligned} \tag{4}$$

(3) and (4) have the same form.

From (3) we have

$$\exp \left[ A_1 + A_2 + 2H + 2 \sum_{j=1}^{m-1} (B_j + D_j) \right] = 1 \tag{5}$$

From (4) we have

$$\begin{aligned}
 \exp \left[ A_1 + A_2 + 2H + 2 \sum_{j=1}^{m-1} (B_j + D_j) \right] &= \begin{vmatrix} S_1 & S_{4m} & \cdots & S_2 \\ S_2 & S_1 & \cdots & S_3 \\ \cdots & \cdots & \cdots & \cdots \\ S_{4m} & S_{4m-1} & \cdots & S_1 \end{vmatrix} \\
 &= \begin{vmatrix} S_1 & (S_1)_1 & \cdots & (S_1)_{4m-1} \\ S_2 & (S_2)_1 & \cdots & (S_2)_{4m-1} \\ \cdots & \cdots & \cdots & \cdots \\ S_{4m} & (S_{4m})_1 & \cdots & (S_{4m})_{4m-1} \end{vmatrix}
 \end{aligned} \tag{6}$$

where

$$(S_i)_j = \frac{\partial S_i}{\partial t_j} \tag{7}$$

From (5) and (6) we have circulant determinant

$$\exp \left[ A_1 + A_2 + 2H + 2 \sum_{j=1}^{m-1} (B_j + D_j) \right] = \begin{vmatrix} S_1 & S_{4m} & \cdots & S_2 \\ S_2 & S_1 & \cdots & S_3 \\ \cdots & \cdots & \cdots & \cdots \\ S_{4m} & S_{4m-1} & \cdots & S_1 \end{vmatrix} = 1 \tag{7}$$

Assume  $S_1 \neq 0, S_2 \neq 0, S_i = 0$ , where  $i = 3, \dots, 4m$ .  $S_i = 0$  are  $(4m-2)$  indeterminate equations with  $(4m-1)$  variables. From (4) we have

$$\begin{aligned}
 e^{A_1} &= S_1 + S_2, & e^{A_2} &= S_1 - S_2, & e^{2H} &= S_1^2 + S_2^2 \\
 e^{2B_j} &= S_1^2 + S_2^2 + 2S_1S_2 \cos \frac{j\pi}{2m}, & e^{2D_j} &= S_1^2 + S_2^2 - 2S_1S_2 \cos \frac{j\pi}{2m}
 \end{aligned} \tag{8}$$

**Example [2].** Let  $4m = 12$ . From (3) we have

$$\begin{aligned}
 A_1 &= (t_1 + t_{11}) + (t_2 + t_{10}) + (t_3 + t_9) + (t_4 + t_8) + (t_5 + t_7) + t_6, \\
 A_2 &= -(t_1 + t_{11}) + (t_2 + t_{10}) - (t_3 + t_9) + (t_4 + t_8) - (t_5 + t_7) + t_6, \\
 H &= -(t_2 + t_{10}) + (t_4 + t_8) - t_6,
 \end{aligned}$$

$$\begin{aligned}
B_1 &= (t_1 + t_{11}) \cos \frac{\pi}{6} + (t_2 + t_{10}) \cos \frac{2\pi}{6} + (t_3 + t_9) \cos \frac{3\pi}{6} + (t_4 + t_8) \cos \frac{4\pi}{6} + (t_5 + t_7) \cos \frac{5\pi}{6} - t_6, \\
B_2 &= (t_1 + t_{11}) \cos \frac{2\pi}{6} + (t_2 + t_{10}) \cos \frac{4\pi}{6} + (t_3 + t_9) \cos \frac{6\pi}{6} + (t_4 + t_8) \cos \frac{8\pi}{6} + (t_5 + t_7) \cos \frac{10\pi}{6} + t_6, \\
D_1 &= -(t_1 + t_{11}) \cos \frac{\pi}{6} + (t_2 + t_{10}) \cos \frac{2\pi}{6} - (t_3 + t_9) \cos \frac{3\pi}{6} + (t_4 + t_8) \cos \frac{4\pi}{6} - (t_5 + t_7) \cos \frac{5\pi}{6} - t_6, \\
D_2 &= -(t_1 + t_{11}) \cos \frac{2\pi}{6} + (t_2 + t_{10}) \cos \frac{4\pi}{6} - (t_3 + t_9) \cos \frac{6\pi}{6} + (t_4 + t_8) \cos \frac{8\pi}{6} - (t_5 + t_7) \cos \frac{10\pi}{6} + t_6, \\
A_1 + A_2 + 2(H + B_1 + B_2 + D_1 + D_2) &= 0, \quad A_2 + 2B_2 = 3(-t_3 + t_6 - t_9). \tag{9}
\end{aligned}$$

From (8) and (9) we have

$$\exp[A_1 + A_2 + 2(H + B_1 + B_2 + D_1 + D_2)] = S_1^{12} - S_2^{12} = (S_1^3)^4 - (S_2^3)^4 = 1 \tag{10}$$

From (9) we have

$$\exp(A_2 + 2B_2) = [\exp(-t_3 + t_6 - t_9)]^3 \tag{11}$$

From (8) we have

$$\exp(A_2 + 2B_2) = (S_1 - S_2)(S_1^2 + S_2^2 + S_1 S_2) = S_1^3 - S_2^3 \tag{12}$$

From (11) and (12) we have Fermat's equation

$$\exp(A_2 + 2B_2) = S_1^3 - S_2^3 = [\exp(-t_3 + t_6 - t_9)]^3 \tag{13}$$

Fermat proved that (10) has no rational solutions for exponent 4 [8].

Therefore we prove we prove that (13) has no rational solutions for exponent 3. [2]

**Theorem** . Let  $4m = 4P$ , where  $P$  is an odd prime,  $(P-1)/2$  is an even number.

From (3) and (8) we have

$$\exp[A_1 + A_2 + 2H + 2 \sum_{j=1}^{P-1} (B_j + D_j)] = S_1^{4P} - S_2^{4P} = (S_1^P)^4 - (S_2^P)^4 = 1 \tag{14}$$

From (3) we have

$$\exp[A_2 + 2 \sum_{j=1}^{\frac{P-1}{4}} (B_{4j-2} + D_{4j})] = [\exp(-t_P + t_{2P} - t_{3P})]^P \tag{15}$$

From (8) we have

$$\exp[A_2 + 2 \sum_{j=1}^{\frac{P-1}{4}} (B_{4j-2} + D_{4j})] = S_1^P - S_2^P \tag{16}$$

From (15) and (16) we have Fermat's equation

$$\exp[A_2 + 2 \sum_{j=1}^{\frac{P-1}{4}} (B_{4j-2} + D_{4j})] = S_1^P - S_2^P = [\exp(-t_P + t_{2P} - t_{3P})]^P \tag{17}$$

Fermat proved that (14) has no rational solutions for exponent 4 [8]. Therefore we prove that (17) has no rational solutions for prime exponent  $P$ .

**Remark**. Mathematicians said Fermat could not possibly had a proof, because they do not understand FLT. In complex hyperbolic functions let exponent  $n$  be  $n = \Pi P$ ,  $n = 2\Pi P$  and  $n = 4\Pi P$ . Every factor of exponent  $n$  has Fermat's equation [1-7]. Using modular elliptic curves Wiles and Taylor prove FLT [9,10]. This is not the proof that Fermat thought to have had. The classical theory of automorphic functions, created by Klein and Poincare, was concerned with the study of analytic functions in the unit circle that are invariant under a discrete

group of transformation. Automorphic functions are the generalization of trigonometric, hyperbolic elliptic, and certain other functions of elementary analysis. The complex trigonometric functions and complex hyperbolic functions have a wide application in mathematics and physics.

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**Jiang's function  $J_{n+1}(\omega)$  in prime distribution**  
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**Abstract**

We define that prime equations

$$f_1(P_1, \dots, P_n), \dots, f_k(P_1, \dots, P_n) \quad (5)$$

are polynomials (with integer coefficients) irreducible over integers, where  $P_1, \dots, P_n$  are all prime. If Jiang's function  $J_{n+1}(\omega) = 0$  then (5) has finite prime solutions. If  $J_{n+1}(\omega) \neq 0$  then there are infinitely many primes  $P_1, \dots, P_n$  such that  $f_1, \dots, f_k$  are primes. We obtain a unite prime formula in prime distribution

$$\begin{aligned} \pi_{k+1}(N, n+1) &= |\{P_1, \dots, P_n \leq N : f_1, \dots, f_k \text{ are } k \text{ primes}\}| \\ &= \prod_{i=1}^k (\deg f_i)^{-1} \times \frac{J_{n+1}(\omega) \omega^k}{n! \phi^{k+n}(\omega) \log^{k+n} N} \frac{N^n}{N} (1 + o(1)). \end{aligned} \quad (8)$$

Jiang's function is accurate sieve function. Using Jiang's function we prove about 600 prime theorems [6]. Jiang's function provides proofs of the prime theorems which are simple enough to understand and accurate enough to be useful.

2000 mathematics subject classification 11P32(primary), 11P99(secondary).

**Keywords:** Jiang function, Prime equations, Prime distribution.

*Mathematicians have tried in vain to discover some order in the sequence of prime numbers but we have every reason to believe that there are some mysteries which the human mind will never penetrate.*

Leonhard Euler

*It will be another million years, at least, before we understand the primes.*

Paul Erdős

Suppose that Euler totient function

$$\phi(\omega) = \prod_{2 \leq P} (P-1) = \infty \quad \text{as } \omega \rightarrow \infty, \quad (1)$$

where  $\omega = \prod_{2 \leq P} P$  is called primorial.

Suppose that  $(\omega, h_i) = 1$ , where  $i = 1, \dots, \phi(\omega)$ . We have prime equations

$$P_1 = \omega n + 1, \dots, P_{\phi(\omega)} = \omega n + h_{\phi(\omega)} \quad (2)$$

where  $n = 0, 1, 2, \dots$ .

(2) is called infinitely many prime equations (IMPE). Every equation has infinitely many prime solutions. We have

$$\pi_{h_i} = \sum_{\substack{P_i \leq N \\ P_i \equiv h_i \pmod{\omega}}} 1 = \frac{\pi(N)}{\phi(\omega)} (1 + o(1)). \quad (3)$$

where  $\pi_{h_i}$  denotes the number of primes  $P_i \leq N$  in  $P_i = \omega n + h_i$   $n = 0, 1, 2, \dots$ ,  $\pi(N)$  the number of primes less than or equal to  $N$ .

We replace sets of prime numbers by IMPE. (2) is the fundamental tool for proving the prime theorems in prime distribution.

Let  $\omega = 30$  and  $\phi(30) = 8$ . From (2) we have eight prime equations

$$\begin{aligned} P_1 = 30n + 1, \quad P_2 = 30n + 7, \quad P_3 = 30n + 11, \quad P_4 = 30n + 13, \quad P_5 = 30n + 17, \\ P_6 = 30n + 19, \quad P_7 = 30n + 23, \quad P_8 = 30n + 29, \quad n = 0, 1, 2, \dots \end{aligned} \tag{4}$$

Every equation has infinitely many prime solutions.

**THEOREM.** We define that prime equations

$$f_1(P_1, \dots, P_n), \dots, f_k(P_1, \dots, P_n) \tag{5}$$

are polynomials (with integer coefficients) irreducible over integers, where  $P_1, \dots, P_n$  are primes. If Jiang's function  $J_{n+1}(\omega) = 0$  then (5) has finite prime solutions. If  $J_{n+1}(\omega) \neq 0$  then there exist infinitely many primes  $P_1, \dots, P_n$  such that each  $f_k$  is a prime.

**PROOF.** Firstly, we have Jiang's function [1-11]

$$J_{n+1}(\omega) = \prod_{3 \leq P} [(P-1)^n - \chi(P)] \tag{6}$$

where  $\chi(P)$  is called sieve constant and denotes the number of solutions for the following congruence

$$\prod_{i=1}^k f_i(q_1, \dots, q_n) \equiv 0 \pmod{P} \tag{7}$$

where  $q_1 = 1, \dots, P-1, \dots, q_n = 1, \dots, P-1$ .

$J_{n+1}(\omega)$  denotes the number of sets of  $P_1, \dots, P_n$  prime equations such that  $f_1(P_1, \dots, P_n), \dots, f_k(P_1, \dots, P_n)$  are prime equations. If  $J_{n+1}(\omega) = 0$  then (5) has finite prime solutions. If  $J_{n+1}(\omega) \neq 0$  using  $\chi(P)$  we sift out from (2) prime equations which can not be represented  $P_1, \dots, P_n$ , then residual prime equations of (2) are  $P_1, \dots, P_n$  prime equations such that  $f_1(P_1, \dots, P_n), \dots, f_k(P_1, \dots, P_n)$  are prime equations. Therefore we prove that there exist infinitely many primes  $P_1, \dots, P_n$  such that  $f_1(P_1, \dots, P_n), \dots, f_k(P_1, \dots, P_n)$  are primes.

Secondly, we have the best asymptotic formula [2,3,4,6]

$$\begin{aligned} \pi_{k+1}(N, n+1) &= |\{P_1, \dots, P_n \leq N : f_1, \dots, f_k \text{ are } k \text{ primes}\}| \\ &= \prod_{i=1}^k (\deg f_i)^{-1} \times \frac{J_{n+1}(\omega) \omega^k}{n! \phi^{k+n}(\omega) \log^{k+n} N} (1 + o(1)). \end{aligned} \tag{8}$$

(8) is called a unite prime formula in prime distribution. Let  $n = 1, k = 0$ ,  $J_2(\omega) = \phi(\omega)$ . From (8) we have prime number theorem

$$\pi_1(N, 2) = |\{P_1 \leq N : P_1 \text{ is prime}\}| = \frac{N}{\log N} (1 + o(1)). \tag{9}$$

Number theorists believe that there are infinitely many twin primes, but they do not have rigorous proof of this old conjecture by any method. All the prime theorems are conjectures except the prime number theorem, because they do not prove that prime equations have infinitely many prime solutions. We prove the following conjectures by this theorem.

**Example 1.** Twin primes  $P, P + 2$  (300BC).

From (6) and (7) we have Jiang's function



$$J_2(\omega) = \prod_{3 \leq P} (P-2) \neq 0$$

Since  $J_2(\omega) \neq 0$  in (2) exist infinitely many  $P$  prime equations such that  $P+2$  is a prime equation. Therefore we prove that there are infinitely many primes  $P$  such that  $P+2$  is a prime.

Let  $\omega = 30$  and  $J_2(30) = 3$ . From (4) we have three  $P$  prime equations

$$P_3 = 30n + 11, \quad P_5 = 30n + 17, \quad P_8 = 30n + 29$$

From (8) we have the best asymptotic formula

$$\begin{aligned} \pi_2(N, 2) &= \left| \{P \leq N : P+2 \text{ prime}\} \right| = \frac{J_2(\omega)\omega}{\phi^2(\omega)} \frac{N}{\log^2 N} (1+o(1)) \\ &= 2 \prod_{3 \leq P} \left( 1 - \frac{1}{(P-1)^2} \right) \frac{N}{\log^2 N} (1+o(1)). \end{aligned}$$

In 1996 we proved twin primes conjecture [1]

Remark.  $J_2(\omega)$  denotes the number of  $P$  prime equations,  $\frac{\omega}{\phi^2(\omega)} \frac{N}{\log^2 N} (1+o(1))$  the number of solutions of primes for every  $P$  prime equation.

**Example 2.** Even Goldbach's conjecture  $N = P_1 + P_2$ . Every even number  $N \geq 6$  is the sum of two primes. From (6) and (7) we have Jiang's function

$$J_2(\omega) = \prod_{3 \leq P} (P-2) \prod_{P|N} \frac{P-1}{P-2} \neq 0$$

Since  $J_2(\omega) \neq 0$  as  $N \rightarrow \infty$  in (2) exist infinitely many  $P_1$  prime equations such that  $N - P_1$  is a prime equation. Therefore we prove that every even number  $N \geq 6$  is the sum of two primes.

From (8) we have the best asymptotic formula

$$\begin{aligned} \pi_2(N, 2) &= \left| \{P_1 \leq N, N - P_1 \text{ prime}\} \right| = \frac{J_2(\omega)\omega}{\phi^2(\omega)} \frac{N}{\log^2 N} (1+o(1)). \\ &= 2 \prod_{3 \leq P} \left( 1 - \frac{1}{(P-1)^2} \right) \prod_{P|N} \frac{P-1}{P-2} \frac{N}{\log^2 N} (1+o(1)) \end{aligned}$$

In 1996 we proved even Goldbach's conjecture [1]

**Example 3.** Prime equations  $P, P+2, P+6$ .

From (6) and (7) we have Jiang's function

$$J_2(\omega) = \prod_{5 \leq P} (P-3) \neq 0$$

$J_2(\omega)$  is denotes the number of  $P$  prime equations such that  $P+2$  and  $P+6$  are prime equations. Since  $J_2(\omega) \neq 0$  in (2) exist infinitely many  $P$  prime equations such that  $P+2$  and  $P+6$  are prime equations.

Therefore we prove that there are infinitely many primes  $P$  such that  $P+2$  and  $P+6$  are primes.

Let  $\omega = 30$ ,  $J_2(30) = 2$ . From (4) we have two  $P$  prime equations

$$P_3 = 30n + 11, \quad P_5 = 30n + 17$$

From (8) we have the best asymptotic formula

$$\pi_3(N, 2) = \left| \{P \leq N : P+2, P+6 \text{ are primes}\} \right| = \frac{J_2(\omega)\omega^2}{\phi^3(\omega)} \frac{N}{\log^3 N} (1+o(1)).$$

**Example 4.** Odd Goldbach's conjecture  $N = P_1 + P_2 + P_3$ . Every odd number  $N \geq 9$  is the sum of three primes. From (6) and (7) we have Jiang's function

$$J_3(\omega) = \prod_{3 \leq P} (P^2 - 3P + 3) \prod_{P|N} \left(1 - \frac{1}{P^2 - 3P + 3}\right) \neq 0$$

Since  $J_3(\omega) \neq 0$  as  $N \rightarrow \infty$  in (2) exist infinitely many pairs of  $P_1$  and  $P_2$  prime equations such that  $N - P_1 - P_2$  is a prime equation. Therefore we prove that every odd number  $N \geq 9$  is the sum of three primes. From (8) we have the best asymptotic formula

$$\begin{aligned} \pi_2(N, 3) &= |\{P_1, P_2 \leq N : N - P_1 - P_2 \text{ prime}\}| = \frac{J_3(\omega)\omega}{2\phi^3(\omega)} \frac{N^2}{\log^3 N} (1 + o(1)) \\ &= \prod_{3 \leq P} \left(1 + \frac{1}{(P-1)^3}\right) \prod_{P|N} \left(1 - \frac{1}{P^3 - 3P + 3}\right) \frac{N^2}{\log^3 N} (1 + o(1)) \end{aligned}$$

**Example 5.** Prime equation  $P_3 = P_1 P_2 + 2$ . From (6) and (7) we have Jiang's function

$$J_3(\omega) = \prod_{3 \leq P} (P^2 - 3P + 2) \neq 0$$

$J_3(\omega)$  denotes the number of pairs of  $P_1$  and  $P_2$  prime equations such that  $P_3$  is a prime equation. Since  $J_3(\omega) \neq 0$  in (2) exist infinitely many pairs of  $P_1$  and  $P_2$  prime equations such that  $P_3$  is a prime equation.

Therefore we prove that there are infinitely many pairs of primes  $P_1$  and  $P_2$  such that  $P_3$  is a prime. From (8) we have the best asymptotic formula

$$\pi_2(N, 3) = |\{P_1, P_2 \leq N : P_1 P_2 + 2 \text{ prime}\}| = \frac{J_3(\omega)\omega}{4\phi^3(\omega)} \frac{N^2}{\log^3 N} (1 + o(1)).$$

Note.  $\deg(P_1 P_2) = 2$ .

**Example 6** [12]. Prime equation  $P_3 = P_1^3 + 2P_2^3$ . From (6) and (7) we have Jiang's function

$$J_3(\omega) = \prod_{3 \leq P} [(P-1)^2 - \chi(P)] \neq 0$$

where  $\chi(P) = 3(P-1)$  if  $2^{\frac{P-1}{3}} \equiv 1 \pmod{P}$ ;  $\chi(P) = 0$  if  $2^{\frac{P-1}{3}} \not\equiv 1 \pmod{P}$ ;  $\chi(P) = P-1$  otherwise.

Since  $J_3(\omega) \neq 0$  in (2) there are infinitely many pairs of  $P_1$  and  $P_2$  prime equations such that  $P_3$  is a prime equation. Therefore we prove that there are infinitely many pairs of primes  $P_1$  and  $P_2$  such that  $P_3$  is a prime. From (8) we have the best asymptotic formula

$$\pi_2(N, 3) = |\{P_1, P_2 \leq N : P_1^3 + 2P_2^3 \text{ prime}\}| = \frac{J_3(\omega)\omega}{6\phi^3(\omega)} \frac{N^2}{\log^3 N} (1 + o(1)).$$

**Example 7** [13]. Prime equation  $P_3 = P_1^4 + (P_2 + 1)^2$ . From (6) and (7) we have Jiang's function

$$J_3(\omega) = \prod_{3 \leq P} [(P-1)^2 - \chi(P)] \neq 0$$

where  $\chi(P) = 2(P-1)$  if  $P \equiv 1 \pmod{4}$ ;  $\chi(P) = 2(P-3)$  if  $P \equiv 1 \pmod{8}$ ;  $\chi(P) = 0$  otherwise.

Since  $J_3(\omega) \neq 0$  in (2) there are infinitely many pairs of  $P_1$  and  $P_2$  prime equations such that  $P_3$  is a prime

equation. Therefore we prove that there are infinitely many pairs of primes  $P_1$  and  $P_2$  such that  $P_3$  is a prime. From (8) we have the best asymptotic formula

$$\pi_2(N, 3) = \left| \{P_1, P_2 \leq N : P_3 \text{ prime}\} \right| = \frac{J_3(\omega)\omega}{8\phi^3(\omega)} \frac{N^2}{\log^3 N} (1 + o(1)).$$

**Example 8** [14-20]. Arithmetic progressions consisting only of primes. We define the arithmetic progressions of length  $k$ .

$$P_1, P_2 = P_1 + d, P_3 = P_1 + 2d, \dots, P_k = P_1 + (k-1)d, (P_1, d) = 1 \tag{10}$$

From (8) we have the best asymptotic formula

$$\begin{aligned} \pi_2(N, 2) &= \left| \{P_1 \leq N : P_1, P_1 + d, \dots, P_1 + (k-1)d \text{ are primes}\} \right| \\ &= \frac{J_2(\omega)\omega^{k-1}}{\phi^k(\omega)} \frac{N}{\log^k N} (1 + o(1)). \end{aligned}$$

If  $J_2(\omega) = 0$  then (10) has finite prime solutions. If  $J_2(\omega) \neq 0$  then there are infinitely many primes  $P_1$  such that  $P_2, \dots, P_k$  are primes.

To eliminate  $d$  from (10) we have

$$P_3 = 2P_2 - P_1, \quad P_j = (j-1)P_2 - (j-2)P_1, \quad 3 \leq j \leq k$$

From (6) and (7) we have Jiang's function

$$J_3(\omega) = \prod_{3 \leq P < k} (P-1) \prod_{k \leq P} (P-1)(P-k+1) \neq 0$$

Since  $J_3(\omega) \neq 0$  in (2) there are infinitely many pairs of  $P_1$  and  $P_2$  prime equations such that  $P_3, \dots, P_k$  are prime equations. Therefore we prove that there are infinitely many pairs of primes  $P_1$  and  $P_2$  such that  $P_3, \dots, P_k$  are primes.

From (8) we have the best asymptotic formula

$$\begin{aligned} \pi_{k-1}(N, 3) &= \left| \{P_1, P_2 \leq N : (j-1)P_2 - (j-2)P_1 \text{ prime}, 3 \leq j \leq k\} \right| \\ &= \frac{J_3(\omega)\omega^{k-2}}{2\phi^k(\omega)} \frac{N^2}{\log^k N} (1 + o(1)) \\ &= \frac{1}{2} \prod_{2 \leq P < k} \frac{P^{k-2}}{(P-1)^{k-1}} \prod_{k \leq P} \frac{P^{k-2}(P-k+1)}{(P-1)^{k-1}} \frac{N^2}{\log^k N} (1 + o(1)) \end{aligned}$$

**Example 9.** It is a well-known conjecture that one of  $P, P+2, P+2^2$  is always divisible by 3. To generalize above to the  $k$ -primes, we prove the following conjectures. Let  $n$  be a square-free even number.

1.  $P, P+n, P+n^2$ ,

where  $3|(n+1)$ .

From (6) and (7) we have  $J_2(3) = 0$ , hence one of  $P, P+n, P+n^2$  is always divisible by 3.

2.  $P, P+n, P+n^2, \dots, P+n^4$ ,

where  $5|(n+b), b=2, 3$ .

From (6) and (7) we have  $J_2(5) = 0$ , hence one of  $P, P+n, P+n^2, \dots, P+n^4$  is always divisible by 5.

3.  $P, P+n, P+n^2, \dots, P+n^6$ ,

where  $7|(n+b), b=2,4$ .

From (6) and (7) we have  $J_2(7)=0$ , hence one of  $P, P+n, P+n^2, \dots, P+n^6$  is always divisible by 7.

4.  $P, P+n, P+n^2, \dots, P+n^{10}$ ,

where  $11|(n+b), b=3,4,5,9$ .

From (6) and (7) we have  $J_2(11)=0$ , hence one of  $P, P+n, P+n^2, \dots, P+n^{10}$  is always divisible by 11.

5.  $P, P+n, P+n^2, \dots, P+n^{12}$ ,

where  $13|(n+b), b=2,6,7,11$ .

From (6) and (7) we have  $J_2(13)=0$ , hence one of  $P, P+n, P+n^2, \dots, P+n^{12}$  is always divisible by 13.

6.  $P, P+n, P+n^2, \dots, P+n^{16}$ ,

where  $17|(n+b), b=3,5,6,7,10,11,12,14,15$ .

From (6) and (7) we have  $J_2(17)=0$ , hence one of  $P, P+n, P+n^2, \dots, P+n^{16}$  is always divisible by 17.

7.  $P, P+n, P+n^2, \dots, P+n^{18}$ ,

where  $19|(n+b), b=4,5,6,9,16,17$ .

From (6) and (7) we have  $J_2(19)=0$ , hence one of  $P, P+n, P+n^2, \dots, P+n^{18}$  is always divisible by 19.

**Example 10.** Let  $n$  be an even number.

1.  $P, P+n^i, i=1,3,5, \dots, 2k+1$ ,

From (6) and (7) we have  $J_2(\omega) \neq 0$ . Therefore we prove that there exist infinitely many primes  $P$  such that  $P, P+n^i$  are primes for any  $k$ .

2.  $P, P+n^i, i=2,4,6, \dots, 2k$ .

From (6) and (7) we have  $J_2(\omega) \neq 0$ . Therefore we prove that there exist infinitely many primes  $P$  such that  $P, P+n^i$  are primes for any  $k$ .

**Example 11.** Prime equation  $2P_2 = P_1 + P_3$

From (6) and (7) we have Jiang's function

$$J_3(\omega) = \prod_{3 \leq P} (P^2 - 3P + 2) \neq 0$$

Since  $J_3(\omega) \neq 0$  in (2) there are infinitely many pairs of  $P_1$  and  $P_2$  prime equations such that  $P_3$  is prime equations. Therefore we prove that there are infinitely many pairs of primes  $P_1$  and  $P_2$  such that  $P_3$  is a prime. From (8) we have the best asymptotic formula

$$\pi_2(N, 3) = |\{P_1, P_2 \leq N : P_3 \text{ prime}\}| = \frac{J_3(\omega)\omega}{2\phi^3(\omega)} \frac{N^2}{\log^3 N} (1 + o(1)).$$

In the same way we can prove  $2P_2^2 = P_3 + P_1$  which has the same Jiang's function.

Jiang's function is accurate sieve function. Using it we can prove any irreducible prime equations in prime distribution. There are infinitely many twin primes but we do not have rigorous proof of this old conjecture by any method [20]. As strong as the numerical evidence may be, we still do not even know whether there are infinitely many pairs of twin primes [21]. All the prime theorems are conjectures except the prime number theorem, because

they do not prove the simplest twin primes. They conjecture that the prime distribution is randomness [12-25], because they do not understand theory of prime numbers.

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Preprint (January 1994).

After Wiles was about to announce his proof of FLT to the world on June 23, 1993. Jiang wrote this paper.

Tepper Gill, Kexi Liu, and Eric Trell, Editors

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**Fermat Last Theorem was Proved in 1991**

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We found out a new method for proving Fermat last theorem (FLT) on the afternoon of October 25, 1991. We proved FLT at one stroke for all prime exponents  $p > 3$ , It led to the discovery to calculate  $n = 15, 21, 35, 105, \dots$ . To this date, no one disprove this proof. Anyone can not deny it, because it is a simple and marvelous proof. It can fit in the margin of Fermat book.

In 1974 we found out Euler formula of the cyclotomic real numbers in the cyclotomic fields [1].

$$\exp\left(\sum_{i=1}^{n-1} t_i J^i\right) = \sum_{i=1}^n S_i J^{i-1} \tag{1}$$

where  $J$  denotes a  $n$ -th root of unity,  $J^n = 1$ ,  $n$  is an odd number,  $t_i$  are the real numbers.

$S_i$  is called the complex hyperbolic functions of order  $n$  with  $n-1$  variables,

$$S_i = \frac{1}{n} \left[ e^A + 2 \sum_{j=1}^{\frac{n-1}{2}} (-1)^{(i-1)j} e^{B_j} \cos\left(\theta_j + (-1)^j \frac{(i-1)j\pi}{n}\right) \right] \tag{2}$$

where

$$A = \sum_{\alpha=1}^{n-1} t_\alpha, \quad B_j = \sum_{\alpha=1}^{n-1} t_\alpha (-1)^{\alpha j} \cos \frac{\alpha j \pi}{n}, \quad \theta_j = (-1)^{j+1} \sum_{\alpha=1}^{n-1} t_\alpha (-1)^{\alpha j} \sin \frac{\alpha j \pi}{n}$$

$$A + 2 \sum_{i=1}^{\frac{n-1}{2}} B_i = 0 \tag{3}$$

Using (1) the cyclotomic theory may extend to totally real number fields. It is called the hypercomplex variable theory [1]. (2) may be written in the matrix form

$$\begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ \dots \\ S_n \end{bmatrix} = \frac{1}{n} \begin{bmatrix} 1 & 1 & 0 & \dots & 0 \\ 1 & -\cos \frac{\pi}{n} & -\sin \frac{\pi}{n} & \dots & -\sin \frac{(n-1)\pi}{2n} \\ 1 & \cos \frac{2\pi}{n} & \sin \frac{2\pi}{n} & \dots & -\sin \frac{(n-1)\pi}{n} \\ \dots & \dots & \dots & \dots & \dots \\ 1 & \cos \frac{(n-1)\pi}{n} & \sin \frac{(n-1)\pi}{n} & \dots & -\sin \frac{(n-1)^2 \pi}{2n} \end{bmatrix} \begin{bmatrix} e^A \\ 2e^{B_1} \cos \theta_1 \\ 2e^{B_1} \sin \theta_1 \\ \dots \\ 2 \exp\left(\frac{B_{\frac{n-1}{2}}}{2}\right) \sin\left(\frac{\theta_{\frac{n-1}{2}}}{2}\right) \end{bmatrix} \tag{4}$$

where  $(n-1)/2$  is an even number.

From (4) we may obtain its inverse transformation

$$\begin{bmatrix} e^A \\ e^{B_1} \cos \theta_1 \\ e^{B_1} \sin \theta_1 \\ \dots \\ \exp\left(\frac{B_{n-1}}{2}\right) \sin\left(\frac{\theta_{n-1}}{2}\right) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ 1 & -\cos \frac{\pi}{n} & \cos \frac{2\pi}{n} & \dots & \cos \frac{(n-1)\pi}{n} \\ 0 & -\sin \frac{\pi}{n} & \sin \frac{2\pi}{n} & \dots & \sin \frac{(n-1)\pi}{n} \\ \dots & \dots & \dots & \dots & \dots \\ 0 & -\sin \frac{(n-1)\pi}{2n} & -\sin \frac{(n-1)\pi}{n} & \dots & -\sin \frac{(n-1)^2\pi}{2n} \end{bmatrix} \begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ \dots \\ S_n \end{bmatrix} \quad (5)$$

From (5) we have

$$\begin{aligned} e^A &= \sum_{i=1}^n S_i, e^{B_j} \cos \theta_j = S_1 + \sum_{i=1}^{n-1} S_{1+i} (-1)^{ij} \cos \frac{ij\pi}{n}, \\ e^{B_j} \sin \theta_j &= (-1)^{j+1} \sum_{i=1}^{n-1} S_{1+i} (-1)^{ij} \sin \frac{ij\pi}{n}. \end{aligned} \quad (6)$$

In (3) and (6)  $t_i$  and  $S_i$  have the same formulas such that every factor of  $n$  has a Fermat equation. Assume  $S_1 \neq 0, S_2 \neq 0, S_i = 0$  where  $i = 3, 4, \dots, n, S_i = 0$  are  $n-2$  indeterminate equations with  $n-1$  variables. From (6) we have

$$e^A = S_1 + S_2, e^{2B_j} = S_1^2 + S_2^2 + 2S_1S_2(-1)^j \cos \frac{j\pi}{n}. \quad (7)$$

From (3) and (7) we may obtain the Fermat equation

$$\exp\left(A + 2 \sum_{j=1}^{\frac{n-1}{2}} B_j\right) = (S_1 + S_2) \prod_{j=1}^{\frac{n-1}{2}} (S_1^2 + S_2^2 + 2S_1S_2(-1)^j \cos \frac{j\pi}{n}) = S_1^n + S_2^n = 1. \quad (8)$$

**Theorem.** Fermat last theorem has no rational solutions with  $S_1S_2 \neq 0$  for all odd exponents.

**Proof.** The proof of FLT is difficult when  $n$  is an odd prime. We consider  $n$  is a composite number.

Let  $n = \prod n_i$ , where  $n_i$  ranges over all odd number. From (3) we have

$$\exp\left(A + 2 \sum_{j=1}^{\frac{f-1}{2}} B_{\frac{n}{f^j}}\right) = \left[\exp\left(\sum_{\alpha=1}^{\frac{n-1}{f}} t_{f\alpha}\right)\right]^f \quad (9)$$

From (7) we have

$$\exp\left(A + 2 \sum_{j=1}^{\frac{f-1}{2}} B_{\frac{n}{f^j}}\right) = S_1^f + S_2^f \quad (10)$$

where  $f$  is a factor of  $n$ . From (9) and (10) we may obtain Fermat equation

$$\exp\left(A + 2 \sum_{j=1}^{\frac{f-1}{2}} B_{\frac{n}{f^j}}\right) = S_1^f + S_2^f = \left[\exp\left(\sum_{\alpha=1}^{\frac{n-1}{f}} t_{f\alpha}\right)\right]^f \quad (11)$$

Every factor of  $n$  has a Fermat equation. From (11) we have

$$f = 1, B_n = B_0 = 0, \quad e^A = S_1 + S_2 = \exp\left(\sum_{\alpha=1}^{n-1} t_\alpha\right) \quad (12)$$

$$f = n, t_n = t_0 = 0, \quad \exp\left(A + 2 \sum_{i=1}^{n-1} B_j\right) = S_1^n + S_2^n = 1 \quad (13)$$

$$f = 3, \exp\left(A + 2B_{\frac{n}{3}}\right) = S_1^3 + S_2^3 = \left[\exp\left(\sum_{\alpha=1}^{\frac{n-1}{3}} t_{3\alpha}\right)\right]^3 \quad (14)$$

If  $S_1 = 1, S_2 = 0$  and  $S_1 = 0, S_2 = 1$ , then  $A = B_j = 0$ . Euler proved (13), therefore (11) has no rational solutions with  $S_1 S_2 \neq 0$  (and so no integer solutions with  $S_1 S_2 \neq 0$ ) for all odd exponents  $f$ . (11) and (13) can fit in the margin of Fermat book.

Let  $n = 3p$  where  $p$  is an odd prime. From (3) and (7) we may derive Fermat equations

$$\exp\left(A + 2 \sum_{i=1}^{\frac{3p-1}{2}} B_j\right) = S_1^{3p} + S_2^{3p} = (S_1^p)^3 + (S_2^p)^3 = 1 \quad (15)$$

$$\exp(A + 2B_p) = S_1^3 + S_2^3 = \left[\exp\left(\sum_{\alpha=1}^{p-1} t_{3\alpha}\right)\right]^3 \quad (16)$$

$$\exp\left(A + 2 \sum_{i=1}^{\frac{p-1}{2}} B_{3j}\right) = S_1^p + S_2^p = \left[\exp(t_p + t_{2p})\right]^p \quad (17)$$

Euler proved (15) and (16), therefore (17) have no rational solutions with  $S_1 S_2 \neq 0$  (and so no integer solutions with  $S_1 S_2 \neq 0$ ) for any odd prime  $p > 3$ . (15)-(17) can fit in the margin

Let  $n = 5p$  where  $p$  is an odd prime. From (3) and (7) we may derive Fermat equations

$$\exp\left(A + 2 \sum_{j=1}^{\frac{5p-1}{2}} B_j\right) = S_1^{5p} + S_2^{5p} = 1 \quad (18)$$

$$\exp(A + 2B_p + 2B_{2p}) = S_1^5 + S_2^5 = \left[\exp\left(\sum_{\alpha=1}^{p-1} t_{5\alpha}\right)\right]^5 \quad (19)$$

$$\exp\left(A + 2 \sum_{j=1}^{\frac{p-1}{2}} B_{5j}\right) = S_1^p + S_2^p = \left[\exp\left(\sum_{\alpha=1}^4 t_{p\alpha}\right)\right]^p \quad (20)$$

(18)-(20) can fit in the margin.

Let  $n = 7p$  where  $p$  is an odd prime. From (3) and (7) we may derive Fermat equations

$$\exp\left(A + 2 \sum_{i=1}^{\frac{7p-1}{2}} B_j\right) = S_1^{7p} + S_2^{7p} = 1 \quad (21)$$

$$\exp(A + 2B_p + 2B_{2p} + 2B_{3p}) = S_1^7 + S_2^7 = \left[\exp\left(\sum_{\alpha=1}^{p-1} t_{7\alpha}\right)\right]^7 \quad (22)$$

$$\exp(A + 2 \sum_{i=1}^{\frac{p-1}{2}} B_{7j}) = S_1^p + S_2^p = [\exp \sum_{\alpha=1}^6 t_{p\alpha}]^p \quad (23)$$

(21)-(23) can also fit in the margin.

Using this method we proved FLT in 1991 [2-5].

Let  $n = p$  where  $p$  is an odd prime. From (3) and (7) we have

$$\exp(A + 2 \sum_{i=1}^{\frac{p-1}{2}} B_j) = S_1^p + S_2^p = 1, e^{2B_1} = S_1^2 + S_2^2 - 2S_1S_2 \cos \frac{\pi}{p} \quad (24)$$

Let  $a = S_1 e^{-B_1}$  and  $b = S_2 e^{-B_1}$  From (24) we have

$$a^p + b^p = (e^{-B_1})^p \quad (25)$$

$$a^2 + b^2 - 2ab \cos \frac{\pi}{p} = 1 \quad (26)$$

The proof of (25) is transformed into studying (26). (26) has no rational solutions with  $ab \neq 0$ ,

because  $\cos \frac{\pi}{p}$  is an irrational number for  $p > 3$ . Therefore (25) has no rational solutions for any odd prime  $p > 3$ . (25) and (26) can also fit in the margin.

**Remark.** If  $S_i \neq 0$ , where  $i = 1, 2, 3, \dots, n$ , then (11)-(23) have infinitely many rational solutions [1].

#### Note:

Let one knew the important results, we gave out about 600 preprints in 1991-1992. There were my preprints in Princeton, Harvard, Berkeley, MIT, Uchicago, Columbia, Maryland, Ohio, Wisconsin, Yale, ... .., England, Canada, Japan, Poland, Germany, France, Finland, ... .., Ann. Math., Mathematika, J. Number Theory, Glasgow Math. J., London Math. Soc., In. J. Math. Math. Sci., Acta Arith., Can. Math. Bull. (They refused the publicaitons of my papers). Both papers were published in Chinese. FLT is as simple as Pythagorean theorem. This proof can fit in the margin of Fermat book. We think the game is up. We sent dept of math (Princeton University) a preprint on Jan. 15, 1992. Wiles claims the second proof of FLT in England (not in U. S. A.) after two years. We wish Wiles and his supporters disprove my proof, otherwise Wiles work is only the second and complex proof of FLT. We believe that the Princeton is the fairest University and history will pass the fairest judgment on proofs of FLT and other problems. We are waiting for word from the experts who are studying this paper.

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