Evaluation of Empirical Formulae for Determination of Hydraulic Conductivity based on Grain-Size Analysis

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Abstract: Several empirical equations to calculate hydraulic conductivity using grain size distribution of unconsolidated aquifer materials have been evaluated in this study. Grading analysis of soil samples extracted from test holes during groundwater investigation project was performed to determine their classification and particle size distribution characteristics; from which hydraulic conductivities were computed. Results showed that all the seven empirical formulae reliably estimated hydraulic conductivities of the various soil samples well within the known ranges. Kozeny-Carman formula proved to be the best estimator of most samples analyzed, and may be, even for a wide range of other soil types. However, some of the formulae underestimated or overestimated hydraulic conductivity; even of the same soils. Alyamani and Sen formula in particular is very sensitive to the shape of the grading curve, hence the need to be very careful when using. Most importantly, all these empirical formulae are to be used strictly within their domains of applicability. [The Journal of American Science. 2007;3(3):54-60]. (ISSN: 1545-1003).

Keywords: Hydraulic Conductivity, Empirical formula, Grain-size analysis.

Introduction

It has long been recognized that hydraulic conductivity is related to the grain-size distribution of granular porous media (Freeze and Cherry 1979). This interrelationship is very useful for the estimation of conductivity values where direct permeability data are sparse such as in the early stages of aquifer exploration .In groundwater hydrology, the knowledge of saturated hydraulic conductivity of soil is necessary for modeling the water flow in the soil, both in the saturated and unsaturated zone, and transportation of water-soluble pollutants in the soil. It also an important parameter for designing of the drainage of an area and in construction of earth dam and levee. Furthermore, it is of paramount importance in relation to some geotechnical problems, including the determination of seepage losses, settlement computations, and stability analyses (Boadu 2000). Above all, hydrogeologists always look for reliable techniques to determine the hydraulic conductivity of the aquifers with which they are concerned, for better groundwater development, management and conservation. Many different techniques have been proposed to determine its value, including field methods (pumping test of wells, auger hole test and tracer test), laboratory methods and calculations from empirical formulae(Todd and Mays 2005). However, accurate estimation of hydraulic conductivity in the field environment by the field methods is limited by the lack of precise knowledge of aquifer geometry and hydraulic boundaries (Uma et al. 1989). The cost of field operations and associated wells constructions can be prohibitive as well. Laboratory tests on the other hands, presents formidable problems in the sense of obtaining representative samples and, very often, long testing times. Alternatively, methods of estimating hydraulic conductivity from empirical formulae based on grain-size distribution characteristics have been developed and used to overcome these problems. Grain-size methods are comparably less expensive and do not depend on the geometry and hydraulic boundaries of the aquifer. Most importantly, since information about the textural properties of soils or rock is more easily obtained, a potential alternative for estimating hydraulic conductivity of soils is from grain-size distribution. Although in hydromechanics, it would be more useful to characterize the diameters of pores rather than those of the grains, the pore size distribution is very difficult to determine, so that approximation of hydraulic properties are mostly based on the easy-to-measure grain size distribution as a substitute(Cirpka 2003). Consequently, Groundwater professionals have tried for decades to relate hydraulic conductivity to grain size. The tasks appear rather straight forward but it found that this correlation is not easily established (Pinder and Celia 2006).

Numerous investigators have studied this relationship and several formulae have resulted based on experimental work. Kozeny (1927) proposed a formula which was then modified by Carman (1937, 1956) to become the Kozeny-Carman equation. Other attempts were made by Hazen (1892), Shepherd (1989), Alyamani and Sen (1993), Terzaghi and Peck (1964). The applicability of these formulae depends on the type of soil for which hydraulic conductivity is to be estimated. Moreover, few formulas give reliable estimates of results because of the difficulty of including all possible variables in porous media. Vukovic and Soro (1992) noted that the applications of different empirical formulae to the same porous medium material can yield different values of hydraulic conductivity, which may differ by a factor of 10 or even 20. The objective of this paper therefore, is to evaluate the applicability and reliability of some of the commonly used empirical formulae for the determination of hydraulic conductivity of unconsolidated soil/rock materials.

Established Empirical Formulae

Hydraulic conductivity (K) can be estimated by particle size analysis of the sediment of interest, using empirical equations relating either K to some size property of the sediment. Vukovic and Soro (1992) summarized several empirical methods from former studies and presented a general formula:

$$K = \frac{g}{v} \cdot C \cdot f(n) \cdot d_e^2 \qquad (1)$$

where K = hydraulic conductivity; g = acceleration due to gravity; v = kinematic viscosity ; C = sorting coefficient; $f(n)$ = porosity function, and d_e = effective grain diameter. The kinematic viscosity (v) is related to dynamic viscosity (μ) and the fluid (water) density (ρ) as follows:

$$v = \frac{\mu}{\rho} \qquad (2)$$

The values of C, $f(n)$ and de are dependent on the different methods used in the grain-size analysis. According to Vukovic and Soro (1992), porosity (n) may be derived from the empirical relationship with the coefficient of grain uniformity (U) as follows:

$$n = 0.255\left(1 + 0.83^U\right) \qquad (3)$$

where U is the coefficient of grain uniformity and is given by:

$$U = \left(\frac{d_{60}}{d_{10}}\right) \qquad (4)$$

Here, d_{60} and d_{10} in the formula represent the grain diameter in (mm) for which, 60% and 10% of the sample respectively, are finer than.
Former studies have presented the following formulae which take the general form presented in equation (1) above but with varying ***C, f(n)*** and ***de*** values and their domains of applicability.

Hazen: $K = \frac{g}{v} \times 6 \times 10^{-4}\left[1 + 10(n - 0.26)\right]d_{10}^2 \qquad (5)$

Hazen formula was originally developed for determination of hydraulic conductivity of uniformly graded sand but is also useful for fine sand to gravel range, provided the sediment has a uniformity coefficient less than 5 and effective grain size between 0.1 and 3mm.

Kozeny-Carman: $K = \frac{g}{v} \times 8.3 \times 10^{-3}\left[\frac{n^3}{(1-n)^2}\right]d_{10}^2 \qquad (6)$

The Kozeny-Carman equation is one of the most widely accepted and used derivations of permeability as a function of the characteristics of the soil medium. This equation was originally proposed by Kozeny (1927) and was then modified by Carman (1937, 1956) to become the Kozeny-Carman equation .It is not appropriate for either soil with effective size above 3mm or for clayey soils (Carrier 2003).

Breyer: $$K = \frac{g}{v} \times 6 \times 10^{-4} \log \frac{500}{U} d_{10}^2 \qquad (7)$$

This method does not consider porosity and therefore, porosity function takes on value 1. Breyer formula is often considered most useful for materials with heterogeneous distributions and poorly sorted grains with uniformity coefficient between 1 and 20, and effective grain size between 0.06mm and 0.6mm.

Slitcher: $$K = \frac{g}{v} \times 1 \times 10^{-2} n^{3.287} d_{10}^2 \qquad (8)$$

This formula is most applicable for grain-size between 0.01mm and 5mm.

Terzaghi: $$K = \frac{g}{v} \cdot C_t \cdot \left(\frac{n - 0.13}{\sqrt[3]{1-n}} \right)^2 d_{10}^2 \qquad (9)$$

where the C_t = sorting coefficient and $6.1\times10^{-3} < C_t < 10.7\times10^{-3}$. In this study, an average value of C_t is used. Terzaghi formula is most applicable for large-grain sand (Cheng and Chen 2007.)

USBR: $$K = \frac{g}{v} \times 4.8 \times 10^{-4} d_{20}^{0.3} \times d_{20}^2 \qquad (10)$$

U.S. Bureau of Reclamation (USBR) formula calculates hydraulic conductivity from the effective grain size (d_{20}), and does not depend on porosity; hence porosity function is a unity. The formula is most suitable for medium-grain sand with uniformity coefficient less than 5 (Cheng and Chen 2007)

Alyamani & Sen: $$K = 1300\left[I_o + 0.025\left(d_{50} - d_{10}\right)\right]^2 \qquad (11)$$

where K is the hydraulic conductivity (m/day), I_o is the intercept (in mm) of the line formed by d_{50} and d_{10} with the grain-size axis, d_{10} is the effective grain diameter (mm), and d_{50} is the median grain diameter (mm). It should be noted that the terms in the formula above bear the stated units for consistency. This formula therefore, is exceptionally different from those that take the general form of equation (1) above. It is however, one of the well known equations that also depends on grain-size analysis. The method considers both sediment grain sizes $\mathbf{d_{10}}$ and $\mathbf{d_{50}}$ as well as the sorting characteristics

Materials and Methods

Samples Test: Four different soils samples were extracted from test holes during an ongoing borehole drilling project aimed at establishing the geological profile of an aquifer system. Samples from the cuttings were collected in containers and taken to the laboratory for further analysis. From the laboratory, the samples were treated and tested for grain size distribution according to the standard procedures of BS1377. The samples (1&3) with coarser particles were tested by the method of dry sieve analysis using a series of sorted BS sieves. The finer samples (2&4) on the other hand, were tested by Hydrometer method.

Grain-size Distribution Analysis: Table1 below shows the results of the particle size distribution analyses of the four soil samples studied. To further analyze the distribution of the particles and to help classify the samples, the test results were then plotted on a semi-logarithmic graph to obtain the grain-size distribution curves for each sample as shown in figure1 below. From the grain-size distribution curves, soil samples were classified according to particle size using a standard *British Soil Classification System*, detailed in *BS 5930: Site Investigation*. In this system, soils are classified into named basic soil-type groups according to size, and the groups further divided into coarse, medium and fine sub-groups. The classifications based on the grain-size distribution curves were as follows:

Sample1 - comprised 4% medium gravel, 19%fine gravel, 32% coarse sand, 28% medium sand, 17% fine sand and is therefore classified as *gravelly sand*.

Sample2 - comprised 4% coarse sand, 82% medium sand, 14% fine sand; and classified as *medium sand*

Sample3 - comprised 13% fine gravel, , 37% coarse sand, 43% medium sand, 02% fine sand; with overall classification as *coarse sand*.

Sample4 - comprised 02% coarse sand, 76% medium sand, 22% fine sand and is classified as *medium sand*.

Determination of K- values from Grain-Size Analysis: From the grain-size distribution curves in figure1 below, the samples were classified, diameters of soil particles at 10%, 20% and 50% cumulative weight determined, and the coefficients of uniformity, intercepts and porosity values were calculated. All these results, from which hydraulic conductivities were calculated using the seven empirical formulae discussed above, are presented in table2 below. Since the kinematic coefficient of viscosity is also necessary for the estimation of hydraulic conductivity, a value of $0.0874 m^2/day$ derived for a water temperature of 20^oC is used in this study.

Results of Different Approaches

The hydraulic conductivity for gravelly sand is not available for Hazen and USBR methods since $U > 5$, a condition for which both methods are inapplicable. Hydraulic conductivities for medium sand samples are not available for Terzaghi method because the formula is only suitable for large-grain sand. On the other hand, conductivity value for coarse sand is not available for USBR since the method is only relevant for medium-grain sand.

Overall results showed that the hydraulic conductivities calculated by the USBR and Slitcher methods are in all cases lower than for the other methods, which is consistent with the conclusions by (Vukovic and Soro 1992) and (Cheng and Chen 2007). These two methods are always considered inaccurate. Likewise, Terzaghi method gave similar low values, may be due to the use of an average value ($8.4x10^{-3}$) of sorting coefficient(C) in the formula. Breyer method is most useful for analyzing heterogeneous sample with poorly sorted (well-graded) grains (Pinder and Celia 2006). It was therefore the best estimator for sample1 and a good one too, for sample3. However, for less heterogeneous (poorly graded) samples (2&4), the method underestimated the hydraulic conductivities values. Hazen formula which is based only on the d_{10} particle size is less accurate than the Kozeny- Carman formula which is based on the entire particle size distribution and particle shape (Carrier 2003). Therefore, the estimations by Kozeny-Carman for samples (2, 3, &4) were more accurate than hazen, and possibly the best estimations in this study and others. Kozeny-Carman however, underestimated sample1 since the formula is not appropriate if the particle distribution has a long, flat tail in the fine fraction (Carrier 2003). Alyamani and Sen method is very sensitive to the shape of the grading curve and is more accurate for well-graded sample. Consequently, it was a fairly good estimator of samples (1&3) but underestimated samples (2&4) due to their poor grading.

Conclusion

Based on the aforementioned analysis and results, the following conclusions can be drawn:

a) Estimating the hydraulic conductivity of soils in terms of grading characteristics can relatively lead to underestimation or overestimation unless the appropriate method is used.

b) For the studied samples, and consequently may be for a wide range of soil type, the best overall estimation of permeability is reached based on Kozeny-Carman's formula followed by Hazen formula. However, Breyer formula is the best for estimation of highly heterogeneous soil sample.

c) Slitcher, USBR and Terzaghi formulae grossly underestimated the hydraulic conductivities in comparison to the other evaluated formulae.

d) Alyamani and Sen formula is very sensitive to shape of the grading curve and as such should be used with care.

e) Therefore, the most suitable formulae for the estimation of hydraulic conductivities in this study were as follows:

f)

- Sample1(Breyer formula) = 114.009m/day,
- Sample2 (Kozeny-Carman) = 56.882m/day,
- Sample3(Kozeny-Carman) =112.495m/day; with Hazen and Breyer formulae acceptable
- Sample4(Kozeny-Carman) = 45.591m/day

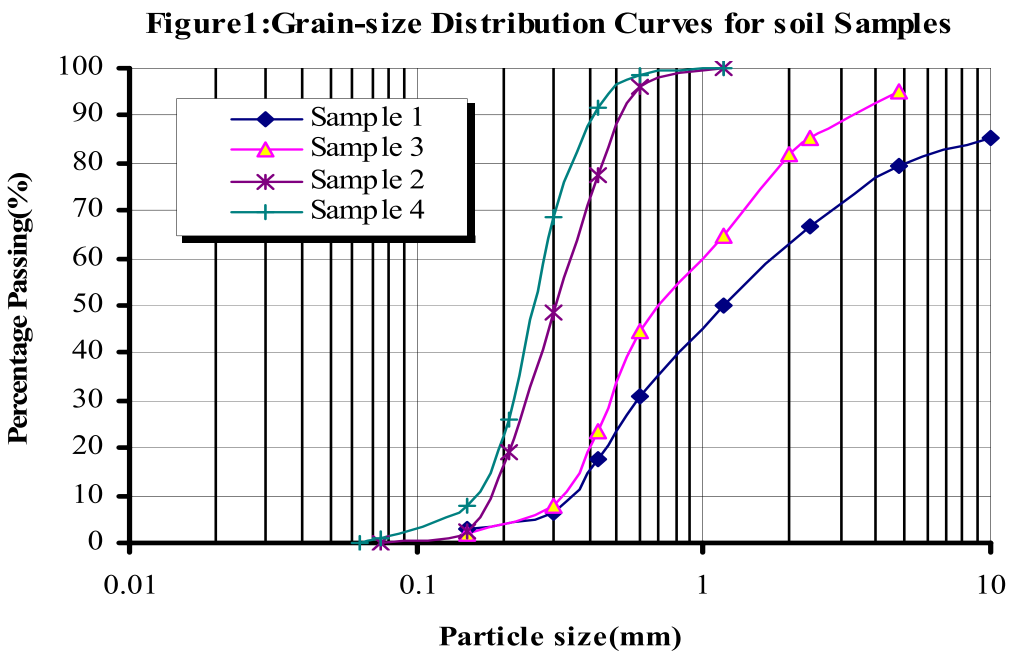
Table1: summary results of soil particle size distribution tests

Sample-1								
Particle size (mm)	10	4.75	2.36	1.18	0.600	0.425	0.300	0.150
Percentage Passing (%)	85.23	79.57	66.73	49.99	30.75	17.55	6.28	2.84
Sample-2								
Particle size (mm)	1.18	0.600	0.425	0.300	0.212	0.150	0.075	-
Percentage Passing (%)	100	96.20	77.65	48.41	18.88	2.30	0.00	-
Sample-3								
Particle size (mm)	4.75	2.36	2.00	1.18	0.600	0.425	0.300	0.150
Percentage Passing (%)	95.05	85.08	82.02	64.66	44.76	23.42	7.96	2.11
Sample-4								
Particle size (mm)	1.18	0.600	0.425	0.300	0.212	0.150	0.075	0.063
Percentage Passing (%)	100	98.44	91.55	68.78	25.83	7.89	0.93	0.00

Table2: Hydraulic conductivities calculated from grain-size analysis using empirical formulae

Sample & its classification	d_{10} (mm)	d_{20} (mm)	d_{50} (mm)	(U)	(n)	(I_o) (mm)	Hazen (m/day)	K-C (m/day)	Breyer (m/day)	Slitcher (m/day)	Terzaghi (m/day)	USBR (m/day)	A/S (m/day)
1-Gravelly sand	0.339	0.468	1.180	5.309	0.349	0.249	NA	80.139	114.009	30.249	51.630	NA	94.788
2-Medium sand	0.180	0.220	0.330	1.917	0.433	0.157	44.454	56.882	39.347	17.327	NA	12.356	33.593
3-Coarse sand	0.310	0.400	0.720	3.226	0.395	0.254	113.500	112.495	105.787	38.001	66.381	NA	90.776
4-Medium sand	0.157	0.189	0.258	1.783	0.438	0.139	34.439	45.591	30.324	13.689	NA	8.713	26.038

Key: K-C = Kozeny-Carman; A/S = Alyamani & Sen



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