# Fuzzy Variable Model for Lake Eutrophication Evaluation and Sustainable Development Countermeasure

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**Abstract:** Considering that lake eutrophication evaluation involves many indexes, many classifications and interval values, fuzzy variable sets theory is applied. Making reference to eutrophication standard of Chinese lakes, Eutrophication degree of lake is divided into 6 classifications. Evaluation indexes include Chl-a,TP,TN and $COD_{Mn}$. According to data measured, index feature value matrix of sample set evaluated can be built. Based on the premise that the same mathematics model is used, index weights are determined by means of entropy value method, equal weight method and pure threshold value method. Relative membership degree of each index to each classification can be calculated with relative difference function model. Then comprehensive relative membership degree of each sample to each classification can be obtained. And then feature value of classification of each sample can be got. At last, the stability of feature values of classification corresponding to three methods is analyzed and grade of each sample is determined. This model is applied to Dongchang Lake. Three lake regions all belong to the fifth classification eutrophic. It corresponds to reality. So it can be indicated that the proposed model is reasonable and practical. Finnally, suggestions are made for the protection and sustainable development of Dongchang Lake. [The Journal of American Science. 2008;4(3):11-17]. (ISSN: 1545-1003).

**Keywords:** lake eutrophication evaluation; fuzzy variable sets; index weights; Dongchang Lake; features of classification; sustainable development

## 1 Introduction

Eutrophication is one of the most serious water quality problems in the world. The contradiction of supply and demand of water resources is getting more and more serious with the rapid development of economy and society. Eutrophication is a sharp problem to lakes. Lake eutrophication evaluation is such a problem that involves many indexes, many classifications and interval values. Fuzzy variable evaluation method can effctively deal with the influence of evaluation standard interval values and set up comprehensive evaluation model to fulfill the comprehensive evaluation to water environment. Based on the premise that the same mathematics model is used, this paper analyzes the influence of the change of index weights to evaluation result.

## 2 Method

### 2.1 Fuzzy variable model for lake eutrophication

Suppose that there is a set that includes n water-quality samples whose features are showed by m features of indexes. Then index feature matrix of samples can be summarised as follows:

$$X = \left( x_{ji} \right) \quad (1)$$

Where $x_{ji}$ is the feature of index i of sample j, and $i = 1, 2, \cdots, m;\ j = 1, 2, \cdots, n$.

Standard value interval matrix of indexes to classifications ( $I_{ab}$ ) can be determined by:

$$I_{ab} = \left( \left[ a, b \right]_{ih} \right) \quad (2)$$

Where $h = 1, 2, \cdots, c$.

The main evaluation steps are as follows: first, referring to $I_{ab}$, range value matrix with variable intervals ( $I_{cd}$ ) can be structured by:

$$I_{cd} = \left( [c, d]_{ih} \right) \qquad (3)$$

Secondly, according to physical analysis to index i, point value matrix M to index i and classification h can be determined by:

$$M = \left( M_{ih} \right) \qquad (4)$$

Fig. 1 shows the relation of point x, point M, interval [a,b] and interval [c,d].

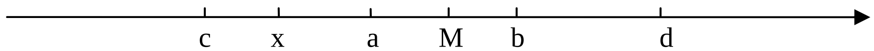


Fig.1 Relation of point x, point M, interval [a,b] and interval [c,d]

Thirdly, relative membership degree matrix ( $\mu_{\underset{\sim}{A}}(u)$ ) can be summarized by

$$\mu_{\underset{\sim}{A}}(u) = \left( \mu_{\underset{\sim}{A}}(u)_{ih} \right) \qquad (5)$$

Where $\mu_{\underset{\sim}{A}}(u)$ can be calculated using formula (6) to (8) and the corresponding data in matrix $I_{ab}$, $I_{cd}$ and M.

$$\begin{cases} D_{\underset{\sim}{A}}(u) = \left( \dfrac{x-a}{M-a} \right)^{\beta} & x \in [a, M] \\ D_{\underset{\sim}{A}}(u) = -\left( \dfrac{x-a}{c-a} \right)^{\beta} & x \in [c, a] \end{cases} \qquad (6)$$

$$\begin{cases} D_{\underset{\sim}{A}}(u) = \left( \dfrac{x-b}{M-b} \right)^{\beta} & x \in [M, b] \\ D_{\underset{\sim}{A}}(u) = -\left( \dfrac{x-b}{d-b} \right)^{\beta} & x \in [b, d] \end{cases} \qquad (7)$$

$$\mu_{\underset{\sim}{A}}(u) = \left[ 1 + D_{\underset{\sim}{A}}(u) \right] / 2 \qquad (8)$$

Among formula (6) and (7), $\beta$ is an non-negative index and usually is endowed with 1. Forthly, index weight vector ( $w = (w_i)$ ) determined, comprehensive relative membership degree vector $\left( \nu_{\underset{\sim}{A}}(u) \right)$ can be calculated using the following approach:

$$\nu_{\underset{\sim}{A}}(u)_h = \frac{1}{1 + \left\{ \dfrac{\sum_{i=1}^{m} \left[ w_i \left( 1 - \mu_{\underset{\sim}{A}}(u)_{ih} \right) \right]^p}{\sum_{i=1}^{m} \left( w_i \mu_{\underset{\sim}{A}}(u)_{ih} \right)^p} \right\}^{\frac{\alpha}{p}}} \qquad (9)$$

Making both $\alpha$ and $p$ equal 1 and normalizing $\left( \nu_{\underset{\sim}{A}}(u) \right)$, comprehensive relative membership degree vector can be got as follows:

$$\nu^{o}_{\underset{\sim}{A}}(u) = \left(\nu^{o}_{\underset{\sim}{A}}(u)_h\right) \quad (10)$$

Fifthly, eutrophication degree of each sample can be evaluated. If $\nu^{o}_{\underset{\sim}{A}}(u)_h > 0.5$, then it belongs to classification h. If $\nu^{o}_{\underset{\sim}{A}}(u)_h \le 0.5$, then it can be evaluated with feature of classification of the same sample (H), which is calculated:

$$H = (1 \quad 2 \quad \Lambda \quad c) \cdot \nu^{0}_{\underset{\sim}{A}}(u)^{T} \quad (11)$$

And then we can differentiate classification of sample j according to its classification features. The method is as follows. If $c-0.5 < H_j \le c$,sample j can be differentiate to classification c. If $1 < H_j \le 1.5$, it can be differentiate to classification 1. If $h-0.5 < H_j \le h+0.5$, it can be differentiate to classification h ($h = 2,3,\mathrm{L}\ ,c-1$). Sixthly, duplicating step 4 and 5, classification features corresponding to different weight are calculated. Analyzing their stability, eutrophication classification of each sample can be determined.

### 2.2 Index weight determination

Index weight determination is focal and difficult point in multi-index evaluation. At present, there are many methods, such as entropy value method, equal weight method and pure threshold value method. Among them, the former two belong objective method and the latter subjective. This paper combines them to improve rationality and reliability of evaluation results.

#### *2.2.1 Entropy value method*

Calculation steps are as follows. First, judging matrix of m indexes corresponding to n samples is constructed( $R = \left(x_{ji}\right)_{n\times m}$ ). Secondly, judging matrix is normalized to matric B, whose arbitrary element is calculated from:

$$b_{ji} = \frac{x_{ji} - x_{\min}}{x_{\max} - x_{\min}} \quad (12)$$

where $x_{\max}$ and $x_{\min}$ are separately the most satisfied and the most unsatisfied to the same index of different samples. Thirdly, entropy of index i is calculated from:

$$H_i = -\frac{1}{\ln n}\left(\sum_{j=1}^{n} f_{ji} \ln f_{ji}\right) \quad (13)$$

where $f_{ji} = \dfrac{1+b_{ji}}{\sum_{j=1}^{n}\left(1+b_{ji}\right)}$. Forthly, entropy weight of index i is calculated from:

$$w_i = \frac{1-H_i}{m - \sum_{i=1}^{m} H_i} \quad (14)$$

where $\sum_{i=1}^{m} w_i = 1$ should be satisfied.

***2.2.2 Equal weight method***

Weight of each index is thought as equal. That is to say:

$$w_i = \frac{1}{m} \quad (i = 1, 2, \cdots, m) \tag{15}$$

***2.2.3 Pure threshold value method***

Reciprocal method reflects the idea that the bigger the threshold value is, the smaller the relative weight is. Weight is calculated from:

$$w_{ih} = \frac{1}{y_{ih}} \Bigg/ \sum_{i=1}^{m} \frac{1}{y_{ih}} \tag{16}$$

where $y_{ih}$ is the value non-dimensioned.

**3 Applied Example**

Dongchang Lake lies in southwest of Liaocheng, shandong, China. Its area is 4.2 square kilometers and its depth is 2-3 meters. With the rapid development of economy and society of Liaocheng City, eutrophication occurs to a certain extent for Dongchang Lake. So eutrophication degree should be evaluated for it in order to supply basis for pollution control and ecology environment management.

The measured data for water quality to Dongchang Lake are provided in table 1. Chl-a,TP,TN and $COD_{Mn}$ are chosen as evaluation indexes to evaluate nutrient states of its three regions.

Table 1. Eutrophication measured data of Dongchang Lake

| Evaluation indexes | Northwest region | Southwest region | Southeast region |
|---|---|---|---|
| Chl-a(mg/$m^3$) | 21.25 | 22.02 | 22.54 |
| TP(mg/$m^3$) | 180 | 161 | 173 |
| TN(mg/$m^3$) | 1980 | 1760 | 1840 |
| $COD_{Mn}$(mg/l) | 14.10 | 13.48 | 13.36 |

According to eutrophication classification standard of lakes and reseriours in China, that with interval values is given in table 2.

Table 2. Eutrophication classification standard of lakes and reseriours in China

| Evaluation indexes | Oligotrophic | Lower-mesotrophic | Mesotrophic | Upper-mesotrophic | Eutrophic | Hypereutrophic |
|---|---|---|---|---|---|---|
| Chla(mg/$m^3$) | 0-1.0 | 1.0-2.0 | 2.0-4.0 | 4.0-10.0 | 10.0-65 | 65-160 |
| TP(mg/$m^3$) | 0-2.5 | 2.5-5.0 | 5.0-25.0 | 25.0-50.0 | 50.0-200 | 200-600 |
| TN(mg/$m^3$) | 0-30 | 30-50 | 50-300 | 300-500 | 500-2000 | 2000-6000 |
| $COD_{Mn}$(mg/l) | 0-0.3 | 0.3-0.4 | 0.4-2.0 | 2.0-4.0 | 4.0-10 | 10-25 |

Then matrix $I_{ab}$, $I_{cd}$ and $M$ can be expressed as follows:

$$I_{ab}=\begin{bmatrix} [0,1.0] & [1.0,2.0] & [2.0,4.0] & [4.0,10.0] & [10.0,65] & [65,160] \\ [0,2.5] & [2.5,5.0] & [5.0,25] & [25,50] & [50,200] & [200,600] \\ [0,30] & [30,50] & [50,300] & [300,500] & [500,2000] & [2000,6000] \\ [0,0.3] & [0.3,0.4] & [0.4,2.0] & [2.0,4.0] & [4.0,10] & [10,25] \end{bmatrix}=\left([a,b]_{ih}\right)$$

$$I_{cd}=\begin{bmatrix} [0,2.0] & [0,4.0] & [1.0,10] & [2.0,65] & [4.0,160] & [10,160] \\ [0,5.0] & [0,25] & [2.5,50] & [5.0,200] & [25,600] & [50,600] \\ [0,50] & [0,300] & [30,500] & [50,2000] & [300,6000] & [500,6000] \\ [0,0.4] & [0,2.0] & [0.3,4.0] & [0.4,10] & [2.0,25] & [4.0,25] \end{bmatrix}=\left([c,d]_{ih}\right)$$

$$M=\begin{bmatrix} 0 & 1.0 & 3.0 & 7.0 & 65 & 160 \\ 0 & 2.5 & 15 & 37.5 & 200 & 600 \\ 0 & 30 & 175 & 400 & 2000 & 6000 \\ 0 & 0.3 & 1.2 & 3.0 & 10 & 25 \end{bmatrix}=\left(M_{ih}\right)$$

Northwest region taken as an example, calculation process are as follows. According to the upper matrixes and formula (6) to (8), $\mu_{\tilde{A}}(u)$ can be calculated.

$$\mu_{\tilde{A}}(u)_{ih\,4\times6}=\begin{bmatrix} 0 & 0 & 0 & 0.398 & 0.602 & 0.102 \\ 0 & 0 & 0 & 0.067 & 0.933 & 0.433 \\ 0 & 0 & 0 & 0.007 & 0.993 & 0.493 \\ 0 & 0 & 0 & 0 & 0.363 & 0.637 \end{bmatrix} \tag{17}$$

### 3.1 Classification feature corresponding to entropy weight

Entropy weight is calculated using formula (12) to (14):

$$w=\begin{pmatrix}0.23 & 0.231 & 0.246 & 0.293\end{pmatrix} \tag{18}$$

Formula (17) to (18) substituted into formula (9) and normalized, then comprehensive relative membership degree vector ($u=\begin{pmatrix}0 & 0 & 0 & 0.087 & 0.566 & 0.347\end{pmatrix}$) is got. The classification feature of the sample can be calculated using formula (11): $H_1=5.259$.

Similarily, the classification features of the southwest and southeast regions are seperately 5.210 and 5.234.

### 3.2 Classification feature corresponding to equal weight

Weight vector is given by:

$$w=\begin{pmatrix}0.25 & 0.25 & 0.25 & 0.25\end{pmatrix} \tag{19}$$

Formula (17) to (19) substituted into formula (9) and normalized, then comprehensive relative membership degree vector ( $u = \begin{pmatrix} 0 & 0 & 0 & 0.094 & 0.575 & 0.331 \end{pmatrix}$ ) is got. The classification feature of the sample can be calculated using formula (11): $H_1 = 5.237$.

Similarily, the classification features of the southwest and southeast regions are seperately 5.187 and 5.212.

**3.3 Classification feature corresponding to weight determined by pure threshold value method**

Weight is calculated using formula (16):

$$w_{ih} = \begin{bmatrix} 0.232 & 0.209 & 0.413 & 0.345 & 0.222 & 0.247 \\ 0.347 & 0.306 & 0.241 & 0.253 & 0.264 & 0.240 \\ 0.286 & 0.306 & 0.203 & 0.253 & 0.265 & 0.241 \\ 0.135 & 0.179 & 0.143 & 0.149 & 0.249 & 0.272 \end{bmatrix} \quad (20)$$

Formula (17) to (20) substituted into formula (21),

$$v_{\tilde{A}}(u)_h = \frac{1}{1 + \left\{ \frac{\sum_{i=1}^{m} \left[ w_{ih} \left( 1 - \mu_{\tilde{A}}(u)_{ih} \right) \right]^p}{\sum_{i=1}^{m} \left( w_{ih} \mu_{\tilde{A}}(u)_{ih} \right)^p} \right\}^{\frac{\alpha}{p}}} \quad (21)$$

Making both $\alpha$ and $p$ equal 1 and normalizing, comprehensive relative membership degree vector ( $u = \begin{pmatrix} 0 & 0 & 0 & 0.119 & 0.559 & 0.321 \end{pmatrix}$ ) is got. The classification feature of the sample can be calculated using formula (11): $H_1 = 5.202$.

Similarily, the classification features of the southwest and southeast regions are seperately 5.166 and 5.185.

**4 Conclusions and Countermeasure for Sustainable Development**

To sum up, the change of the value of H is very small when index weights change. So we use the mean value of H calculated by the three methods as the ultimate value. And we can get the evaluation result that the three lake regions all belong to classification 5(eutrophic).

Combine with the reality of Dongchang Lake, we can promote its sustainable development mainly by water diversion, control of pollution source and restoration and restruction of aquatic vegetation.

(1) The degree of eutrophication of Dongchang Lake is closely relevant to water diversion from Tanzhuang Reseriour. So the quantity and time of water diversion should be studied with the aim of guaranteeing good cycle of the water in Dongchang Lake.

(2) The improvement degree of water environment quality in Dongchang Lake also relates to the site and number of ehtrances of water diversion, which should be determined by calculating scientificly on the premise of feasibility to construction and economy in order to make the yield of limit water resources better.

(3) To improve the water quality, we should control the discharge of domestic wastewater, retrieve tour garbage and decrease or eliminate nets for aquaculture.

(4) Ecological restoration is a way usually used to improve the water quality of city lake. Restoration and Restruction experiment of aquatic vegetation of Wuli Lake, East Lake in Wuhan and Dianchi Lake

produced a certain effect. It is thus clear that the perfect water ecosystem and the higher degree of hydrobiological diversity can be gradually set up.

By means of adopting measures and methods above mentioned, the water environment quality of Dongchang Lake will be obviously improved. Thus environment grade of Liaocheng City will be heightened.

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