Electroweak Interaction in the $(\mu^- - \mu^+)$ and $(\tau^- - \tau^+)$ - leptonic Pair production at High Energies

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Abstract: The theoretical treatment of the process of the annihilation of $(e^- - e^+)$ - pair into $(\mu^- - \mu^+)$ - or $(\tau^- - \tau+$ - pairs at high regions of energy can be studied in details. The Electroweak interaction in the processes can also be studied obtaining analytical formulae for the cross-sections of the process in different cases. Applying the obtained formulae for the case of high energy to obtain the energy distribution for the processes. The effect of polarization of the particles on the process can be also studied, obtaining the formulae for the cross-sections in both weak $(V \pm A)$ and electromagnetic interactions. The comparison between the two types of interactions e.g. weak and electromagnetic, and the interference between them is performed, showing that the study of particle polarization in the process is very important and give us a result compatible with the Salam-Weinberg model of electroweak interactions and the experimental studies.

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1. Introduction

The study of pair-production processes was done firstly by **Dirac in 1928** [1], in his famous paper: The quantum theory of the electron, part II. And subsequent cloud-chamber observations by **Anderson** [2], in which he observed the presence of positron.

Bhabha [3] was the first one who study the process of meunic $(\mu^- - \mu^+)$-pair production due to the scattering of $(e^- - e^+)$ -pair in ultrarelativistic case.

The inverse processes, i.e. the $(e^- - e^+)$ -pair creation due to the $(\mu^- - \mu^+)$-scattering process was studied in details by **Ginsburg** [4].

In 1996, **Buskulic** and others [5] were studied the $(\tau^- - \tau^+)$-pair production due to the annihilation of $(e^- - e^+)$-pair at very high energies (130, 136 Gev.), and in 1997, **Bernreuther** and others [6] studied the same process using the standard model of Salam-Weinberg.

The study of polarization in the pair production processes was done in many works [7-9]. **Haug** [9], studied the linear polarization of the photon in the pair-production process in details.

The spin correlation in the pair production processes by electromagnetic field was studied also in 2006 by **Kruglov** [10], and the$(e^- - e^+)$ pair by an electron in a magnetic field at different values of energies from threshold (1.022 Mev) to 100 Gev was studied recently by **Novac** and others [11].

$2 -$ Theory

In Salam-Weinberg standard model of the unified electroweak interactions [12,13], the processes: $e^-e^+ \rightarrow \mu^-\mu^+$ and $e^-e^+ \rightarrow \tau^-\tau^+$ can be studied successfully. The model contains four fields: two charged boson fields (W^-, W^+) , a neutral electromagnetic field $\vec{A}$ represented by a photon, and a neutral boson field (Z°).

With these four fields associated four generators for abelian symmetric group $SU(2)\otimes U(1)$. The generators τ_1, τ_2, τ_3 are the generators of the group $SU(2)$ and the identity I is for the group $U(1)$ [14].

The Feynman diagrams of the processes:

$$e^- + e^+ \rightarrow \mu^-(\tau^-) + \mu^+(\tau^+) \qquad (1)$$

are shown in fig. 1(a) and fig. 1(b).

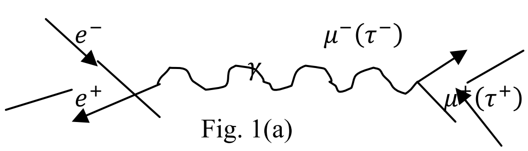

Fig. 1(a)

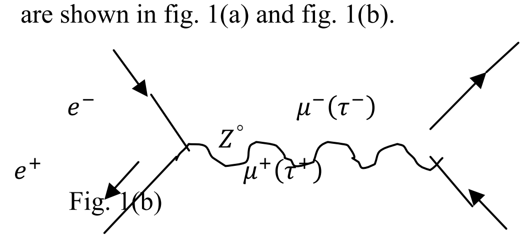

Fig. 1(b)

Where the intermediate state is a virtual photon (γ) represent the electromagnetic interaction (Fig. 1a), or a neutral Z°-boson represent the weak interaction (Fig.1b). According to Salam-Weinberg model, the total scattering amplitude of the processes (1) as an electroweak processes may be represented by the sum of two amplitudes A_γ and A_Z, where:

$$A_\gamma = (ie^2)\,[\bar{u}_e + \gamma_\alpha V_{e^-}]\,[G_\gamma]\,[\bar{V}_{\ell^-}\,\gamma_\beta\,u_{\ell^+}] \quad (2)$$

$$A_Z = \frac{(ig)^2}{16\cos^2\theta_\omega}\,[\bar{u}_e + \gamma_\alpha(\acute{g} - \gamma_5)\,V_{e^-}]\,[G_Z]\,[\bar{V}_{\ell^-}\,\gamma_\beta(\acute{g} - \gamma_5)\,u_{\ell^+}] \quad (3)$$

The function (V_{e^-}) represent the annihilation of e^- (with momentum P_{e^-}), and $(\bar{u}_{e^+})$ represent the annihilation of e^+, with momentum $(-P_{e^+})$, while the function $(\bar{V}_{\ell^-})$ represent the creation of ℓ^- $(\mu^-\ and\ \tau^-)$ with momentum P_{ℓ^-}, and (u_{ℓ^+}) represent the creation of ℓ^- $(\mu^+\ and\ \tau^+)$ with momentum $(-P_{\ell^+})$.

Also G_γ and G_Z represent the Feynman propagators for the virtual photon (γ) and the vector neutral boson (Z°) and they have the forms [14]:

$$G_\gamma = \frac{g_{\alpha\beta} - \frac{q_\alpha q_\beta}{q^2}}{q^2} \quad (4)$$

$$G_Z = \frac{g_{\alpha\beta} - \frac{q_\alpha q_\beta}{M_Z^2}}{q^2 - M_Z^2} \quad (5)$$

$q = P_{e^-} + P_{e^+}$ represent the momentum of the intermediate state, M_z is the mass of Z° (which is of order 90 Gev).

The two factors g and $\acute{g}$ represent the coupling factors in the $SU(2)\otimes U(1)$ model, where g represent the coupling factor for the group $SU(2)$ and $\acute{g}$ represent the coupling factor for the group $U(1)$.

The factors g and $\acute{g}$ are related to the factor e (which is in fact represent the electron charge) by the relations:

$$g = \frac{e}{\sin\theta_\omega}\,,\quad g' = \frac{e}{\cos\theta_\omega}\,,\qquad g' = g\tan\theta_\omega \quad (6)$$

Where the Weinberg mixing or coupling angle θ_ω is given in terms of the factors g and $\acute{g}$ by the relations:

$$\sin\theta_\omega = \frac{\acute{g}}{\sqrt{g^2 + \acute{g}^2}}\,,\qquad \cos\theta_\omega = \frac{g}{\sqrt{g^2 + \acute{g}^2}} \quad (7)$$

Also from (6) and (7) we see that:

$$\sin\theta_\omega = \frac{e}{g}\,,\qquad \cos\theta_\omega = \frac{e}{\acute{g}}\,,\quad \tan\theta_\omega = \frac{\acute{g}}{g} \quad (8)$$

The value of the angle θ_ω is given by [14]:

$$\sin^2\theta_\omega = 0.224 \pm 0.015 \quad (9)$$

Also we note the following:

(i) The two processes (1) can be written as a current-current neutral interactions: $(\bar{e}\,e)\,(\bar{\mu}\,\mu)$ and $(\bar{e}\,e)\,(\bar{\tau}\,\tau)$.

(ii) The weak interaction can take place into two chanels:
the vector (V)-chanel and the axial vector (A)-chanel.
The coupling constants for the two chanels of interactions are given by:

$$g_V = 1 - 4\sin^2\theta_\omega\,,\qquad g_A = 1 \quad (10)$$

(iii) Feynman and **Gell-Mann** [15], showed that the charged weak leptonic current in the leptonic interaction take the form $(V - A)$, i.e the difference between the vector (V) and the axial vector (A) currents.

(iv) At very high energy $E_e > 100\ Gev$, we have the following formula for the total cross-section of the proess: $e^-e^+ \rightarrow \mu^-\mu^+$ [14]:

$$\sigma = \frac{4\pi\alpha^2}{3E_e^2}\left[1 + \frac{g_V^2}{2\sin^2(2\theta_\omega)} + \frac{(g_V^2+1)^2}{16\sin^4(2\theta_\omega)}\right] \quad (11)$$

3 − The basic formulae for the cross-sections of the processes (1):

The differential cross-section of the processes (1) depends on the square of the amplitude i.e on the quantity $|A_\gamma + A_z|^2$, and if we take the interaction term between the two amplitudes, we have the term: $(A_\gamma A_z{}^* + A_\gamma{}^* A_z)$ which is known as the interference term.

The theoretical treatment of the process $e^-e^+ \rightarrow \mu^-\mu^+$ shows that the cross-section of the electromagnetic interaction between the particles is inversely proportional to the square of the total energy (E) of the pair (e^-, e^+). Moreover, the cross-section of the weak interaction between the particles is directly proportional to E^2. Consequently, we can say that the weak interaction becomes more effective with the increasing total energy of the scattering particles. For high energy values $(E > 10Gev)$, the effect of weak interaction seems to be stronger than the effect of electromagnetic one. If we take the polarization (or helicities) of the interacting particles into considerations, and if h_{e^-}, h_{e^+}; h_{ℓ^-}, h_{ℓ^+} are the helicities of the particles e^-, e^+, $\ell^-(\mu^-, \tau^-)$, and $\ell^+(\mu^+, \tau^+)$ respectively, where:

$$h_i = +1 \quad (for\ e^-, \mu^-, \tau^-)$$
$$h_i = -1 \quad (for\ e^+, \mu^+, \tau^+) \quad (12)$$

We can calculate the scattering cross-section of the processes of $(\bar{\mu}\,\mu)$ and $(\bar{\tau}\,\tau)$-polarized pairs produced from the annihilation of the polarized $(\bar{e}\,e)$-pair. After a long calculations, we finally obtain the following relations for the total cross-section of the processes:

$$\sigma = \sigma_{elect.} + \sigma_{weak} + \sigma_{interf} \quad (13)$$

Where:

$$\sigma_{elect.} = \frac{\alpha^2}{96\pi}\,\frac{f}{m_\ell^2 T_{e\ell}^2}\,(1 - h_{e^-}\,h_{e^+})\,(1 - h_{\ell^-}\,h_{\ell^+}) \quad (14)$$

$$\sigma_{weak} = \xi\left(\frac{m_\ell^2}{3\pi}\right) f(1-h_{e^-}\ h_{e^+})\left(1-h_{\ell^-}\ h_{\ell^+}\right)[T_{e\ell}^2(1-$$
$$\eta 1fhe- h\ell- +\eta \qquad (15)$$

$$\sigma_{interf.} = \xi_1\left(\frac{\alpha}{16\pi}\right) f(1-h_{e^-}\ h_{e^+})\left(1-h_{\ell^-}\ h_{\ell^+}\right)[1 - \eta_2 f h_{e^-} h_{\ell^-}] \qquad (16)$$

Where:

$$T_{e\ell} = \frac{E_e}{m_\ell} \quad i.e. \quad T_{e\mu} = \frac{E_e}{m_\mu} \ , \quad T_{e\tau} = \frac{E_e}{m_\tau} \qquad (17)$$

E_e is the energy of electron (positron) in the C.M. system, and m_ℓ is the rest mass of the lepton ($m_\ell = m_\mu, m_\tau$).

$$f = \sqrt{1-\frac{1}{T_{e\ell}}} = \sqrt{1-\frac{m_\ell}{E_e}} = \frac{1}{E_e}\sqrt{E_e(E_e-m_\ell)} \qquad (18)$$

The factors $\xi, \xi_1, \eta, \eta_1, \eta_2$ for the (V,A)-weak interaction are given by:

$$\xi = |g_V|^2 + |g_A|^2, \xi\eta = |g_V|^2 - |g_A|^2, \xi\eta_1 = g_V\, g_A{}^* + g_V{}^* g_A$$
$$\xi_1 = g_V + g_V{}^* \quad , \quad \xi_1\eta_2 = g_A + g_A{}^* \qquad (19)$$

4 – Study of (V+A) and (V-A) types of interactions in the processes (1):

We consider now the two cases of the (V ± A)-interactions of the neutral weak currents: $(\bar{e}\, e)$, $(\bar{\mu}\, \mu)$, and $(\bar{\tau}\, \tau)$.

(1) In the (V+A)-interaction (or cpupling) of electronic $(\bar{e}\, e)$ and leptonic $(\bar{\ell}\, \ell)$ currents:

$$g_A = g_V \quad \text{i.e.} \quad \frac{g_A}{g_V} = 1$$

From (19) we have:

$$\xi = 2{g_V}^2, \quad \xi\eta = 0, \quad \xi\eta_1 = 2{g_V}^2, \quad \xi_1 = 2g_V$$
$$\xi_1\eta_2 = 2g_V, \quad \eta_1 = 1, \quad \eta_2 = 1$$

And noting that: $g_V = \sqrt{2}\, G$, where G is the Fermi universal coupling constant ($G = 1.41 \times 10^{-49}\, erg.cm^3$) [14] then: $\xi = 4\, G^2$, $\xi_1 = 2\sqrt{2g}$

And the relations (14),(15),(16) reduced to the following relations at high energies where:

$$f = \sqrt{1-\frac{1}{T_{e\ell}}} \approx 1, \qquad \frac{f}{m_\ell^2\, {T_{e\ell}}^2} \approx \frac{1}{E_e^2}$$

$$\sigma_{elect.} = \frac{\alpha^2}{96\pi}\left(\frac{1}{E_e^2}\right)(1-h_{e^-}\ h_{e^+})\left(1 - h_{\ell^-}\ h_{\ell^+}\right) \qquad (21)$$

$$\sigma_{weak} = \frac{4\, G^2}{3\,\pi}(E_e^2)(1-h_{e^-}\ h_{e^+})\left(1-h_{\ell^-}\ h_{\ell^+}\right)(1 - h_{e^-}\ h_{\ell^-}) \qquad (22)$$

$$\sigma_{interf.} = \frac{\sqrt{2}\,\alpha\, G}{8\,\pi}(1-h_{e^-}\ h_{e^+})\left(1-h_{\ell^-}\ h_{\ell^+}\right)(1 - h_{e^-}\ h_{\ell^-}) \qquad (23)$$

(2) In the (V-A)-interaction (or coupling) of electronic $(\bar{e}\, e)$ and leptonic $(\bar{\ell}\, \ell)$ currents:

$$g_A = -g_V \quad i.e \quad \frac{g_A}{g_V} = -1$$

From (19) we have:

$$\xi = 2{g_V}^2, \quad \xi\eta = 0, \quad \xi\eta_1 = -2{g_V}^2, \quad \xi_1 = 2g_V$$
$$\xi_1\eta_2 = -2g_V, \quad \eta_1 = -1, \quad \eta_2 = -1 \qquad (24)$$

And if $g_V = \sqrt{2}\, G$, then: $\xi = 4\, G^2$, $\xi_1 = 2\sqrt{2g}$ and the relations (14),(15),(16) reduced to the following relations at high energies (where $f \approx 1$, $\frac{f}{m_\ell^2\, {T_{e\ell}}^2} \approx \frac{1}{E_e^2}$)

$$\sigma_{elect.} = \frac{\alpha^2}{96\pi}\left(\frac{1}{E_e^2}\right)(1-h_{e^-}\ h_{e^+})\left(1 - h_{\ell^-}\ h_{\ell^+}\right) \qquad (25)$$

$$\sigma_{weak} = \frac{4\, G^2}{3\,\pi}(E_e^2)(1-h_{e^-}\ h_{e^+})\left(1-h_{\ell^-}\ h_{\ell^+}\right)(1 + h_{e^-}\ h_{\ell^-}) \qquad (26)$$

$$\sigma_{interf.} = \frac{\sqrt{2}\,\alpha\, G}{8\,\pi}(1-h_{e^-}\ h_{e^+})\left(1-h_{\ell^-}\ h_{\ell^+}\right)(1 + h_{e^-}\ h_{\ell^-}) \qquad (27)$$

5- Results and conclusion

From the relations (21)-(23) for (V+A) interaction and the relations (25)-(27) for the (V-A) interaction in the processes (1), we see the following results (at high energies):

(1) The electromagnetic cross-section ($\sigma_{elect.}$) in the two cases of (V-A) and (V+A) are identical, i.e the electromagnetic interaction not depends on the (vector) V- or (axial vector) A- types of interactions.

(2) The weak and electroweak (interference) cross-sections are proportional to $(1 - h_e\ h_\ell)$ in the (V+A)- coupling, and to $(1 + h_e\ h_\ell)$ in the (V-A)-coupling.

(3) At high energies, the electromagnetic cross-section is inversely proportional to (E_e^2), while the cross-section of the weak interaction is directly proportional to (E_e^2), consequently, we see that the weak interaction becomes mor effective with the increasing total energy of the (e^-, e^+)-pair. This is in agreement with the experimental results for the processes $e^+e^+ \rightarrow \mu^-\mu^+$ [3].

(4) the electroweak (interference)-term i.e. $\sigma_{interf.}$ is not depends on the total energy of the (e^-, e^+)-pair.

(5) The term $(1 + h_{e^-}\ h_{\ell^-})$ has a value if we take: $h_{e^-} = 1$ and $h_{\ell^-} = 1$ i.e. if we take $h_{e^-}\ h_{\ell^-} = 1$, wherease the term $(1 - h_{e^-}\ h_{\ell^-})$ disappears in this case.

This means that the cross-sections σ_{weak} and $\sigma_{interf.}$ for (V-A) coupling hase a value, while for (V+A) coupling these cross-sections are zero, i.e. the possibility of occurring the processes (1) takes place only for the (V-A) interaction, which is in agreement with the experimental studies.

(6) If we take the values of $h_{e^-}, h_{e^+}, h_{\ell^-}, h_{\ell^+}$ [formula (12)], we see that the terms $(1 - h_{e^-}\ h_{e^+})$,

$(1 - h_{\ell^-}\ h_{\ell^+})$, and $(1 + h_{e^-}\ h_{\ell^-})$ are equal the value 2, and consequently the cross-sections $\sigma_{elect.}, \sigma_{weak}$, and $\sigma_{interf.}$ for the (V-A)-interaction can take the following forms, for high energies:

$$\sigma_{elect.} = \frac{\alpha^2}{96\pi}\left(\frac{1}{E_e^2}\right)(4) = \left(\frac{1}{24\pi}\right)\alpha^2\left(\frac{1}{s}\right) \quad (28)$$

$$\sigma_{weak} = \frac{4\,G^2}{3\,\pi}(E_e^2)(8) = \left(\frac{32}{3\pi}\right)G^2(s) \quad (29)$$

$$\sigma_{interf.} = \frac{\sqrt{2}\,\alpha\,G}{8\,\pi}(8) = \left(\frac{\sqrt{2}}{\pi}\right)\alpha G \quad (30)$$

Where $s = E_e^2$ i.e. $\square_{\square} = \sqrt{\square}$

(7) If we neglect the polarization of the particles, and at high energies, we can get the following formulae for the cross-sections of the processes (1):

$$\square_{\square\square\square\square\square.} = \frac{4\square\square^2}{3}\left(\frac{1}{\square}\right) \quad (31)$$

$$\square_{\square\square\square\square} = \frac{\square\,\square^2}{12}\frac{(\square_{\square}^2 + 1)^2}{\sin^4(2\square_{\square})}\left[\frac{\square}{(\square - \square_{\square}^2)^2}\right] \quad (32)$$

$$\square_{\square\square\square\square\square\square.} = \frac{2\square\square^2}{3}\left[\frac{\square_{\square}^2}{\sin^2(2\square_{\square})}\right]\left(\frac{1}{\square}\right) \quad (33)$$

Where $\square_{\square} = 1 - 4\sin^2\square_{\square}$

If we take $\sin^2\square_{\square} \approx 0.25$, then $\square_{\square} \approx 0$, and for very high energies: $\square \gg \square_{\square}$, the formulae (32), (33) takes the forms:

$$\square_{\square\square\square\square} \cong \frac{\square\,\square^2}{12}\left[\frac{1}{\sin^4(2\square_{\square})}\right]\left(\frac{1}{\square}\right) \quad (34)$$

$$\square_{\square\square\square\square\square\square.} \cong 0 \quad (35)$$

From the relations (31), (34), (35) and the relations (28)-(30) for the polarized particles, we see that:

1. The cross-section $\square_{\square\square\square\square\square.}$ is inversely proportional to $\square$ or $\square_{\square}^2$, in the two cases, i.e. the case of polarization and the case without studieng the polarization of the particles, which is inagreement with the experimental results for the electromagnetic interaction of the process $\square^+\square^- \rightarrow \square^-\square^+$
2. The cross-section $\square_{\square\square\square\square}$ is directly proportional to ($\square$) if we take the polarization into account (equation (29)), while it oppositely proportional to ($\square$) if we neglect this polarization (equation (34)). The experimental results show that: there is a large increasing in the cross-section with increasing the energy in this process $\square^+\square^- \rightarrow \square^-\square^+$. Thus we can see that, the study of polarization gives us results compatible with the experimental results.
3. The interference term $\square_{\square\square\square\square\square\square.}$ between the electromagnetic and weak interactions in the ease of neglecting the polarization is vanishes (equation (35)), which is not in agreement with Salam-Weinberg theorem which confirm the existence of interference (electroweak) term between the two interactions, otherwise in the case of taking polarization into account, the interference term ($\square_{\square\square\square\square\square\square\square\square\square\square\square}$) exist and has a constant value (equation (30). Thus we conclude that the study of particle polarization in the processes (1) is very important and give us a result compatible with Salam-Weinberg theorem and the experimental studies.

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