The Elimination of 50 Hz Power Line Interference from ECG Using a Variable Step Size LMS Adaptive Filtering Algorithm

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Abstract: This article demonstrates "a new variable step size LMS adaptive filtering algorithm" to eliminate 50 Hz power line interference. This approach can provide faster convergence rate and smaller mean square error (MSE). [Life Science Journal. 2006;3(4):90-93] (ISSN: 1097-8135).

Keywords: adaptive filter; LMS algorithm; 50 Hz power line interference

Abbreviations: ECG: electrocardiogram; LMS: least mean square; MSE: mean square error; SNR: signal to noise ratio; NLMS: normalized least mean square; SVSLMS: S-function variable step least mean square

1 Introduction

ECG signal is always interrupted by mixed interferences, such as 50 Hz power line interference, respiratory and muscle electric interference etc. Specifically, since signal to noise ratio (SNR) of electrocardiogram (ECG) signal is very low, the interfering frequency at 50 Hz may overwhelm the source signal. Methodologically, elimination of 50 Hz interference has been discussed a lot[1,2]. Majority of the approaches suppose the 50 Hz power line interference frequency not to fluctuate. In this short report, we use a variable step size least mean square (LMS) adaptive filtering algorithm to eliminate the 50 Hz power line interference whose frequency has small fluctuation from ECG signal. The LMS algorithm and stochastic gradient algorithm[3,4], introduced by Widrow and Hoff in 1960, is widely used in practice because of its simplicity, computational efficiency, and good performance under a variety of operating conditions. LMS algorithm is based on mathematical method of the steepest decline, which is to define a performance function along the vector in the direction toward negative gradient of the steepest value to get itself recovery. Figure 1 shows the basic of LMS algorithm. The LMS algorithm can be summarized as

$$y(n)=\sum_{i=0}^{L} W_i x(n-1)=W^{\mathrm{T}}X \tag{1}$$

where $X(k)=[x(k),\ x(k-1),\cdots,x(k-L+1)]^{\mathrm{T}}$ and filtering coefficient W is defined as $W(k)=[w_0(k),\ w_1(k),\cdots,w_{L-1}(k)]^{\mathrm{T}}$, L is the number of filtering order; W_i is the filtering coefficient. The optimization can be determined by using LMS algorithm as OP procedures:

$$\mathrm{OP}=\begin{cases} y(n)=w^{\mathrm{T}}(n)*x(n) & (\textit{Filtering}) \\ e(n)=d(n)-y(n) & (\textit{Error formation}) \\ w(n)=w(n-1)+2*u*e(n)*x(n) & (\textit{Confficient updation}) \end{cases}$$

where u is the step-size control parameter.

The adaptation parameter, step size u selected, must be small enough to ensure that the algorithm converges to the optimum point. However, this small step size causes slow convergence. As a matter of fact, the convergence condition can be satisfied by choosing the range $0 \leqslant u \leqslant \frac{1}{\mathrm{Tr}[R]}$. Where $\mathrm{Tr}[R]$ denotes the trace of R and R is the auto-correlation of X. The convergence rate is proportional to the variation of parameter u[5]. If u increases, the y will converge faster but increases mean square error (MSE) and decreases the stability margin. In order to resolve this contradiction, we proposed a new algorithm to modify the variable step size LMS adaptive algorithms.

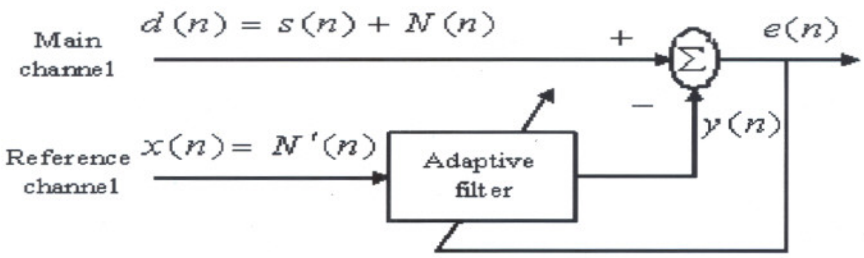


Figure 1. The structure of LMS algorithm

instead of using numbers, normalized least mean square (NLMS) algorithm[6] modify u by defining variables α and β

$$u(n)=\frac{\alpha}{\beta+x_n^{\mathrm{T}}x_n}=\frac{\alpha}{\beta+r(n)} \tag{2}$$

where $r(n)=x_n^{\mathrm{T}}x_n$ is defined as the interior product of the input vector $x(n)$.

The value of $u(n)$ has to be put small value to avoid the decreasing stability, therefore, α must be positive number in $0<\alpha<1$ and $\beta\approx 0.0001$. NLMS algorithm can effectively reduce amplifying gradient noise in the process of convergence, and keep the good convergence rate eventually.

In order to improve the performance of the filter, Tan *et al*[7] proposed a new variable step size LMS adaptive algorithm, S-function variable step least mean square (SVSLMS) algorithm, which define $u(n)$ as a Sigmoid function of $e(n)$,

$$u(n)=\beta\left(\frac{1}{1+E(-a^{\| e(n)x(n)\|})}-0.5\right) \tag{3}$$

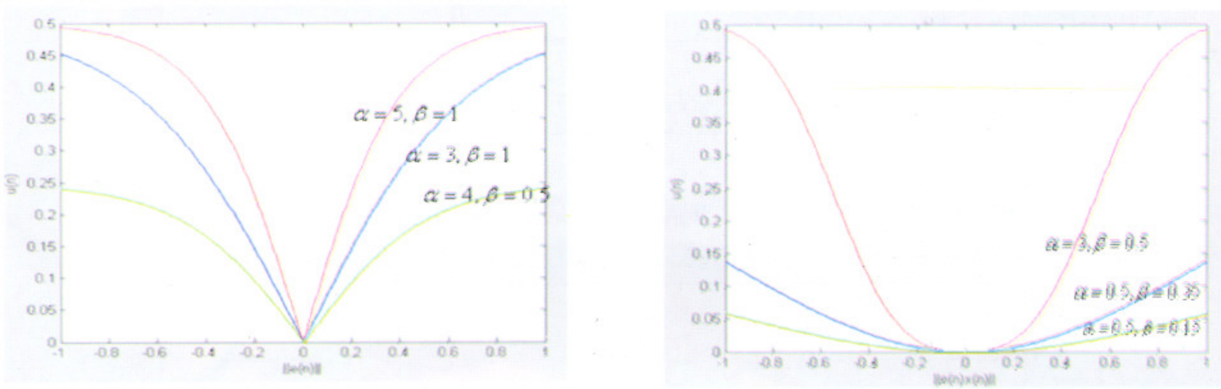
The advantage of this algorithm is that the step size is bigger in preliminary stage of unknown system, so having faster convergence rate. When the algorithm has already converged, no matter how big the input interference was, it can keep very small step size to obtain very small MSE.

2 Experiment and Method

To overcome the imperfection of SVSLMS algorithm[8], we use a new variable step size LMS adaptive filtering algorithm.

$$u(n)=\beta(1-e^{(\| e(n)x(n)\|^2)}) \tag{4}$$

Figure 2 (a) shows the result of using SVSLMS algorithm for analysis. The error function $e(n)$ alternated near zero where the algorithm has steadied or will steady. But $u(n)$ changes too fast. The step size of SVSLMS algorithm varies in phase at adaptive steady state. This is our major concern. When β is in the rang of $0<\beta<2/\lambda_{\max}$, in contrary, the proposed algorithm controls the shape of the function, further more $\beta>0$ controls the value range of the function of $u(n)$ if $\alpha>0$. Obviously, we should conclude $0<\mu<1/\lambda_{\max}$ and $0<\beta<1/\lambda_{\max}$.



(a) (b)

Figure 2. (a)The graph of $u(n)$ and $\| e(n)\|$ (SVSLMS algorithm) (b)The graph of $u(n)$ and $\| e(n)x(n)\|$ (proposed algorithm)

3 Result and Discussion

In equation (1), $\| e(n)X(n) \|$ is very big and $\mu(n) \approx \beta$ initially as well as obtained the biggest convergence rate. Being close to steady state, $\| e(n)X(n) \|$ will reduce. When it reaches the steady state, both $\| e(n)X(n) \|$ and $u(n)$ becomes very small, and the MSE is also very small. When the input signal is non-stationary random signals, the instantaneous change of input signal causes $\| e(n)X(n) \|$ to change very much. It causes the algorithm automatically in fast convergence condition. The relational graph of $u(n)$ and $\| e(n)X(n) \|$ is shown in Figure 2 (b). From the Figure 2, in order to accelerate convergence rate, α and β should be big. In contrary, make the MSE smaller, α and β should be smaller.

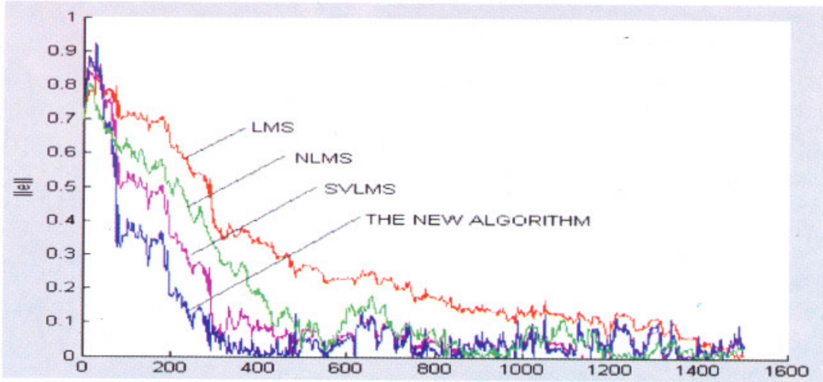


Figure 3. The convergence rate

Figure 3 is the result of examining the convergence performance of algorithms through the computer simulation. It shows the performance of SVSLMS and LMS as well as NLMS. Comparing with the SVSLMS, the convergence rate of the proposed algorithm is faster, and makes the MSE smaller enough.

This report presents new variable step size LMS adaptive filtering algorithm to eliminate the 50 Hz interference from the ECG. The block diagram is shown in Figure 1. Performance of the noise cancellation was tested by using stationary input signal ECG with 50 Hz sine wave added but frequency is undulating between 50 + 1 Hz and 50 − 1 Hz that models the input of main channel. Sine wave whose frequency fluctuates between 50 − 1 Hz and 50 + 1 Hz models the input of reference channel. The result of simulation is shown in Figure 4.

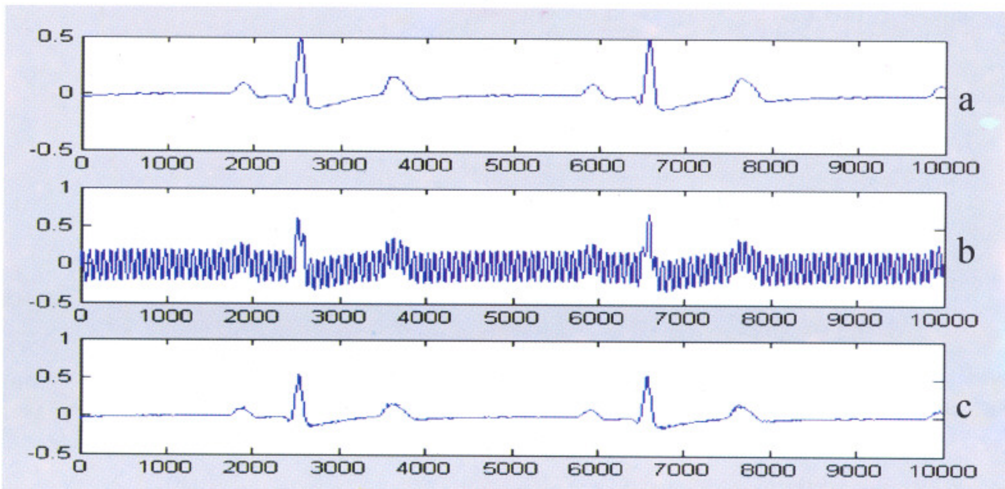


Figure 4. The result of simulation

(a): The ECG signal; (b): The ECG signal with 50 Hz interference; (c) The result of proposed elimination processing

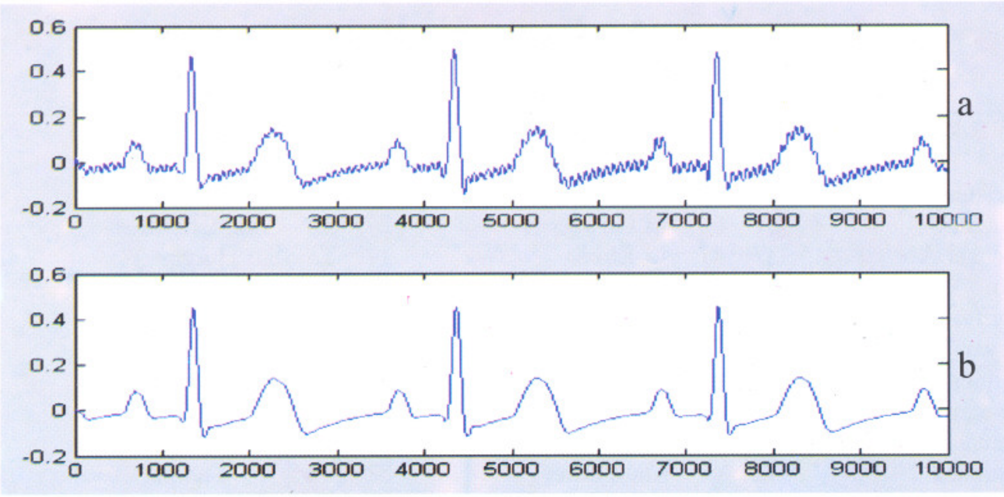


Figure 5. The real ECG signal
(a): The real ECG signal with 50 Hz interference; (b): The result of processing

In practice, we choose the reference channel which contains the 50 Hz interference and its harmonic. The common-mode signal recorded at the right leg reference electrode[9] which is truly correlated with the noise in ECG recording. The result of processing is shown in Figure 5. We see that 50 Hz interference to ECG is violent before processing and restrained after processing.

4 Conclusion

The approach proposed improves the convergence rate to enhance the calculation speed, cause the algorithms being more advantageous to apply in the real-time processing and reduce the MSE so as to improve the quality of eliminating interference. The result obtained by theory and simulation shows that this algorithm, compared with traditional LMS algorithm and other improved LMS algorithm, is much more effective to eliminate the 50 Hz interference from ECG signal.

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