Theoretical Analysis of Potential Runoff Energy from a Grid DEM

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Abstract: A simple potential runoff index (PRI) concept and its utility for distributed hydrological model is developed to compare a relative degree of potential runoff energy between different watersheds. In general, the actual response of hydrologic processes, while the time of precipitation processes from beginning to end, has closely correlations with the terrain relief on these watersheds, which usually can be expressed through a hypsometric curve. A traditional hypsometric curve was established by a linear regression analysis based on some relief data such as basin area, landform, topographical relief, drainage pattern and so on, but it absent a physically-based distributed hydrological model system and yet cannot be reflected the basin heterogeneities of topography, land cover and soil types so that only be used for local catchment area. Therefore, a new hypsometric curve based on digital elevation model (DEM) is produced in this study. In according with the new hypsometric curve a theoretical lag time of flow (TLTF) was computed, and then the PRI is defined an index which multiply TLTF by the relative catchment area, which the calculation of PRI from a grid DEM is finished automatically through GIS system using ARC/INFO functions. Furthermore, the PRI is used to estimate the potential runoff energy of several sub-basins within the Susquehanna River Basin in Pennsylvania, and the spatially distributive results of PRI are in good agreement with the historical runoff investigations. In addition, the PRI would be used to estimate a synthetic roughness (SR) of watershed, and further to analyze some correlations between SR and flood event, between runoff magnitude and soil moisture, as well as between runoff magnitude and land use or cropping pattern in agriculture, etc. [Nature and Science, 2004,2(1):17-23].

Key words: hypsometric curve; potential runoff index (PRI); distributed hydrological model; a grid DEM

1 Introduction

As well-know the topography of a basin has fundamental effects on its hydrologic response characteristics, the actual response of hydrologic processes has a closely correlation with the terrain relief in anywhere river basin, but both catchment area and landform are two crucial impact parameters (Black, 1996). The both important parameters were skillfully composed by a kind of hypsometric curve (Strahler, 1952) at the same time, which included a lot of topographical features in catchment area of the river basin and provides a visual representation of the watershed's profile too (Jenson, 1988; Verdin, 1999). Once the hypsometric curve is as a kind of cumulative frequency distribution these geomorphologic differences in the watershed can be computed by mathematical statistical analysis of hydrology (Harlin, 1978), for example, a coefficient of skewness will exits a significance correlation with a time of hydrograph peak (Harlin, 1984).

Such a correlation parameters, however, were produced with empirical data during the past decades, and still were incapable of reflecting the reality of water flow in catchment networks until now. Moreover, a traditional hypsometric curve has been created by regression equation based on some relief data such as basin area, landform, topographical relief, drainage pattern and so on, which the hypsometric curve only be used for local catchment area and is improper to apply to distributed hydrological models directly. Consequently, a new hypsometric curve based on digital elevation model in distributed hydrologic model system is developed.

The extraction of potential runoff energy from a grid DEM is one of the essential components of most physically based distributed hydrological models. On the one hand, the spatial distribution of topographic index may be derived from the DEM of the basin

(Jenson, 1993; Quinn, 1995; Yu, 2001a). On the other hand, because land surface of study area is not a homogeneous, a basic strategy to solve this problem is to subdivide the watershed into some different relatively homogeneous parts (Ao, 1999). The main aim of this study focused on the development of a new hypsometric curve using distributed hydrological model system and the assessment of potential runoff energy in the catchment area through the PRI. Meanwhile, in order to make it suitable to handle the spatial heterogeneities of factors such as vegetation, soil properties, etc., a block-wise method use of the new hypsometric curve was proposed for runoff generation in this study.

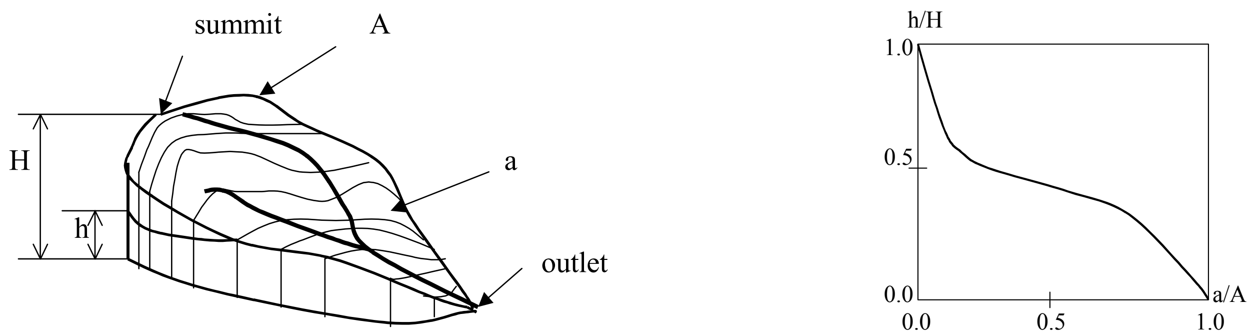


Figure 1 A Catchment area and its general longitudinal section profile

2 Methods

2. 1 Hypsometric curve

Due to hydrological properties of a basin are relative to the geological and topographical features of the ground surface, a new hypsometric curve based on digital elevation model usually describes a correlation both elevation and catchment area in river basin, which can be expressed by a general longitudinal section profile in the watershed as shown in Figure 1. For regional analytical geomorphic research, the axis is plotted as rations of each zone to the total relief or area. Thus, the hypsometric curve is defined by multinomial equation as following

$$y = c_0 + c_1 x + c_2 x^2 + \cdots + c_n x^n \qquad (1)$$

where y is the relative height, $y = h/H$; x is the relative area, $x = a/A$; c_i is multinomial constant coefficient, $0 \le i \le n$, in general, n=3~5 (Harlin, 1978).

2. 2 Theoretical lag time of flow

We assume, when a unit quantity water droplet flows along the longitudinal section profile from top to bottom in the whole hydrological processes, that the time of flowing-through period is called theoretical lag time of flow (TLTF), i.e. so-called lag time of watershed. If any one point $x = x_0$, on the curve of the longitudinal section profile (Figure 2), it can be written into one order integral form (Lou, 1998)

$$\tan\theta = y' = c_1 + 2c_2 x + \cdots + nc_n x^{n-1} \qquad (2)$$

While a unit quantity water droplet m flows down along the longitudinal section profile (impervious), the water droplet has an equilibrant equation at $x=x_0$, namely,

$$mg\sin\theta - \mu mg\cos\theta = ma \qquad (3)$$

Where g is gravity accelerated coefficient, g=9.8 m/s^2; a is a tangential acceleration at $x=x_0$; μ is a

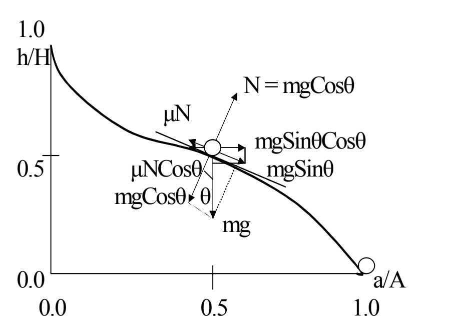


Figure 2 The new hypsometric curve

friction factor, that is as a considerable index reflecting synthetic roughness (SR) of catchment area either.

Furthermore, a horizontal component of the tangential acceleration will be expressed as

$$a_x = a\cos\theta = (g\sin\theta - \mu g\cos\theta)\cos\theta$$
$$= g\left(\frac{\tan\theta}{1+\tan^2\theta} - \mu\frac{1}{1+\tan^2\theta}\right) \quad (4)$$

At the moment *t*, if the water droplet has a rate *$v_x(t)$* and moves a distance dx in a interval time, from t to *t+dt*, the changing rate is given by

$$dx = v_x dt \quad \text{and} \quad dv_x = a_x dt \quad (5)$$

$$\text{or} \quad dt = \frac{dx}{v_x} \quad (6)$$

by using (5), equation (6) becomes

$$v_x dv_x = a_x dx \quad (7)$$

and integrating equation (7)

$$\frac{v_x^2}{2} = \int_0^x a_x dx \quad (8)$$

combining equation (6) with substituting (8), thus

$$t = \int_0^t dt = \int_0^1 \frac{dx}{v_x} = \int_0^1 \frac{dx}{\sqrt{2\int_0^x a_x dx}} \quad (9)$$

In here, t is called TLTF, which the TLTF is easy to be automatically computed using Newton's law and numerical integral method from a grid DEM in Unix Workstation System (Jenson, 1993; Yu, 2000b). Within equation (9), the gravity acceleration constant has been regularity divided by a difference from summit to outlet in the study watershed.

2. 3 Potential runoff index

If that a "regular lag time" (RLT) of the watershed to be defined by a time of practical observation time divided by relative catchment area, while the actual response of hydrologic processes from summit to outlet on the river basin, the difference between TLTF and RLT exist usually a positive linear correlation, shown in reference (Saghafian, 1995). Therefore, it is possible that a simple potential runoff index (PRI) with multiply TLTF by the catchment area can be as a special index using for compare relative degree of runoff potential energy between different watersheds (Lou, 1997). The PRI is defined as follows:

$$\text{PRI} = \text{TLTE} \times \text{a catchment area} \quad (10)$$

We will pay attention to that TLTF does not directly equal practical observation value (i.e. a lag time of hydrograph peak (LTHP)), but a difference between TLTF and LTHP still performs some important differences of the topographical features in different catchment area. Meanwhile, under the same condition of precipitation the short this TLTF is, the larger the potential runoff energy is, that is, the more the possibility of runoff or flood appears in the watershed.

3 A Grid DEM

For this study, the DEM was generalized to the grid. To extracted topographical features of the study area, an interactive command system called "GRID" in the ARC/INFO package performs such tasks, and the features data obtained from the ARC/INFO processing procedures are in the form of grids, which the elevation at each 3-arc second grid point was assumed to be the average elevation over a 3×3 arc-second rectangle (every grid cell is about 100 m by 100 m).

To improve accuracy for predicting hydrological responses in watershed area, a study basin will be divided into different relatively homogeneous blocks, which size of the blocks depends on the heterogeneity of topography and land cover as well as the basin scale. Each block size is comprised of numerous grid cells but this manageable block size is one in which the friction factor (i.e. SR) can be identified by different soil type or land cover. Based on the subdivision of the basin, the friction factor is calibrated for each block rather than the whole catchment, and synthetic roughness index is also calculated for each block. Through this improvement the heterogeneities of topography, land cover and soil type in a large basin can be approximated. Certainly, this subdividing method is not perfect, but it has the advantages of simplicity and flexibility in using GIS information (Gurmell, 2000).

4 Primary Application

4. 1 Researching basins (Yu, 1999, 2000a, 2001b)

The area of these applications is the Susquehanna River Basin (SRB), which is 80,300 km^2 watershed covering portions of New York, Pennsylvania, and Maryland of the United States. The SRB flows south into the Chesapeake Bay at Baltimore, Maryland (Lakhtakia, 1998). The applicability of the new hypsometric curve and PRI are examined in four sub-basins within the SRB, in which there are different sizes and various terrain characteristics and show in Table 1 and Figure 3.

Table 1 General description of the four sub-basins in SRB

	Drainage Area (km^2)	H(m)	Streamflow (m^3/sec)	M
WE-38	7.5	267	0.69	0.088
Mt	422	396	1.52	0.085
UWB	7,952	495	2100	0.035
UNB	10,016	495	2810	0.033

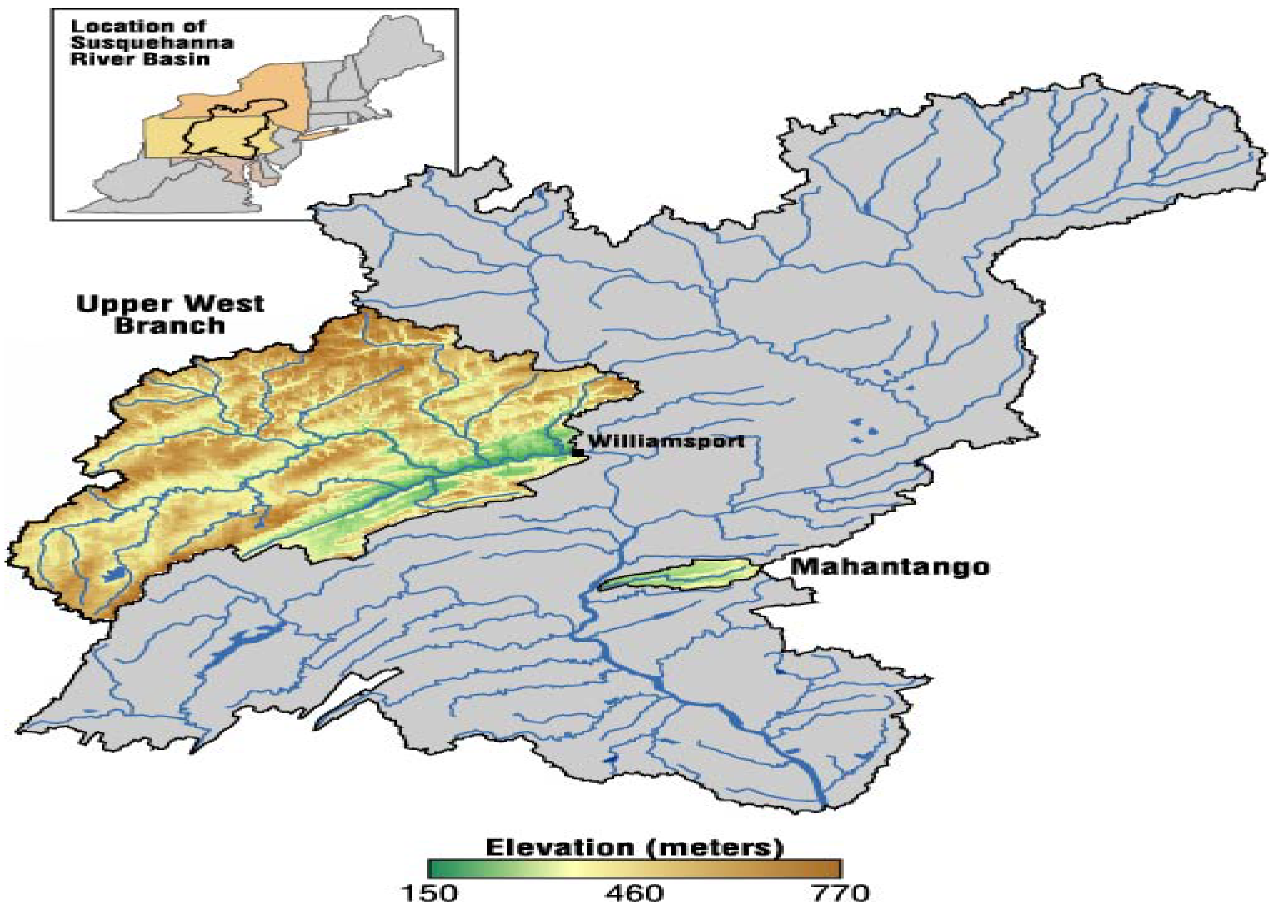


Figure 3 Location map of four sub-basins in susquehanna river basin

The Upper West Branch (UWB) watershed is a sub-basin of the SRB and located in north-central Pennsylvania with an area of 14,710 km^2. The watershed lies within the Appalachian Plateau Physiographic Province and is mainly covered by forest. The stream generally flows east and merges with the Susquehanna River at Williamsport, Pennsylvania. Another sub-basin of the SRB is the upper north branch (UNB) with an area of 27,518 km^2, which lies within the glaciated Appalachian Plateau Physiographic Province. The lower portion of the SRB is in the Appalachian Mountain section of the valley and ridge Physiographic Province. The topography in this region is controlled by a succession of narrow, step-sided ridges and valleys, trending northeast to southwest, and is prone to runoff. Detailed information on climate, soil, vegetation, topography, surface hydrologic parameters, and subsurface hydrology is provided in Yu et al (2000a). One of example is implemented in the WE-38 watershed. The WE-38 is a typical upland agricultural sub-watershed with a catchment area of 7.29 km^2 in the East Mahantago Creek of the SRB. Elevation within the watershed ranges from 230 m at the watershed outlet to 490 m above sea level (msl) at the top of the watershed divided, and its dip is about 22^o-30^o from south to north.

4.2 Block-wises use of distributed hydrologic model

The previous generation of hypsometric curve cannot reflect the effects of catchment changes on hydrological responses, and it is incapable of analyzing general and specific hydrological processes, one of main reasons is that the friction factor of the basin has be regardless. Theoretically, since the catchment was divided into blocks by referencing different land cover and soil types in the large watershed a synthetic roughness index can be reflected, while the friction factors were identified for each block rather than the whole basin.

Through this improvement the method simplicity is maintained, the new hypsometric curve can be used to provide a tool to explore general hydrological phenomena or specific runoff processes and to assess the impacts of anthropogenic basin changes on hydrological responses. And then its use of parameters which has a physical interpretation and the representation of spatial variability in parameter values, namely, further to reflect the influences of watershed changes on hydrological responses and be utilized for analyzing hydrologic processes. In the Table 1, for further consider land cover changes and /or influences, the four sub-basins with different friction factor -- natural and spurious pits or sinks, have to be divided into different blocks while used different μ value (assuming μ=0.015-0.250, Beven, 1997).

5 Results and Discussions

The grid DEMs used for the UWB is the USGS 3-arc second data set. Due to efforts of the scale varied in space have influence to the distribution of hypsometric curve; we compare different space scale effects with grid cell sizes or grid spacing of 15m, 50m,100m and 200 m.

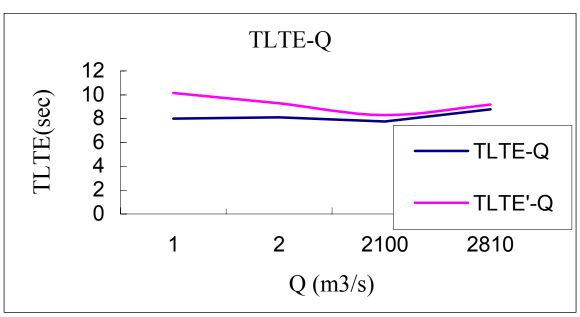
5. 1 Effects of grid spacing

From Table 2, it seems that overall hydrological responses with various grid spacing from 15 m to 100 m give the same values both of TLTE and PRI in SRB. This is because the scale of grid size cannot change the distribution of hypsometric curve (Kalma, 1995), yet the potential runoff energy on the catchment area has not relationship with the scale of grid size in different distributed hydrologic model.

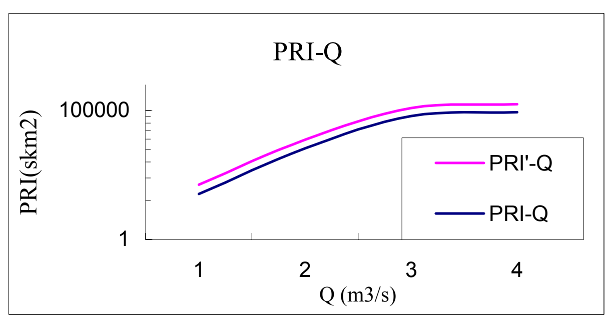
All of calculation results of PRI and relative streamflow of the catchments are shown in Figure 4. The practical application results in Susquehanna River Basin indicated that the response of runoff has marked correlation with topographical relief and that PRI compare well with the data of historical runoff record, and the relative coefficient of squared value on the chart is calculated about 0.992.

5. 2 Validation of land cover

Figure 4 or Table 3 shows the calculate resu-lts of TLTE and PRI of μ=0 are larger than that of μ≠0, which reflect the storage effect of land cover in the watershed. Obviously, the land cover is a very important factor to decide runoff processes in the catchment area. In the traditional hypsometric curve, hydrologic heterogeneity-ies such as topography, land use and soil properties have been considered as homogeneous, therefore traditional



(a) Relationship of TLTE with different μ value (Scale: 100 m×100 m)



(b) Relationship of PRI with different μ value (Scale: 100 m×100 m)

Figure 4 The correlation between TLTE or PRI and streamflow of study areas

Table 2 TLTE and PRI of the Four Sub-basins in SRB

	TLTE				PRI			
Scale	15m	50m	100m	200m	15m	50m	100m	200m
WE-38	8.04	8.04	7.88	7.96	58.52	58.77	59.12	62.05
Mt	8.12	8.12	8.12	8.12	3424	3424	3424	3426
UWB	7.77	7.77	7.77	7.77	61773	61771	61771	61774
UNB	8.78	8.78	8.78	8.78	87942	87943	87915	87911

Table 3 Comparison of the values of TLTE and PRI

	M	TLTE	PRI	M	TLTE’	PRI’
WE-38	0.0	7.99	59	0.088	10.15	74
Mt	0.0	8.12	3425	0.085	9.29	3921
UWB	0.0	7.77	61772	0.035	8.01	63712
UNB	0.0	8.78	87928	0.033	9.18	91918

approaches are unable to provide insights into the understanding of the effects of hydrologic processes. On the contrary, a new distributed hypsometric curve emphasize spatially heterogeneity within individual grid cells and employ digital elevation model data to account for heterogeneity, so that a more realistic representation of spatial variations of various hydrologic processes is obtained.

5. 3 Treatment of special areas within grid DEM

Most pits are considered to be spurious, and large-scale pits are generally rare, it is, a fact that the percentage of pits in existing DEM is relatively low. We also compare the results both of original and filled terrain, the rations of removing or filling pit is usually small than 3% of the whole grid cells in SRB, so whether or not the pits and depressions have be filled do not yield deviation significantly either (Figure 4 (c)). In fact, whatever the creating methods or data sources of available a grid DEM, such as by interpolation from contour maps, i.e. by image correlation devices from aerial photographs including satellite imagines and regardless of its resolution. In natural landscapes, particularly in large scale, pits and flat surfaces are relatively rare (Garbercht, 1997) but in currently available various grid spacing, about 5% of grid points are pits (limited investigation).

The PRI, however, is also considered an index of hydrologic similarity and expressed as a distribution function. Because whether any watershed has a higher PRI will depend on some elements such as climatic condition, precipitation intensity and duration etc, so that the PRI can be used to compare the relative degree between different watersheds where happening runoff.

By the way, some relative index influencing runoff such as precipitation, infiltration and soil moisture didn’t be considered in this study, but PRI still can be used to express a different potential runoff energy between different watersheds under above mentioned conditions.

6 Summary

From the foregoing discussions, we can see that topography and land cover predominantly affected the hydrological response process and pattern in a basin. The new hypsometric curve from a grid of digital elevation model had been given a simpler and efficient method to describe topographical features in the river basin, which is easy to be used for distribute hydrological modeling. Based on the new hypsometric curve the potential runoff index (PRI) is produced in this study. Under the influences of different terrain relief and relative drainage area, a PRI reflects the response of precipitation in researching watershed, and shows that the hydrologic possess and the potential runoff energy to compare with other watershed. Accordingly, through the conclusions derived from the above applications, it can be concluded that the PRI has broad applicability and can be used for different purposes; it can be applied to different catchments of various sizes and to simulate the hydrological response processes in a basin; it is able to applied to un-gauged basins for hydrological simulations; and it can be used as a basic tool for finding out the relationship between model parameter values and GIS information, as well as for analyzing the effects of human influences on runoff characteristics through runoff simulations.

Acknowledgment

This study was partly funded by NASA and EPA. All computations were performed on the Unix workstation of Hydro-Computing Laboratory in the Department of Geosciences at UNLV.

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