Ideal Interval Method Based Model for Water Quality Evaluation

Juliang Jin [1], Shujuan Wang [1], Yiming Wei [2]

(1.College of Civil Engineering, Hefei University of Technology, Hefei, Anhui 230009, PRC, jinjl66@xinhuanet.com;
2. Institute of Policy and Management, Chinese Academy of Sciences, Beijing 100080, China)

Abstract: The key of water quality evaluation is to establish the nonlinear relation between water quality indexes and water quality grades. Nowadays only evaluation criterion of water quality, which is expressed by interval concept, reflects this relation information during practical evaluation process. For this reason, a new model for water quality evaluation is presented based on improved ideal interval method with using accelerating genetic algorithm (IIM-WQE). Research results show that IIM-WQE is directly driven by the example series produced from evaluation criterion of water quality, and water quality relative grades computed by using all membership degrees information are regarded as evaluation results of water quality, which can avoid distorting when using the principle of maximum membership degree and can heighten precision of water quality evaluation. IIM-WQE is visual, handy and universal, which can widely be applied to different system evaluations if only evaluation criterion or samples of evaluation indexes and evaluation result values are known. [Nature and Science 2004,2(1):24-28].

Key words: water environment systems engineering; evaluation of water quality; ideal interval method; genetic algorithm; relative membership degree

1 Introduction

Water Quality Evaluation (WQE) is to comprehensively evaluate water quality grades according to evaluation criterion of water quality and water quality indexes. It is very important to heighten the scientific level of water planning management and prevention and control of water pollution, to promote sustainable development of regional society, economy, resource and environment (Chen, 1998; Jin, 2002). Due to the incompatibility of evaluation results among individual water quality index of real water, it will lack practice for judgment of evaluate water quality grades by using evaluation criterion table of water quality directly, for this reason, many evaluation methods have been presented now. Each method has its applicability (Chen, 1998; Jin, 2000; 2001a; 2002a; 2002b; Hu, 1998). All the essences of these methods are try to describe the complex nonlinear relation between evaluation indexes and water quality grades, while nowadays only evaluation criterion of water quality which is expressed by interval concept (Chen, 1998; Hu, 1998) reflects this relation information during practical evaluation process. For this reason, in this paper a scheme of ideal interval method applied to evaluation of water quality, named IIM-WQE for short, is presented based on Yang (2003) and accelerating genetic algorithm (AGA) (Jin, 2001b; 2002a). The evaluation results of IIM-WQE continuous are real number values, so its evaluation precise is high. In the end an application case of IIM-WQE was done.

2 Establishment of IIM-WQE

Suppose evaluation criterion of water quality is $\{([a^*(i,j), b^*(i,j)], i)|i=1\sim ni, j=1\sim nj\}$. Where $a^*(i,j)$ and $b^*(i,j)$ are lower limit and upper limit of variation interval, which is named ideal interval (Yang, 2003) of ith water quality grade jth evaluation index respectively; i is criterion grade value of ith water quality grade; ni and nj are the number of grades and evaluation indexes of water quality evaluation criterion respectively. IIM-WQE establishment procedure includes four steps as follows:

Step1: to create sample series of water quality evaluation criterion randomly and standardize them. Produce uniformly and randomly nu index sample values $x^*(k,j)$ in the variation interval of each index , corresponding criterion grade values are $y(k)=i$; in order to fully reflect the importance of border values of each index in evaluation criterion, select each border value of indexes once, and corresponding criterion grade value is the average of the two criterion grades values related to this border value. Thus sample series of water quality evaluation criterion $\{x^*(k,j), y(k)\}|k=1\sim nk, j=1\sim nj\}$ are obtained, where *nk* is the number of the samples. In order to eliminate dimensional effect, and to make IIM-WQE have universality, the indexes are treated as follows:

$$x(k,j) = x^*(k,j)/x_{\max}(j) \quad (k=1\sim nk，j=1,nj) \quad (1)$$

$$a(i,j) = a^*(i,j)/x_{\max}(j) \quad (i=1\sim ni，j=1,nj) \quad (2)$$

$$b(i,j) = b^*(i,j)/x_{\max}(j) \quad (i=1\sim ni，j=1,nj) \quad (3)$$

where $x_{\max}(j)$ is the maximum of the *j*th evaluation index in water quality evaluation criterion, that is $x_{\max}(j) = \max_i\{b(i,j)\}$.

Step 2: to compute the distance $D(k,i)$between criterion sample $x(k,j)$ and ideal interval of criterion grade [$a(i,j)$，$b(i,j)$]:

$$D(k,i) = \sum_{j=1}^{nj} w(j)d(k,i,j) \quad (4)$$

$$d(k,i,j) = \begin{cases} a(i,j) - x(k,j), x(k,j) < a(i,j) \\ 0, x(k,j) \in [a(i,j), b(i,j)] \\ x(k,j) - b(i,j), x(k,j) > b(i,j) \end{cases} \quad (5)$$

where $w(j)$ is the weight of the *j*th index, it can be determined by consulting expert with subjective weight method based on the analytic hierarchy process, by objective weight method based on projection pursuit according to the sample series of water quality criterion (Jin, 2001a; 2002a), or by combining the subjective weight and objective weight; where $k=1\sim nk$, $i=1\sim ni$, $j=1\sim nj$.

Step 3: to compute relative membership degree values $r(k,i)$ of the sample $\{x(k,j)\}$ relative to criterion grade ideal interval:

$$r(k,i) = e^{-cD(k,i)} / \sum_{i=1}^{ni} e^{-cD(k,i)} \quad (6)$$

where *c* is a model parameter, it is a constant more than 1 generally. The more *c* is, the more easily criterion sample $x(k,j)$ tend to criterion grade which has small relative membership degree $D(k,i)$. In order to avoid a distorting judgment by using maximum membership degree rule, to utilize all the relative membership degree information, and to make the judgment more accord with apply the fact, water quality relative grade (Chen, 2002) can be adopted as computed value of water quality criterion grade which corresponds to the criterion sample $x(k,j)$:

$$h(k) = \sum_{i=1}^{ni} r(k,i)i \quad (7)$$

And then *c* can be optimized by solving equations (8)

$$\min \quad f(c) = \sum_{k=1}^{nk} |h(k) - y(k)| \quad (8)$$

AGA is a universal global optimization method, which simulates best selected and bad abandoned principle and chromosome population information exchange mechanism in the creation evaluation process. It is more handy and effective to solve optimization problem like equations (8). The detail algorithm of AGA can be seen in Jin (2002a) or Jin (2001b).

Step 4: to comprehensively evaluate water quality grade. Suppose each water quality evaluation index of researched water sample is $\{z^*(k,j)|k=1\sim nz，j=1\sim nj\}$, where nz is the number of water samples needed evaluation. Replace $x^*(k,j)$ with $z^*(k,j)$, put it into equations (1), and then water quality grade $h(k)$ ($k=1\sim nz$) can be computed corresponding to evaluation example according to equations from (2) to (8).

3 Case Study

Now as is a case, rich nutrient comprehensive evaluation of lake water environment quality illustrates the process of applying IIM-WQE. Evaluation criterion of lake water environment quality (Hu, 1998) is seen in Table 1, where the left end value of ideal interval of the first water quality grade of TP, TOD, TN indexes, and the right end value of that of transparency index are determined by the authors.

Table 1 Evaluation criterion of lake water environment quality (Hu, 1998)

evaluation index	water quality grade				
	even poor nutrition	poor nutrition	middle nutrition	rich nutrition	even rich nutrition
	1st grade	2 nd grade	3 rd grade	4 th grade	5 th grade
TP(μg/L)	[0, 1]	[1, 4]	[4, 23]	[23, 110]	[110, 660]
TOD(mg/L)	[0.00, 0.09]	[0.09, 0.36]	[0.36, 1.80]	[1.80, 7.10]	[7.10, 27.10]
Transparency(m)	[37, 74]	[12, 37]	[2.4, 12]	[0.55, 2.4]	[0.17, 0.55]
TN (mg/L)	[0.00, 0.02]	[0.02, 0.06]	[0.06, 0.31]	[0.31, 1.20]	[1.20, 4.60]

According to Table 1 and step 1 of IIM-WQE modeling, evaluation criterion sample series composed of 54 sample dots from No.1 to No.54 shown in Table 2 can be gained. Transform the 54 indexes sample values $\{x^*(k,j)|k=1\sim54, j=1\sim4\}$ to $\{x(k,j)| k=1\sim54, j=1\sim4\}$ according to equations (1), scale each interval in Table 1 by using equations (2) and equations (3), each index weight is 1/4, calculate evaluation criterion grade of water quality $h(k)$ (k=1~54) by using equations from (4) to (7), put them to equations (8) with $\{y(k)|k=1\sim54\}$, solve equations (8) by using AGA, and obtain the optimum evaluation value of c which is 99.888, and the corresponding minimum of objective function $f(c)$=2.771. Error analysis between evaluation criterion grades of water quality and grades of IIM-WQE are shown in Table 3 when c=99.888. Table 2 and Table 3 show that the nonlinear relation between water quality indexes and water quality grades described by IIM-WQE is satisfied. So IIM-WQE can be used to evaluate rich nutrition grades of the five lake water quality, its result is shown in Table 4, where the evaluation results of other methods are also listed. Table 5 shows the distances and membership degree values between lake water quality samples and ideal intervals of criterion grades.

Table 2 Comparison results between evaluation criterion samples of water quality and values of IIM-WQE

No.	water quality index				water quality grade		No.	water quality index				water quality grade	
	TP (μg/L)	TOD (mg/L)	transparency (m)	TN (mg/L)	criterion	IIM-WQE		TP (μg/L)	TOD (mg/L)	transparency (m)	TN (mg/L)	criterion	IIM-WQE
1	0.658	0.059	50.857	0.001	1.000	1.008	28	12.631	0.479	4.738	0.257	3.000	3.045
2	0.559	0.051	42.650	0.015	1.000	1.120	29	7.879	1.129	4.840	0.249	3.000	3.074
3	0.106	0.034	67.902	0.009	1.000	1.000	30	14.074	1.405	3.723	0.140	3.000	3.102
4	0.978	0.061	42.207	0.010	1.000	1.137	31	21.122	1.442	3.487	0.142	3.000	3.151
5	0.646	0.051	47.659	0.015	1.000	1.025	32	9.431	0.655	11.033	0.091	3.000	2.729
6	0.078	0.056	73.851	0.010	1.000	1.000	33	23.000	1.800	2.400	0.310	3.500	3.499
7	0.742	0.086	54.876	0.013	1.000	1.002	34	36.301	2.753	1.872	0.802	4.000	3.986
8	0.607	0.014	54.330	0.011	1.000	1.003	35	63.988	3.232	1.064	1.159	4.000	4.003
9	0.142	0.029	43.305	0.000	1.000	1.089	36	84.708	4.817	2.045	0.465	4.000	3.998
10	0.316	0.003	48.395	0.008	1.000	1.018	37	95.742	5.358	0.594	0.805	4.000	4.013
11	1.000	0.090	37.000	0.020	1.500	1.500	38	50.872	5.349	0.816	1.151	4.000	4.015
12	2.790	0.195	14.496	0.040	2.000	2.241	39	34.379	2.639	2.191	0.917	4.000	3.990
13	3.862	0.356	18.619	0.049	2.000	2.090	40	87.044	3.024	1.423	0.856	4.000	4.000
14	3.569	0.338	12.537	0.055	2.000	2.440	41	32.149	4.351	1.692	0.382	4.000	3.965
15	1.816	0.300	14.218	0.034	2.000	2.263	42	84.474	1.817	1.552	1.138	4.000	4.001
16	3.011	0.268	29.132	0.057	2.000	1.959	43	67.301	3.868	2.334	0.758	4.000	3.998
17	1.060	0.132	25.247	0.059	2.000	1.994	44	110.000	7.100	0.550	1.200	4.500	4.500
18	3.368	0.187	33.033	0.034	2.000	1.832	45	649.959	21.826	0.210	3.396	5.000	5.000
19	3.842	0.324	20.456	0.036	2.000	2.044	46	161.122	19.802	0.428	3.580	5.000	5.000
20	3.889	0.299	17.987	0.058	2.000	2.109	47	281.367	10.708	0.515	2.934	5.000	5.000
21	3.414	0.270	18.595	0.033	2.000	2.076	48	443.507	26.418	0.364	2.626	5.000	5.000
22	4.000	0.360	12.000	0.060	2.500	2.501	49	355.354	8.808	0.438	2.126	5.000	5.000
23	21.883	1.292	8.447	0.165	3.000	2.999	50	549.721	9.753	0.250	2.602	5.000	5.000
24	10.488	0.797	2.943	0.196	3.000	3.088	51	184.419	11.128	0.263	3.523	5.000	5.000
25	13.009	1.414	9.719	0.197	3.000	2.966	52	342.262	24.807	0.370	2.658	5.000	5.000
26	9.853	1.116	6.878	0.255	3.000	3.026	53	181.062	12.666	0.274	2.875	5.000	5.000
27	9.925	1.479	6.117	0.079	3.000	3.002	54	233.632	26.579	0.250	1.943	5.000	5.000

Table 3 Error analysis between evaluation criterion grades of water quality and grades of different models

Evaluation method	percent of absolute error falling the following range (%) [0, 0.1]	[0, 0.2]	[0, 0.3]	[0, 0.4]	[0, 0.5]	[0, 0.6]	mean absolute error (grade)	mean relative error (%)
pp method[5]	48.00	68.00	92.00	92.00	96.00	100.00	0.15	7.74
interpolation method[6]	89.66	93.10	93.10	94.83	98.28	100.00	0.05	1.58
IIM-WQE	81.48	92.59	98.15	98.15	100.00	100.00	0.05	2.65

Table 4 Computed grades of five lakes water quality by using different model

Lake	water quality index TP (μg/L)	TOD (mg/L)	Transparency (m)	TN (mg/L)	computed water quality grades of different models NN[3]	Logistic curve[4]	interpolation[6]	IIM-WQE
Hangzhou Xi Lake	130	10.30	0.35	2.76	5	4.92	4.860	5.000
Wuhan Dong Lake	105	10.70	0.40	2.00	4	4.80	4.546	4.999
Qinghai Lake	20	1.40	4.50	0.22	3	3.09	3.028	3.150
Chao Lake	30	6.26	0.25	1.67	4	4.59	4.058	4.241
Dian Lake	20	10.13	0.50	0.23	4	3.67	3.638	3.998

Table 5 Distance and membership degree values between lake water quality samples and ideal intervals

lake	distance between lake water quality samples and ideal intervals of criterion grades					membership degree values between lake water qualities samples and ideal intervals of criterion grades				
	1st grade	2nd grade	3rd grade	4th grade	5th grade	1st grade	2nd grade	3rd grade	4th grade	5th grade
Hangzhou Xi Lake	0.416	0.326	0.259	0.123	0.000	0.000	0.000	0.000	0.000	1.000
Wuhan Dong Lake	0.369	0.278	0.212	0.077	0.002	0.000	0.000	0.000	0.001	0.999
Qinghai Lake	0.140	0.050	0.000	0.017	0.153	0.000	0.006	0.838	0.156	0.000
Chao Lake	0.282	0.191	0.125	0.027	0.038	0.000	0.000	0.000	0.759	0.241
Dian Lake	0.235	0.144	0.083	0.034	0.087	0.000	0.000	0.007	0.988	0.005

Table 3, Table 4 and Table 5 show that: 1) The evaluation results of IIM-WQE are the same as other methods basically. 2) According to the evaluation criterion of environment quality in Table 1, the results of IIM-WQE are reasonable. 3) The results of IIM-WQE that the water quality grades of these five lake from big to small are Hangzhou Xi Lake, Wuhan Dong Lake, Chao Lake, Dian Lake, and Qinghai Lake in turn, where the first two lakes can be judged as 5th grade (even rich nutrition), the later lake can be judged as 3rd grade (middle nutrition), the rest can be judged as 4th grade (rich nutrition).

4 Conclusion

In order to describe the nonlinear relation between water quality indexes and water quality grades, a new method for water quality evaluation based on improved ideal interval method and accelerating genetic algorithm (IIM-WQE) is presented. IIM-WQE belongs to non-function model evaluation methods, its evaluation process is directly driven by sample series produced from water quality evaluation criterion, which overcomes the difficulty how evaluation function relation between evaluation indexes and evaluation grades is reasonably structured. The results of IIM-WQE are of real values, high precision. IIM-WQE is visual, handy and universal, which can widely be applied to system evaluation of evaluation criterion or evaluation indexes and comprehensive evaluation values are known.

Acknowledge

Foundation Item: The Excellent Young Teachers Program in Higher Education Institute of MOE, P.R.C.

[Department of Education and Person (2002)350], Anhui Provincial Science and Technology Fund of Excellence Youth, Anhui Provincial Natural Science Foundation (No. 01045102), Chinese National Natural Science Fund Project (No. 40271024).

References

[1] Chen S. Fuzzy optimum recognition theory and application for complex water resource system[M]. Changchun: Jilin University Press, 2002.

[2] Chen S. Theory and Application for Engineering Fuzzy Sets[M]. Beijing: Chinese National Defence Industry Press, 1998.

[3] Hu M, Guo D. Fuzzy Neural Network Method for Evaluation of Lake Water Quality Rich Nutrition[J]. Chinese Environment Science Research, 1998,(4):40-2.

[4] Jin J, Din J, Water Resource Systems Engineering[M]. Chendu: Science and Technology Press of Sichuan, 2002a.

[5] Jin J, Wei Y, Din J. Projection Pursuit Model for Comprehension Evaluation of Water Quality[J]. Chinese Journal of Environment Science, 2001a,21(4):431-434.

[6] Jin J, Wei Y, Fu Q, *et al.* An Interpolation Model for Comprehensive Evaluation of Water Quality [J]. Chinese Journal of Water Hydraulic, 2002b,(12): 91-4, 100.

[7] Jin J, Yang X, Din J. Improvement of Standard Genetic Algorithm-Accelerating Genetic Algorithm[J]. Systems

Engineering—Theory & Practice, 2001b , 21(4):8-13.

[8] Jin J, Yang X, Jin B, et al. New Model for Comprehensive Evaluation of Water Environment Quality[J]. Chinese Environment Monitor ,2000, 16(4):42-7.

[9] Yang X, Yang Z, Li J, et al. Comprehensive Evaluation of Atmosphere, Environment Quality for Multi-objective Decision-IIM[J]. Chinese Environmental Engineering, 2003, 21(3):70-2.