Self-Exciting Threshold Auto-Regressive Model (SETAR) to Forecast the Well Irrigation Rice Water Requirement

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Abstract: Some periodicity was noticed in records of water usage for rice irrigation, which was analyzed using the Self-Exciting Threshold Auto-Regressive (SETAR) model. Using auto-correlation analysis, values of rice evapotranspiration for different growth phases were found to depend on each other. Weather and other factors cause the rice evapotranspiration to change periodically. So the SETAR model was prepared for analysis of groundwater usage to irrigate rice in the Sanjiang Plain. Nine parameters were used to describe the periodicity of weather factors. By comparison with practical values, the precision is high. So, the model can be used to help layout and manage an irrigation area. At the same time, it can be applied to optimizing an irrigation system. [Nature and Science 2004,2(1):36-43].

Key words: SETAR model, well irrigation rice, water requirements

1 Introduction

There are many models to estimate rice crop water requirements based on either simple regressions linked to temperature or radiation and the physically based equations of Penman and Monteith. A sequential analysis method may also be used to identify an auto-regression model $AR(p)$ containing a trend and cyclic component of a water requirement sequence. This article establishes a sectional auto-regression model $AR(p)$ to describe the mean annual sequence of irrigation water requirements. The SETAR model was established by locating threshold values r and selecting suitable delay steps d and threshold ranges L.

2 Brief Introduction on SETAR Model

Tong (1978) first proposed the Self-Exciting Threshold Auto-Regressive Model (SETAR). Its basic idea is to introduce $l-1$ thresholds $r_j\,(j=1,2,3,\cdots,l-1)$ in the range of an observed sequence $\{x(t)\}$, divide time axis into l ranges, distribute observation sequence $\{x(t)\}$ into different threshold ranges according to the value of $\{x(t-d)\}$ by delay steps d and then adopt different AR model to describe the entire system for $x(t)$ in each range. The common form of Threshold Auto-Regressive Model (TAR) is：

$$x(t)=\varphi_j(0)+\sum_{i=1}^{nj}\varphi_j(i)x(t-i)+a_j(t)$$

$$r_{j-1}<x(t-d)<r_j \quad (\mathrm{j}=1,2,3,\cdots,\mathrm{l}) \tag{1}$$

In formula (1), $a_i(t)$ is the white noise list of σ_i^2. σ_j^2 is variance. (i) is the auto-regressive coefficient of model in the threshold region. nj is the rank of model in the threshold region. The key difference between equation (1) and an AR model is that they distribute observation sequence $\{x(t)\}$ into different threshold ranges according to values of $\{x(t-d)\}$ and adopt a different R model to describe all $x(t)$ in each range. becomes an AR model when the value of l equals to 1 and the value of d equals to 0.

Equation (1) indicates that it builds up an $AR(n_j)$ model for n sequential data between $x(t)$ and $x(t-n_j)$ when the value of $x(t-d)$ is in the range of (r_{j-1},r_j), where t is cursor. If the number is K and the value of is in the range of (r_{j-1},r_j), we can use

K groups of data that have n_j+1 sequential data to establish $AR(n_j)$ model. In other words, we may ascertain the AR model at the time of t according to the range of the value of $x(t-d)$.

3 Modeling on Threshold Auto-Regressive Model

Because Threshold Auto-Regressive Model is sectional AR model, it can be adopted for parameter estimating method and model inspection criterion of the AR model during modeling. At present, the least square method of model parameter estimate and AIC criterion of model adaptability inspection has been widely applied. The difficulty of modeling $SETAR$ is to ascertain the number of threshold range l, threshold value $r_1, r_2, \cdots, r_{l-1}$ and retardant steps d. In theory, this is the multi-dimension optimizing problem for l, $r_1, r_2, \cdots, r_{l-1}$ and d. If we adopt AIC criterion to ascertain the model steps ($n_1, n_2, \cdots, n_l$) in each threshold range and the objective function about l, $r_1, r_2, \cdots, r_{l-1}$ and d, AIC value should be the function that has $2l+1$ parameters. That is:

$$AIC \quad {}_{2}, \cdot \quad r \quad)$$

AIC values of the model should be the

space shown in the upper formula. This article will combine H. Tong method and D.D.C method established the $SETAR$ model (Yang, 1996; Jin, 2000; Ding, 1988; 1998; Du, 1991).

3.1 Drawing dot figure by D.D.C method (Yang, 1996)

D.D.C method was put forward by Jinpei Wu. Its idea is with drawing dot figure to ascertain the number of threshold l and search range of threshold value $r_1, r_2, \cdots, r_l$; with duplication method to estimate model parameter and with cross-validation method to confirm retardance steps . The duplication method is to recur the least square method for the parameter matrix of the AR model.

The first step of modeling is to ascertain the number of threshold and the search range of threshold value $r_1, r_2, \cdots, r_{l-1}$.

For time series, we can divide axis $x(t-d)$ into s sections evenly according to the data $(x(t), x(t-d))($ $1,2,3$. If the number of $x(t-d)$ in the first section is N , the mean value of $x(t)$

$$\hat{E}(x(t)/x(t-d))_i = \frac{1}{N_i} \qquad (\mathrm{i}=1,2,\cdots,\mathrm{s};\ \mathrm{d}=1,2,,\cdots). \quad (2)$$

The dot figure can be acquired through drawing the dot $\hat{E}(x(t)/x(t-d))_i$ in the center of each section with the horizontal axis $x(t-$ and vertical axis $\hat{E}(x(t)/x(t-d))$. When the dots show linear distribution in the dot figure, we can use the sectional linear model to describe the time series. This is the basic idea of the $SETAR$ model. Thus, the dot figure can not only judge the property of the model but also provide two notes for the $SETAR$ model: firstly, to ascertain the number of the threshold range according to the number of subsection and decrease a layer nesting in Tong method because it is unnecessary for l to optimize; secondly, to turn the large scale optimization of the threshold to local optimization when ascertain the search range of each threshold value $r_1,$ in the turning point of sectional linearity. Thus, the modeling of D.D.C method has three steps：① dot figure drawing; ② ascertain l and the search range of $\cdots, r_{l-1}$ according to the dot figure; ③ optimize for $\cdots, r_{l-1}$ with any optimizing theory. The article adopts the golden section method and searches the optimum value of $r_1, r_2, \cdots, r_{l-1}$ in three to five steps.

3.2 Optimum retard steps d, r_j and $\varphi_j(i)$ by tong method (Yang, 1996)

Tong method is the method that optimizes in one-dimensional space and layer upon layer nesting for d , l and r_j ($\mathrm{j}=1,2,3,\cdots\mathrm{l}\text{-}1$). Its basic idea is as:

① acquire a $SETAR$ model through fixing a group of d , l , r_j ($\mathrm{j}=1,2,3,\cdots\mathrm{l}\text{-}1$), establishing the AR model in each range respectively and ascertaining the applied model of each range according to AIC criterion; ② change the value of d , l and r_j ($\mathrm{j}=1,2,3,\cdots\mathrm{l}\text{-}1$) respectively and acquire a $SETAR$ model according the first step ; ③ compare with the AIC value of the $SETAR$ model established by d , l and r_j ($\mathrm{j}=1,2,3,\cdots\mathrm{l}\text{-}1$) and consider the model with the minimal AIC value as applied model. Because threshold range has been ascertained by the dot figure, a layer nesting can be decreased. That is to say that l is a known value. The steps in detail are as followed:

①Suppose that retardant steps d equal to one and ascertain the search range of threshold value r_j ($\mathrm{j}=1,2,3,\cdots\mathrm{l}\text{-}1$) by dot figure method. Then divide time series $\{x(t-d)\}(t=d+1,d+2,\cdots N)$ into l subsections according to the range it belongs to among $(-\infty,r_1],(r_1,r_2],\cdots,(r_{l-1},+\infty)$.

② Establish the AR model in each threshold range. Confirm the form of the model by auto-correlation analysis and ascertain the steps n_j of the model primarily through bias-correlation analysis. For example, if for any series the AR model is established, its auto-correlation coefficient is:

$$\rho(k)=\frac{\sum_{t=1}^{n-k}x(t)x(t+k)-\frac{1}{n-k}(\sum_{t=1}^{n-k}x(t))(\sum_{t=1}^{n-k}x(t+k))}{\left[\sum_{t=1}^{n-k}x^2(t)-\frac{1}{n-k}(\sum_{t=1}^{n-k}x(t))^2\right]^{\frac{1}{2}}\left[\sum_{t=1}^{n-k}x^2(t+k)-\frac{1}{n-k}(\sum_{t=1}^{n-k}x(t+k))^2\right]^{\frac{1}{2}}}$$

Where time delay k equals to 0,1,2,…m. When n is bigger than 50 ($n>50$), m must be less than n/4 and m equals to n/10 usually. Draw the allowable range that confidence level of auto-correlation coefficient is 95% in the auto-correlation figure. If α equals to 0.05, allowable range of auto-correlation coefficient $\rho(k)$ is

$$\rho(k)(\alpha=0.05)=\frac{-1\pm1.96\sqrt{n-k-1}}{n-k}$$

If p -step correlation coefficient is prominent to independent sequence, the former random sequence is a mean square contingency sequence and the $AR(p)$ model can be adopted.

The main way of confirming the steps p is to do the statistical analysis for bias-correlation coefficient. The bias-correlation coefficient φ_k can be acquired by recurrence algorithm (Ding, 1988; 1998; Du, 1991).

The same as the former theory, we can establish the AR model of each threshold range respectively. Then inspect each model by AIC criterion:

$$AIC=N_j\ ln\,\sigma_j^2+2(n_j+1)\quad (\mathrm{j}=1,2,3,\cdots,\mathrm{l})$$

By all appearances, when the value of d , l and r_j (j=1,2,3,···l-1) has been known, AIC value is only the function of model steps n_j. When AIC value is the minimal one, the model is the applied model of each section. Its AIC value is:

$$AIC(n_j)=\min_{1\le n_j\le M}\{N_j\ ln\sigma_j^2+2(n_j+1)\}\ (\mathrm{j}=1,2,3,\cdots,\mathrm{l})$$

where M is the uppermost steps.

③ Acquire the $SETAR$ model by combining the applied model $AR^{(j)}(n_j)$ (j=1,2,3,···,l) of each threshold range. AIC value of each section model add up to the AIC value of the model. That is as the

$$AIC(l \qquad (3)$$

Because we acquire the model and the AIC value in the case of l parameters unchanged from d , l and , the AIC value in equation (3)is the function of the l parameters.

④ Fix d and and optimize for threshold value r_j (j=1,2,3, in the known search range by adopting golden section method or accelerating genetic algorithm. Then repeat the former steps and calculate AIC value according to equation (3). Finally confirm the group of whose value is the minimal one as the best threshold value in the case of known d and . The objective function of threshold optimization is:

$$AIC \quad \hat{r}_1 = _{-\infty<r_1<} \quad _{-1} \{ \quad _1)\} \qquad (4)$$

⑤ Fix l and repeat the former steps when d is from d+1 to D, which is the maximal search steps. Then ascertain the best retardant steps d when the AIC value is the minimal one. Because in fact the data of modeling only have N , where $max\{d,M\}$, the AIC value should adopt standard value. That is:

$$\overline{AIC}(l \quad _2, \quad \frac{\quad _{l-1})}{} \qquad (5)$$

Calculate different AIC value for each d according to equation(5). Then confirm the $\hat{d}$ when the AIC value is the minimal one as the best retardant steps in the case of known . The objective function of d optimization is:

$$\overline{AIC}(l \quad \hat{r}_2,\cdots,\hat{r}_{l-1})=\min_{1\le d\le D}\{\overline{AIC}(l;d;\hat{r}_1,\hat{r}_2,\cdots,\hat{r}_{l-1})\}$$

Thus, the process of parameter optimization has finished and the $d,n_1,n_2)$ model has been established.

3.3 SETAR model forecast (the best forecast) (Yang, 1996)

As the model, the best forecast of threshold auto-regressive model is also the forecast with the least

variance of forecast error. The best-forecast value $\hat{x}_k(t)$ at time t for next k step equals to the conditional mathematics expectation $E[x(t+k)]$ at time t in time $t=t+k$. That is: $\hat{x}_k(t)=E[x(t+k)]$. The formula of the best forecast of threshold auto-regress-
ive model is:

$$\hat{x}_k(t)=\begin{cases} \varphi_j(0)+\sum_{i=1}^{n_j}\varphi_j(i)x(t+k-i) & \text{(k=1)} \\ \varphi_j(0)+\sum_{i=1}^{k-1}\varphi_j(i)\hat{x}_{k-i}(t)+\sum_{i=k}^{n_j}\varphi_j(i)x(t+k-i) & (\text{k}\le \text{n}_\text{j}) \\ \varphi_j(0)+\sum_{i=1}^{n_j}\varphi_j(i)\hat{x}_{k-i}(t) & (\text{k}>\text{n}_\text{j}) \\ & (r_{j-1}<x(t+k-d)\le r_j, j=1,2,3,\cdots,l) \end{cases} \qquad (6)$$

4 A Case Study

The Sanjiang Plain is an important crop commodity base in China. By the end of 1998, 80% of irrigation water for rice was believed to be drawn from groundwater. Since people have excessively exploited ground water the ground water tables have fallen beneath the depth of farmers wells. In the spring of 1996, the Jiansanjiang administration bureau had recorded the incidence of more than 600 of such dry wells. This ground water resources crisis has constrained further development in the Sanjiang Plain. Precise forecasting of water requirements is expected to have a significant effect on establishing reasonable water use, thereby saving ground water resources and renewing a sustainable ground water balance that still benefits agriculture in the Sanjiang Plain.

Using the fifteen-year observation data (1984-1998) in Fu Jin, which is the hinterland in the Sanjiang Plain, the well irrigation to rice can be divided into six phases according to its breeding period. Rice planting was around May 20 every year based on climate records in past years in San Jin Plain. Therefore, the rice growth phases are: rice planting stage (May 20-May 29, 10 days in total), turning green stage (May 30-June 5, 7 days in total), tillering stage（6.6～7.10，35 days in total）, booting stage (July 11-July 20, 10 days in total), spiking and blooming stage (July 21-July 27, 7 days in total) and grain filling and mature stage (July 28-Agust 31, 35 days in total) (Fu, 2000).

Table 1 Water requirements rice of Fu Jin City in the Sanjiang Plain (daily mean, mm)

Year	rice planting stage	returning green stage	Tillering	booting stage	spiking and blooming stage	grain filling and mature stage
1984	4.86	7.32	8.18	9.54	7.34	5.71
1985	5.28	7.62	9.88	10.20	12.89	5.93
1986	4.10	5.93	9.77	10.03	11.35	5.08
1987	5.44	5.85	10.02	11.52	12.45	6.05
1988	4.90	5.16	7.77	10.82	8.26	7.28
1989	4.60	5.55	8.53	12.78	9.97	5.99
1990	5.48	5.14	7.55	9.85	7.14	5.50
1991	4.25	5.14	9.64	12.06	9.83	6.04
1992	4.87	4.12	9.06	12.23	6.80	4.91
1993	4.88	5.29	6.72	11.21	10.30	6.26
1994	3.53	5.60	8.27	10.21	8.70	5.52
1995	4.58	6.20	6.90	10.25	9.88	5.65
1996	5.18	6.12	9.06	10.05	10.46	6.90
1997	4.50	6.53	8.35	12.96	9.46	5.72
1998	4.59	5.15	9.90	12.64	11.27	7.54

4.1 Drawing dot figure

Firstly, consider $x(t-d)(d=1,2,3,\cdots 6)$ as the horizontal axis and $\hat{E}(x(t)/x(t-d))$ as the vertical axis and divide the horizontal axis into fifteen sections. Then line out the dot $\hat{E}(x(t)/x(t-d))_i$ in the center of each section and draw the dot figure for water requirement sequence of well irrigation rice when retardant steps are from one to six. Finally, ascertain the number of threshold range and the search range of threshold value. Because the dot figure is two-section linear distribution from Figure 1 to Figure 6, the number of threshold range is two. From the turning point of two straight lines in Figure 1 to Figure 6, the search range of threshold value r is between seven and eight approximately. So, golden section method can be used to optimize.

4.2 Optimizing step by step

Firstly, suppose that the retardant steps d are one to six and the search range of threshold value r is between seven and eight. Then calculate the AIC value in the different case respectively by adopting golden section method, applying two-layer cycle mechanically and utilizing Matlab 5.3 software program. Consequently ascertain the best threshold value r and retardant steps d (Table 2, Figure 7). It is known from Table 2 that AIC value is the minimal one when d equals to 6.0 and the best threshold value r equals to 7.55. In the case, there are three AR model steps correspondingly. AIC inspection of the AR model is in Table 2 and Table 3.

The independent inspection of the AR model is：

$AR(n_1)$： $Q=6.5899<\chi^2_{0.05}=9.488$

$AR(n_2)$： $Q=3.3696<\chi^2_{0.05}=9.488$

All of them pass the independent inspection.

Table 2 Step by step optimize ($l = 2$)

	d =1	d =2	d =3	d =4	d =5	d =6
AIC	115.4904	108.9723	86.7864	120.2671	108.5283	60.9523
Threshold Value r	7.60	7.57	7.55	7.58	7.58	7.55
Model Steps n_1， n_2	3，3	3，3	2，2	2，3	2，5	3，3

Table 3 AIC Testing of $AR(n_1)$ model

$AIC(0)$	$AIC(1)$	$AIC(2)$	$AIC(3)$	$AIC(4)$	$AIC(5)$	$AIC(6)$	$AIC(7)$
	20.4682	15.3375	12.4302	16.2228	13.2245	15.2274	18.6218

Table 4 AIC Testing of $AR(n_2)$ model

$AIC(0)$	$AIC(1)$	$AIC(2)$	$AIC(3)$	$AIC(4)$	$AIC(5)$	$AIC(6)$	$AIC(7)$
	58.3743	56.2078	54.5222	57.1001	59.0141	60.0505	61.0984

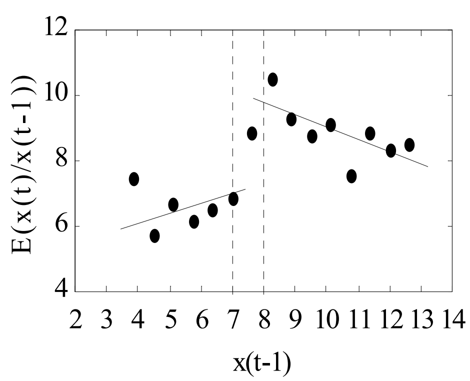

Figure 1 Dot Figure (d =1)

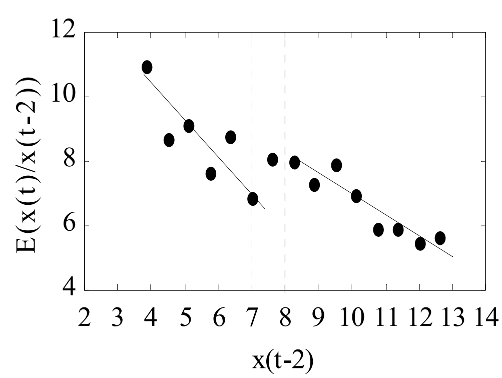

Figure 2 Dot Figure (d =2)

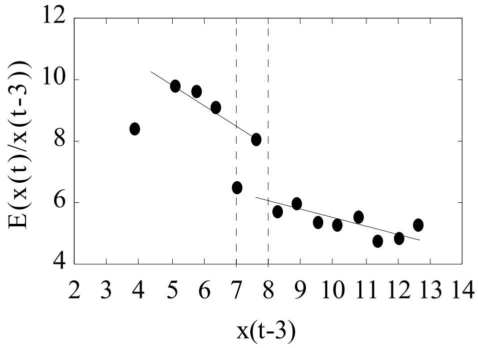

Figure 3 Dot Figure (d =3)

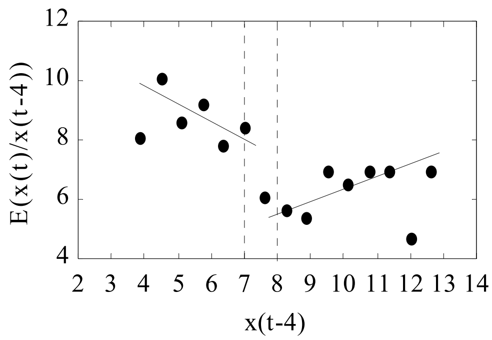

Figure 4 Dot Figure (d =4)

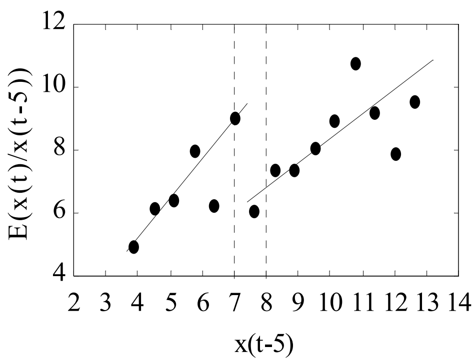

Figure 5 Dot Figure (d =5)

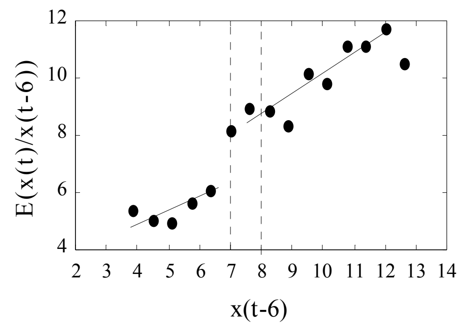

Figure 6 Dot Figure (d =6)

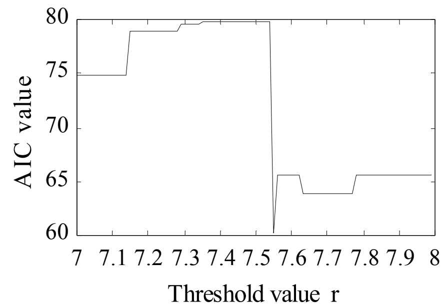

Figure 7 Seeking the Optimum Value of r (d =6, l =2)

4.3 Establish the threshold auto-regressive model of water requirement

$$x(t) = \begin{cases} aver\ 1 + \varphi(1,1)(x(t-1) - aver\ 1) \\ + \varphi(1,2)(x(t-2) - aver\ 1) \\ + \varphi(1,3)(x(t-3) - aver\ 1) \quad x(t-6) \le r \\ aver\ 2 + \varphi(2,1)(x(t-1) - aver\ 2) \\ + \varphi(2,2)(x(t-2) - aver\ 2) \\ + \varphi(2,3)(x(t-3) - aver\ 2) \quad x(t-6) > r \end{cases}$$

where $aver1$ is the mean of $AR(n_1)$ model and $aver2$ is the mean of $AR(n_2)$ model. $\varphi(1,1), \varphi(1,2), \varphi(1,3)$ is the auto-regressive coefficient of $AR(n_1)$ model and $\varphi(2,1), \varphi(2,2), \varphi(2,3)$ is the auto-regressive coefficient of $AR(n_2)$ model. r is threshold value. The optimizing value of each parameter is shown in Table 5.

Table 5 Optimizing the parameters of $SETAR$ model (r =7.55, d =6, l =2)

	$\varphi(1,1)$	$\varphi(1,2)$	$\varphi(1,3)$	$aver1$
$AR(n_1)$	-0.3657	-0.5268	0.2906	5.6466
	$\varphi(2,1)$	$\varphi(2,2)$	$\varphi(2,3)$	$aver2$
$AR(n_2)$	-0.3736	-0.2786	0.1758	10.0922

4.4 Rice water requirement forecast

According to the former theory, forecast water requirement of well irrigation rice during each phase in Fu Jin in 1999 and 2000 and compare with the observed value (Figure 6). Forecast precision of the model is very high and its mean relative error is in 10% (Table 6).

Table 6 Using SETAR Model to forecast the mean daily water requirement of well irrigation rice during each phase in Fu Jin City in 2000

Year	Item	rice planting stage	returning green stage	Tillering stage	booting stage	spiking and blooming stage	grain filling and mature stage
1999	Observed value	4.85	5.98	9.72	10.15	11.04	6.12
	Forecast value	4.82	6.17	9.85	9.75	10.46	5.96
	Absolute error	-0.03	+0.19	+0.13	-0.40	-0.58	-0.16
	Relative error（%）	-0.62	+3.18	+1.34	-3.94	-5.25	-2.61
2000	Observed value	5.20	6.06	9.85	10.54	11.01	6.35
	Forecast value	5.23	5.70	10.06	10.04	10.14	5.88
	Absolute error	+0.03	-0.36	+0.21	-0.50	-0.87	-0.77
	Relative error（%）	+0.58	-5.94	+2.13	-4.74	-7.90	-7.4

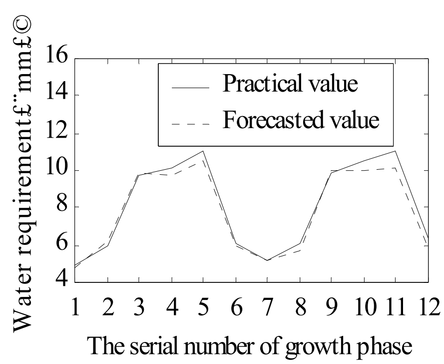

Figure 8 The Water Requirement Curve Fitted by SETAR Model

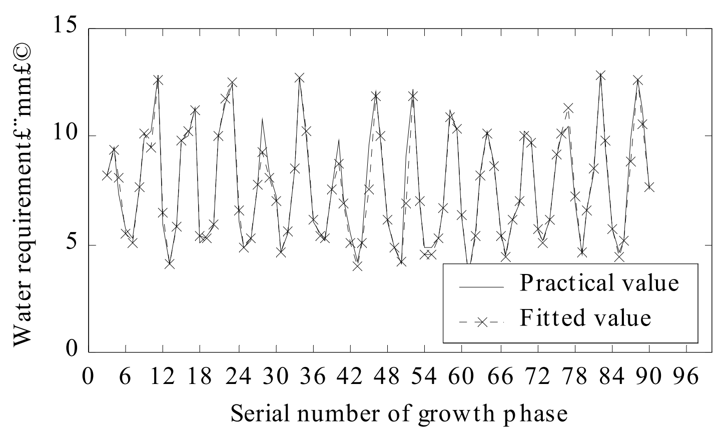

Figure 9 The Contrast Curve Between Practical Value and Forecasting Value Calculated by SETAR Model

5 Conclusion

This article applies the *SETAR* model in well irrigation rice water requirement forecast. It is divided according to the breeding phase through analyzing its property of water requirement. Establish threshold auto-regressive model of daily mean water requirement by optimizing parameters of the model, such as threshold value, threshold range, retardant steps and auto-regressive coefficient etc. Because the control of threshold has its effect, it is possible to describe effectively the non-linear dynamic system of well irrigation rice daily mean water requirement in each phase by using interdependent property of water requirement in retardant steps of one step, two steps and three steps. The high forecast precision shows that the capability of the *SETAR* model is steady and can be applied in irrigation water widely.

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