A Novel H_∞ Output Feedback Controller Design for LPV Systems with a State-delay

Junling Wang [1], Changhong Wang [1], Wei Yuan [2]

(1. Department of Control Science and Engineering, Harbin Institute of Technology, Harbin, Heolongjiang 150001, China, E-mail: jun_ling2003@yahoo.com.cn
2. Education Department of Heilongjiang Province, Harbin, Heolongjiang 150001, China)

Abstract: A H_∞ control for linear parameter-varying (LPV) systems with a parameter-varying state delay is described. A new parameter-dependent H_∞ performance criterion is first established by the introduction of a slack variable, which exhibits a kind of decoupling between Lyapunov functions and system matrices. This kind of decoupling enables us to obtain a more easily tractable condition for analysis and synthesis problems. Then, the corresponding output feedback controller design is investigated upon this new performance criterion, with sufficient conditions obtained for the existence of admissible controllers in terms of parameterized linear matrix inequalities (PLMIs) and a non-convex constraint set. The cone complement linearization idea is employed to convert the controller design into a convex optimization problem. A numerical example is provided to illustrate the feasibility and advantage of the proposed controller design procedure. [Nature and Science, 2004,2(1):53-60].

Key words: linear parameter-varying system; parameterized linear matrix inequality; time delay; H_∞ control.

1 Introduction

Linear parameter-varying (LPV) systems have recently much attention because they provide a systematic means of computing gain-scheduled controllers (Shamma, 1990; Apkarian, 1995, 1998). LPV systems are characterized as linear systems that depend on time-varying real parameters. These parameters are assumed to be exogenous signals that are unknown in advance but are constrained a priori to lie in some known, bounded set, and can be measured in real time. Recently, many researchers examined the stability analysis and gain scheduling control of LPV systems extensively and a great number of important results have been reported to the literature (see, for instance, (Shamma, 1990; Apkarian, 1995, 1998; Wu, 2001; Tan, 2000, 2003; Zhang, 2001, 2002; Bara, 2001a, 2001b) and the references therein).

On the other hand, time delays are often present in engineering systems, which have been generally regarded as a main source of instability and poor performance. Therefore, recent research effort is focused more on the analysis and synthesis problems of LPV time-delay systems. To mention a few, (Wu, 2001; Tan, 2003; Tan, 2000) investigated control problems for LPV systems with parameter-varying delays, and (Zhang, 2001, 2002) with a fixed delay size.

In this note, we extend the results in (Wu, 2001) to output feedback control synthesis problems for LPV systems with a parameter-varying state-delay. We seek to develop controllers that are scheduled based on the measurement of the parameters to guarantee stability and the desired H_∞ performance specification. Firstly, a parameter-dependent H_∞ performance criterion is established based on the Lyapunov approach. Secondly, we further improve the obtained performance by decoupling the product terms involving the positive definite matrices, which is enabled the introduction of an additional slack variable. This resulting new performance condition is more easily tractable for analysis and synthesis problems. Thirdly, upon this new criterion, the corresponding parameter-varying dynamic output feedback controllers are designed, which guarantee the closed-loop system to be asymptotically stable with a prescribed H_∞ disturbance attenuation level. Since the sufficient conditions for the existence of such controllers are not expressed as parameterized linear matrix inequalities (PLMIs), an iterative algorithm involving convex optimization is proposed. Numerical example shows that the effectiveness of the proposed methods. The results obtained in this note can be easily extended to LPV systems with multiple delays.

Notations: Throughout this note the superscript T stands for matrix transposition, R^n denotes the n dimensional Euclidean space, $R^{m\times n}$ is the set of all $m\times n$ real matrices, and the notation $P>0$ for $P\in R^{n\times n}$ means that P is symmetric and positive definite. In addition, in symmetric block matrices or long matrix expressions, we use * as an ellipsis for the terms that are induced by symmetry, $diag\{\cdots\}$ stands for a block-diagonal matrix, and $trace\{H\}$ denotes the trace of the matrix H.

2 Problem Set-up

Consider the following LPV system with a parameter-varying delay:

$$\begin{aligned}\dot{x}(t) &= A(\rho(t))x(t) + A_h(\rho(t))x(t-h(\rho(t))) + B_1(\rho(t))\omega(t) + B_2(\rho(t))u(t)\\ z(t) &= C_1(\rho(t))x(t) + C_{1h}(\rho(t))x(t-h(\rho(t))) + D_{11}(\rho(t))\omega(t) + D_{12}(\rho(t))u(t)\\ y(t) &= C_2(\rho(t))x(t) + C_{2h}(\rho(t))x(t-h(\rho(t))) + D_{21}(\rho(t))\omega(t)\end{aligned} \tag{1}$$

with initial condition

$$x(\theta)=\phi(\theta),\ \ \forall\theta\in[-h(\rho(0)),0] \tag{2}$$

where $x(t)\in R^n$ is the state vector; $y(t)\in R^m$ is the measured output vector; $\omega(t)\in R^l$ is the exogenous disturbance signal; $z(t)\in R^p$ is the control output vector; $u(t)\in R^q$ is control input. The state-space matrices $A(\cdot)$, $A_h(\cdot)$, $B_1(\cdot)$, $B_2(\cdot)$, $C_1(\cdot)$, $C_2(\cdot)$, $C_{1h}(\cdot)$, $C_{2h}(\cdot)$ $D_{11}(\cdot)$, $D_{12}(\cdot)$, $D_{21}(\cdot)$ and the delay $h(\cdot)$ are assumed to be bounded continuous functions of a time-varying parameter vector $\rho(t)\in\Re_\psi^\nu$. The set $\Re_\psi^\nu$ is the set of allowable parameter trajectories

$$\Re_\psi^\nu=\{\rho:\rho(t)\in\psi,|\dot{\rho}_i|\le\upsilon_i,i=1,2,\cdots,s,\forall t\in R_+\} \tag{3}$$

where ψ is a compact set of R^s, $\{\upsilon_i\}_{i=1}^s$ are nonnegative numbers and it is assumed that the parameter trajectories are bounded with bounded variation rates. And the delay $h(\cdot)$ is assumed to be a differentiable function such that $0<h(t)\le H<\infty$, $\dot{h}(t)\le\sigma<1$, $\forall t\ge 0$ .For simplicity, ρ denotes time-varying parameters $\rho(t)$ respectively throughout this note.

Construct a dynamic output feedback LPV controller described by

$$\begin{aligned}\dot{x}_c(t) &= A_c(\rho)x_c(t) + B_c(\rho)y(t)\\ u(t) &= C_c(\rho)x_c(t) + D_c(\rho)y(t)\end{aligned} \tag{4}$$

where $x_c(t)\in R^n$ is the state vector and $A_c(\rho)$, $B_c(\rho)$, $C_c(\rho)$, $D_c(\rho)$ are to be determined parameter-varying matrices.

The feedback connection of the system (1) with the controller (4) produces a closed-loop system described by

$$\begin{aligned}\dot{\xi}(t) &= \bar{A}(\rho)\xi(t) + \bar{A}_h(\rho)E\xi(t-h(\rho)) + \bar{B}(\rho)\omega(t)\\ z(t) &= \bar{C}(\rho)\xi(t) + \bar{C}_h(\rho)E\xi(t-h(\rho)) + \bar{D}(\rho)\omega(t)\end{aligned} \tag{5}$$

where

$$\begin{aligned}&\xi(t)=\begin{bmatrix}x(t)\\ x_c(t)\end{bmatrix},\ \ \bar{A}(\rho)=\begin{bmatrix}A(\rho)+B_2(\rho)D_c(\rho)C_2(\rho) & B_2(\rho)C_c(\rho)\\ B_c(\rho)C_2(\rho) & A_c(\rho)\end{bmatrix},\\ &\bar{A}_h(\rho)=\begin{bmatrix}A_h(\rho)+B_2(\rho)D_c(\rho)C_{2h}(\rho)\\ B_c(\rho)C_{2h}(\rho)\end{bmatrix},\quad \bar{B}(\rho)=\begin{bmatrix}B_1(\rho)+B_2(\rho)D_c(\rho)D_{21}(\rho)\\ B_c(\rho)D_{21}(\rho)\end{bmatrix},\\ &\bar{C}(\rho)=\begin{bmatrix}C_1(\rho)+D_{12}(\rho)D_c(\rho)C_2(\rho) & D_{12}(\rho)C_c(\rho)\end{bmatrix},\quad \bar{C}_h(\rho)=C_{1h}(\rho)+D_{12}(\rho)D_c(\rho)C_{2h}(\rho),\\ &\bar{D}(\rho)=D_{11}(\rho)+D_{12}(\rho)D_c(\rho)D_{21}(\rho),\quad E=\begin{bmatrix}I & 0\end{bmatrix}.\end{aligned} \tag{6}$$

Our objective is to seek an output feedback controller that asymptotically stabilizes the closed-loop system and guarantees a prescribed H_∞ performance, that is, it should be guaranteed that

$$\|z\|_2^2 < \gamma^2 \|\omega\|_2^2 \tag{7}$$

for all nonzero $\omega \in l_2[0,\infty)$ under zero initial conditions, where γ is a positive scalar,

$$\|\omega\|_2^2 = \int_0^\infty \omega^T(t)\omega(t)dt\ ,\ \ \|z\|_2^2 = \int_0^\infty z^T(t)z(t)dt\ .$$

3 H_∞ Performance Criterion

In this section, we will establish the H_∞ performance criterion for time-delayed LPV systems.

Lemma 1: Consider system (1) and suppose γ is a given positive constant. Then the closed-loop system (5) is asymptotically stable and has an H_∞ performance level less than γ if there exists a matrix function $0 < P(\rho) = P^T(\rho) \in R^{2n\times 2n}$ and a matrix $0 < Q = Q^T \in R^{n\times n}$ such that for all $\rho \in \Re_\psi^\nu$ and $|\tau_i| \le \upsilon_i$ the following inequality holds

$$\begin{bmatrix} \bar{A}^T(\rho)P(\rho)+P(\rho)\bar{A}(\rho)+\sum_{i=1}^{S}(\tau_i\frac{\partial P}{\partial\rho_i})+E^TQE & P(\rho)\bar{A}_h(\rho) & P(\rho)\bar{B}(\rho) & \bar{C}^T(\rho) \\ * & -(1-\sum_{i=1}^{S}(\tau_i\frac{\partial h}{\partial\rho_i}))Q & 0 & \bar{C}_h^T(\rho) \\ * & * & -\gamma I & \bar{D}^T(\rho) \\ * & * & * & -\gamma I \end{bmatrix} < 0 \tag{8}$$

***Remark* 1:** The above lemma will be obtained by the similar way to (Wu, 2001) using different Lyapunov-Krasovskii type functional

$$V(\xi(t),\rho(t)) = \xi^T(t)P(\rho)\xi(t) + \int_{t-h(\rho)}^{t}\xi^T(s)E^TQE\xi(s)ds \tag{9}$$

where $0 < P(\rho) = P^T(\rho) \in R^{2n\times 2n}$ and $0 < Q = Q^T \in R^{n\times n}$.

***Remark* 2:** It should be noted that the condition presented in *Lemma 1* contain product terms between Lyapunov matrices and system matrices, such that condition (8) is a bilinear matrix inequality when (6) is considered. In the following, we will present an improved version of *Lemma 1* by introducing a slack variable to decouple these product terms, which is more easily tractable for controller synthesis problems.

***Theorem* 1:** Consider system (1) and suppose γ is a given positive constant. Then the closed-loop system (5) is asymptotically stable and has an H_∞ performance level less than γ if there exists matrix function $0 < P(\rho) = P^T(\rho) \in R^{2n\times 2n}$ and three matrices $0 < Q = Q^T \in R^{n\times n}, 0 < Z = Z^T \in R^{n\times n}, W \in R^{2n\times 2n}$ such that for all $\rho \in \Re_\psi^\nu$ and $|\tau_i| \le \upsilon_i$ the following inequality and an non-convex condition hold

$$\begin{bmatrix} -(W+W^T) & W^T\bar{A}(\rho)+P(\rho) & W^T\bar{A}_h(\rho) & W^T\bar{B}(\rho) & 0 & 0 & W^T \\ * & -P(\rho)+\sum_{i=1}^{S}(\tau_i\frac{\partial P}{\partial\rho_i}) & 0 & 0 & \bar{C}^T(\rho) & E^T & 0 \\ * & * & -(1-\sum_{i=1}^{S}(\tau_i\frac{\partial h}{\partial\rho_i}))Q & 0 & \bar{C}_h^T(\rho) & 0 & 0 \\ * & * & * & -\gamma I & \bar{D}^T(\rho) & 0 & 0 \\ * & * & * & * & -\gamma I & 0 & 0 \\ * & * & * & * & * & -Z & 0 \\ * & * & * & * & * & * & -P(\rho) \end{bmatrix} < 0 \tag{10}$$

$$QZ = I \tag{11}$$

Proof **:** The proof is based on the generalization of the stability results of (Apkarian, 2001). The inequality (10) is equivalent to:

$$\begin{bmatrix} 0 & P(\rho) & 0 & 0 & 0 & 0 & 0 \\ * & -P(\rho)+\sum_{i=1}^{S}(\tau_i \frac{\partial P}{\partial \rho_i}) & 0 & 0 & \bar{C}^T(\rho) & E^T & 0 \\ * & * & -(1-\sum_{i=1}^{S}(\tau_i \frac{\partial h}{\partial \rho_i}))Q & 0 & \bar{C}_h^T(\rho) & 0 & 0 \\ * & * & * & -\gamma I & \bar{D}^T(\rho) & 0 & 0 \\ * & * & * & * & -\gamma I & 0 & 0 \\ * & * & * & * & * & -Z & 0 \\ * & * & * & * & * & * & -P(\rho) \end{bmatrix} + \begin{bmatrix} -I \\ \bar{A}^T(\rho) \\ \bar{A}_h^T(\rho) \\ \bar{B}^T(\rho) \\ 0 \\ 0 \\ I \end{bmatrix} W \begin{bmatrix} I & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} + (*) < 0$$

Now we can drop the matrix W by using the Projection Lemma. The null-spaces of $\begin{bmatrix} I & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ and $\begin{bmatrix} I & \bar{A}(\rho) & \bar{A}_h(\rho) & \bar{B}(\rho) & 0 & 0 & I \end{bmatrix}$ are respectively

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ I & 0 & 0 & 0 & 0 & 0 \\ * & I & 0 & 0 & 0 & 0 \\ * & * & I & 0 & 0 & 0 \\ * & * & * & I & 0 & 0 \\ * & * & * & * & I & 0 \\ * & * & * & * & * & I \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} \bar{A}(\rho) & \bar{A}_h(\rho) & \bar{B}(\rho) & 0 & 0 & I \\ I & 0 & 0 & 0 & 0 & 0 \\ * & I & 0 & 0 & 0 & 0 \\ * & * & I & 0 & 0 & 0 \\ * & * & * & I & 0 & 0 \\ * & * & * & * & I & 0 \\ * & * & * & * & * & I \end{bmatrix},$$

thus the projection conditions yield to:

$$\begin{bmatrix} -P(\rho)+\sum_{i=1}^{S}(\tau_i \frac{\partial P}{\partial \rho_i}) & 0 & 0 & \bar{C}^T(\rho) & E^T & 0 \\ * & -(1-\sum_{i=1}^{S}(\tau_i \frac{\partial h}{\partial \rho_i}))Q & 0 & \bar{C}_h^T(\rho) & 0 & 0 \\ * & * & -\gamma I & \bar{D}^T(\rho) & 0 & 0 \\ * & * & * & -\gamma I & 0 & 0 \\ * & * & * & * & -Z & 0 \\ * & * & * & * & * & -P(\rho) \end{bmatrix} < 0 \tag{12}$$

$$\begin{bmatrix} P(\rho)\bar{A}(\rho)+\bar{A}^T(\rho)P(\rho)-P(\rho)+\sum_{i=1}^{S}(\tau_i \frac{\partial P}{\partial \rho_i}) & P(\rho)\bar{A}_h(\rho) & P(\rho)\bar{B}(\rho) & \bar{C}^T(\rho) & E^T & P(\rho) \\ * & -(1-\sum_{i=1}^{S}(\tau_i \frac{\partial h}{\partial \rho_i}))Q & 0 & \bar{C}_h^T(\rho) & 0 & 0 \\ * & * & -\gamma I & \bar{D}^T(\rho) & 0 & 0 \\ * & * & * & -\gamma I & 0 & 0 \\ * & * & * & * & -Z & 0 \\ * & * & * & * & * & -P(\rho) \end{bmatrix} < 0 \tag{13}$$

By Schur complement (Boyd, 1994), the inequality (13) is equivalent to (8), and (12) is equivalent to the constraint $-P(\rho)+\sum_{i=1}^{S}(\tau_i \frac{\partial P}{\partial \rho_i})<0, \quad (1-\sum_{i=1}^{S}(\tau_i \frac{\partial h}{\partial \rho_i}))>0$

This means that the domain of solution given by (10-11) is included in the domain of solutions satisfying (8) and thus the condition (10-11) is sufficient to ensure the closed-loop system asymptotically stable and guarantee the prescribed H_∞ performance level.

4 H_∞ Output Feedback Synthesis

In this section, the H_∞ performance criterion presented in the above section will be used to design the parameter-dependent H_∞ output feedback controllers.

First we introduce a partition of the slack matrix W and its inverse $V=W^{-1}$ in the form

$$W=\begin{bmatrix} W_{11} & W_{12} \\ W_{21} & W_{22} \end{bmatrix}, \quad V=W^{-1}=\begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{bmatrix} \tag{14}$$

There is no loss of generality in assuming that V_{21} and W_{21} are invertible. Then we introduce the notation

$$J_W=\begin{bmatrix} W_{11} & I \\ W_{21} & 0 \end{bmatrix}, \quad J_V=\begin{bmatrix} I & V_{11} \\ 0 & V_{21} \end{bmatrix} \tag{15}$$

then $WJ_V=J_W, VJ_W=J_V$. Multiplying the righthand and left-hand sides of the inequality (10) by $J=diag\{J_V, J_V, I, I, I, I, J_V\}$ and its transpose, respectively, we obtain:

$$\begin{bmatrix} -J_V^TJ_W-J_W^TJ_V & J_W^T\bar{A}(\rho)J_V+J_V^TP(\rho)J_V & J_W^T\bar{A}_h(\rho) & J_W^T\bar{B}(\rho) & 0 & 0 & J_W^TJ_V \\ * & J_V^T(-P(\rho)+\sum_{i=1}^{S}(\tau_i\frac{\partial P}{\partial \rho_i}))J_V & 0 & 0 & J_V^T\bar{C}^T(\rho) & J_V^TE^T & 0 \\ * & * & -(1-\sum_{i=1}^{S}(\tau_i\frac{\partial h}{\partial \rho_i}))Q & 0 & J_V^T\bar{C}_h^T(\rho) & 0 & 0 \\ * & * & * & -\gamma I & J_V^T\bar{D}^T(\rho) & 0 & 0 \\ * & * & * & * & -\gamma I & 0 & 0 \\ * & * & * & * & * & -Z & 0 \\ * & * & * & * & * & * & -J_V^TP(\rho)J_V \end{bmatrix}<0 \tag{16}$$

Defining the following matrices

$$\begin{aligned} &X(\rho)=J_V^TP(\rho)J_V, \quad R=W_{11}, \quad F=V_{11}, \quad U=W_{11}^TV_{11}+W_{21}^TV_{21}, \\ &\tilde{A}(\rho)=W_{11}^TA(\rho)V_{11}+W_{21}^TA_c(\rho)V_{21}+W_{21}^TB_c(\rho)C_2(\rho)V_{11}+W_{11}^TB_2(\rho)C_c(\rho)V_{21} \\ &\qquad +W_{11}^TB_2(\rho)D_c(\rho)C_2(\rho)V_{11}, \\ &\tilde{B}(\rho)=W_{21}^TB_c(\rho)+W_{11}^TB_2(\rho)D_c(\rho) \\ &\tilde{C}(\rho)=D_c(\rho)C_2(\rho)V_{11}+C_c(\rho)V_{21} \\ &\tilde{D}(\rho)=D_c(\rho) \end{aligned} \tag{17}$$

and considering (6) and (15), we have

$$
\left[\begin{array}{cccccccccc}
-(R+R^T) & -(I+U) & X_1(\rho)+R^T A(\rho)+\tilde{B}(\rho)C_2(\rho) & \tilde{A}(\rho)+X_2(\rho) & R^T A_h(\rho)+\tilde{B}(\rho)C_{2h}(\rho) & R^T B_1(\rho)+\tilde{B}(\rho)D_{21}(\rho) & 0 & 0 & R^T & U \\
* & -(F+F^T) & X_2^T(\rho)+A(\rho)+B_2(\rho)\tilde{D}(\rho)C_2(\rho) & X_3(\rho)+A(\rho)F+B_2(\rho)\tilde{C}(\rho) & A_h(\rho)+B_2(\rho)\tilde{D}(\rho)C_{2h}(\rho) & B_1(\rho)+B_2(\rho)\tilde{D}(\rho)D_{21}(\rho) & 0 & 0 & I & F \\
* & * & -X_1(\rho)+\sum_{i=1}^{S}(\tau_i \frac{\partial X_1}{\partial \rho_i}) & -X_2(\rho)+\sum_{i=1}^{S}(\tau_i \frac{\partial X_2}{\partial \rho_i}) & 0 & 0 & C_1^T(\rho)+C_2^T(\rho)\tilde{D}^T(\rho)D_{12}^T(\rho) & I & 0 & 0 \\
* & * & * & -X_3(\rho)+\sum_{i=1}^{S}(\tau_i \frac{\partial X_3}{\partial \rho_i}) & 0 & 0 & F^T C_1^T(\rho)+\tilde{C}^T(\rho)D_{12}^T(\rho) & F^T & 0 & 0 \\
* & * & * & * & -(1-\sum_{i=1}^{S}(\tau_i \frac{\partial h}{\partial \rho_i}))Q & 0 & C_{1h}^T(\rho)+C_{2h}^T(\rho)\tilde{D}^T(\rho)D_{12}^T(\rho) & 0 & 0 & 0 \\
* & * & * & * & * & -\gamma I & D_{11}^T(\rho)+D_{21}^T(\rho)\tilde{D}^T(\rho)D_{12}^T(\rho) & 0 & 0 & 0 \\
* & * & * & * & * & * & -\gamma I & 0 & 0 & 0 \\
* & * & * & * & * & * & * & -Z & 0 & 0 \\
* & * & * & * & * & * & * & * & -X_1(\rho) & -X_2(\rho) \\
* & * & * & * & * & * & * & * & * & -X_3(\rho)
\end{array}\right] < 0 \qquad (18)
$$

$$
\begin{bmatrix} X_1(\rho) & X_2(\rho) \\ * & X_3(\rho) \end{bmatrix} > 0 \qquad (19)
$$

Summarizing the above way, we can derive the following theorem.

Theorem 2: Consider system (1) and suppose γ is a given positive constant. Then an admissible H_∞ output feedback controller exists if there exist matrices functions

$0 < X_1(\rho) = X_1^T(\rho) \in R^{n\times n}$, $X_2(\rho) \in R^{n\times n}$, $0 < X_3(\rho) = X_3^T(\rho) \in R^{n\times n}$, $\tilde{A}(\rho) \in R^{n\times n}$, $\tilde{B}(\rho) \in R^{n\times m}$, $\tilde{C}(\rho) \in R^{q\times n}$, $\tilde{D}(\rho) \in R^{q\times m}$ and matrices $R \in R^{n\times n}$, $F \in R^{n\times n}$, $U \in R^{n\times n}$ such that for all $\rho \in \Re_\psi^\nu$ and $\left|\tau_i\right| \le \upsilon_i$ (11), (18) and (19) hold.

Furthermore, if (11), (18) and (19) have feasible solution, an admissible output feedback can be carried out by two steps:

a. Compute a factorization $V_{21}^T W_{21}$ of $U - V_{11}^T W_{11}$ and deduce V_{21} and W_{21}.
b. Compute the controller data $A_c(\rho), B_c(\rho), C_c(\rho), D_c(\rho)$ by reversing the formulas in (17).

Remark 3: Notice that the PLMI conditions (18) and (19) correspond to infinite-dimensional convex problems due to their parametric dependence. Using the gridding technique and the appropriate basis functions (Apkarian, 1998; Wu, 2001; Tan, 2000, 2003), infinite-dimensional PLMIs can be transformed to finite-dimensional ones, which can be solved numerically using convex optimization techniques. We choose the appropriate basis functions as $\sum_{j=1}^{n_j} f_j(\rho)$, then, for example, we have

$$X_1(\rho)=\sum_{j=1}^{n_f} f_j(\rho)X_{1j} \tag{20}$$

Remark 4: The condition (11) is a non-convex constraint. We can readily modify the algorithm proposed in (El Ghaoui, 1997) to solve the above nonlinear problem to obtain the suboptimal minimum H_∞ performance level γ.

Algorithm 1:

i) Find a feasible solution to (18-21). Set

$$k=0,\gamma_0=\gamma. \quad \text{where } \begin{bmatrix} Q & I \\ I & Z \end{bmatrix} \geq 0 \tag{21}$$

ii) Find a feasible set $(R,F,U,X_1,X_2,X_3,\tilde{A}_1,\tilde{A}_2,\cdots\tilde{A}_{nf},\tilde{B}_1,\tilde{B}_2,\cdots\tilde{B}_{nf},\tilde{C}_1,\tilde{C}_2,\cdots\tilde{C}_{nf},\tilde{D}_1,\tilde{D}_2,\cdots\tilde{D}_{nf})^0$.

iii) Choose a sufficiently small initial $0<\varepsilon \ll 1$ and solve the following LMI problem.

Minimize $Trace(QZ^k+Q^kZ)$ subject to (18-21).

Set
$$\begin{aligned}&(R,F,U,X_1,X_2,X_3,\tilde{A}_1,\tilde{A}_2,\cdots\tilde{A}_{n_f},\tilde{B}_1,\tilde{B}_2,\cdots\tilde{B}_{n_f},\tilde{C}_1,\tilde{C}_2,\cdots\tilde{C}_{n_f},\tilde{D}_1,\tilde{D}_2,\cdots\tilde{D}_{n_f})^k\\ &=(R,F,U,X_1,X_2,X_3,\tilde{A}_1,\tilde{A}_2,\cdots\tilde{A}_{n_f},\tilde{B}_1,\tilde{B}_2,\cdots\tilde{B}_{n_f},\tilde{C}_1,\tilde{C}_2,\cdots\tilde{C}_{n_f},\tilde{D}_1,\tilde{D}_2,\cdots\tilde{D}_{n_f})\end{aligned}$$

iv) If $Trace(QZ)\leq n+\varepsilon$ holds, then set $\gamma_0=\gamma$ and return to ii) after decreasing γ to some extent. If $Trace(QZ)>n+\varepsilon$ within a specified number of iterations, then exit. Otherwise, set $k=k+1$, and go to iii).

5 Numerical Example

Consider the following time-delay LPV system:

$$\begin{aligned}\dot{x}(t)&=\begin{bmatrix}0 & 1+0.2\rho(t)\\ -2 & -3+0.1\rho(t)\end{bmatrix}x(t)+\begin{bmatrix}0.2\rho(t) & 0.1\\ -0.2+0.1\rho(t) & -0.3\end{bmatrix}x(t-(1+0.5\rho(t))+\begin{bmatrix}0.2\\0.2\end{bmatrix}\omega(t)+\begin{bmatrix}1\\0.2\end{bmatrix}u(t)\\ z(t)&=\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}x(t)+\begin{bmatrix}0\\1\end{bmatrix}u(t)\\ y(t)&=\begin{bmatrix}0 & 1\end{bmatrix}x(t)\end{aligned} \tag{22}$$

where $\rho(t)=\sin(t)$ satisfies $\rho(t)\in[-1,1],\dot{\rho}(t)\in[-1,1]$ and the time delay $h(\rho(t))=1+0.5\rho(t)$ is varying from 0.5 to 1.5 and the condition $dh/dt<1$ holds. To design a parameter-dependent output feedback controller to guarantee a prescribed H_∞ performance level γ, we choose appropriate basis functions

$$f_1(\rho(t))=1,\ \ f_2(\rho(t))=\rho(t)$$

and grid the parameter space using 9 points grid. By Algorithm 1, for $\gamma=0.4061$, an admissible parameter--dependent output feedback controller is given by

$$\begin{aligned}A_c(\rho(t))&=\begin{bmatrix}-277.6489-0.1781\rho(t) & 89.7425+0.0921\rho(t)\\ -44.6239-0.5853\rho(t) & 14.7158+0.0273\rho(t)\end{bmatrix},\quad B_c(\rho(t))=\begin{bmatrix}-345.9074-0.2329\rho(t)\\ -60.4045-0.8487\rho(t)\end{bmatrix},\\ C_c(\rho(t))&=\begin{bmatrix}0.0965+0.0062\rho(t) & 0.2957-0.0050\rho(t)\end{bmatrix},\quad D_c(\rho(t))=-0.1054-0.0313\rho(t)\end{aligned}$$

and for $\gamma=0.4141$, the controller is described by

$$A_c(\rho(t)) = \begin{bmatrix} -264.9895 - 0.1852\rho(t) & 85.8323 + 0.0946\rho(t) \\ -39.3147 - 0.5791\rho(t) & 12.9948 + 0.0282\rho(t) \end{bmatrix}, \quad B_c(\rho(t)) = \begin{bmatrix} -325.8208 - 0.2406\rho(t) \\ -53.0215 - 0.8355\rho(t) \end{bmatrix},$$

$$C_c(\rho(t)) = \begin{bmatrix} 0.0975 + 0.0064\rho(t) & 0.3014 - 0.0051\rho(t) \end{bmatrix}, \quad D_c(\rho(t)) = -0.1057 - 0.0315\rho(t)$$

Even for $\gamma = 0.105$ and $\varepsilon = 0.0008$, we can still find feasible output feedback controller which produces relatively larger gain than the above results. Therefore, we can choose appropriate γ to design feasible output feedback controller.

6 Concluding Remarks

In this note, a new H_∞ performance criterion for time-delayed LPV systems is presented, upon which the parameter-dependent H_∞ output feedback controller design problem is investigated. An iterative output feedback controller design procedure is described. A numerical example has shown the feasibility and applicability of the proposed designs.

References

[1] Apkarian P, Adams RJ. Advanced gain-scheduling techniques for uncertain systems. IEEE Trans Control System Technology , 1998, 6:21-32.

[2] Apkarian P, Adams RJ. Continuous-time analysis, eigen-structure assignment, and H_2 synthesis with enhanced linear matrix inequalities (LMI) characterizations. IEEE Trans Automatic Control , 2001, 12:1941-6.

[3] Apkarian P, Gahinet PA. Convex characterization of gain-scheduled H_∞ controllers. IEEE Trans Automatic Control , 1995 , 40:853-64.

[4] Bara G.I, Daafouz J, Kratz F. Advanced gain scheduling techniques for the design of parameter-dependent observers [A]. Proceedings of the 40th Conference on Decision & Control, Florida USA, 2001a , 3892-7.

[5] Bara G.I, Daafouz J, Kratz F. Parameter-dependent control with γ -performance for affine LPV systems [A]. Proceedings of the 40th Conference on Decision & Control, Florida USA, 2001b , 2378-9.

[6] Boyd SP, El Ghaoui L, Feron E, Balakrishnan V. Linear matrix inequalities in system and control theory. Philadelphia: SIAM, 1994.

[7] El Ghaoui L, Oustry F, Rami A. A cone complementarity linearization algorithm for static output-feedback and related problems. IEEE Trans Automat Control , 1997 , 42(8):1171-6.

[8] Shamma JS, Athans M. Analysis of gain scheduled control for nonlinear plants. IEEE Trans Automatic Control, 1990, 35:898-907.

[9] Tan K, Grigoriadis KM, L_2-L_2 and L_2-L_∞ output feedback control of time-delayed LPV systems. Proceedings of the 39th IEEE Conference on Decision and Control, Sydney, Australia, 2000 , 4422-7.

[10] Tan K, Grigoriadis KM, Wu F. H_∞ and L_2-to-L_∞ gain control of linear parameter-varying systems with parameter-varying delays. IEE Proc Control Theory Appl , 2003 , 150(5):509-17.

[11] Wu F, Grigoriadis KM. LPV systems with parameter-varying time delays: analysis and control. Automatica , 2001 , 37:221-9.

[12] Zhang LW, Guo LS, Liu XP. H_∞ control for a class of linear parameter-varying systems with time-delay. Control and Decision , 2001 , 5:595-8.

[13] Zhang XP, Tsiotras P, Knospe C. Stability analysis of LPV time-delayed systems. *International Journal on control* , 2002 , 75:538-58.