Study on the Mathematic Model of Product Modular System Orienting the Modular Design

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Abstract: Modular design is a key technology of implementing virtual organization and mass customization. Orienting the modular design a mathematic model of modular system is presented and is used to realize the mathematic description of modular decomposing "Top-Down" process. Extending Boolean matrix of modular interface relationship is gotten by reverse reasoning strategy of modular interface relationship. The application based on the mathematic model of modular system and extending Boolean matrix of modular interface relationship is concluded, which provides theory foundation for the researching of modular design software system. [Nature and Science, 2004,2(1):61-67].

Keywords: modularization; modular design; modular interface; mass customization

1 Introduction

Modular design technology, as the advanced form of standardization, is a key technology of implementing virtual organization and mass customization. At present, the research of modular design is concentrating on the realization of design, production, assembly and maintaining of the specific product using modular design, and doesn't pay attention to the theory study. As a result, modular design is still a philosophy discussion and a thinking perfection, and can't form a general theory that instruct industry to implementing modular design. Through researching on the process of modular design, this paper presents a mathematical model of product modular system and realizes the mathematical description of the Top-Down module partition process. The application based on the mathematic model of modular system and extending Boolean matrix of modular interface relationship is concluded, which provides theory foundation for the researching of modular design software system (Tong, 2000; Huang, 2000).

2 Product Modular System Modeling

2.1 Product modular system mathematical model

Modular product consisted of basic modules, which have independent function, independent structure and suitable granularity (Figure 1). Basic module refers to the most basic unit that constitutes modular product

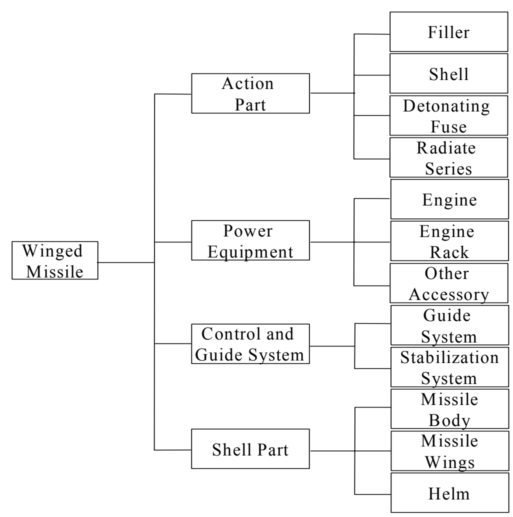


Fig. 1. Modular tree of winged missile

and constructional relationship, or the route of input and output of stream, energy, information, and so on. The superiority of modular design is using existing standard modules to manufacture new product by interchange and combination. From the point of view of systems engineering if we take the basic modules make up of the product as input of the modular design and take the modular product as the output of modular design, modular product is a system in which basic modules associating and effecting each other in a certain exterior environment (Chen, 1988). We call it product modular system, and its mathematical model can be:

$$P = f(M_n, W_{n\times n}) \tag{1}$$

Where P represents the product modular system, M_n represents the basic modular vector which make up of the modular product, $M_n = (m_1, m_2, \cdots, m_n)$. $W_{n\times n}$ represents the Boolean matrix of interfaces relationship in basic modules, which shows the relationship of the product basic modules. $W_{n\times n} = (\omega_{kl})_{n\times n}$, where $k, l = 1, 2, \cdots, n$, and correlative coefficient ω_{kl} determines if there is a relationship between m_k and m_l,

$$\omega_{kl} = \omega_{lk} = \begin{cases} 0, & k = l \\ 0, & k \neq l,\ m_k \text{ and } m_l \text{ have not relationship} \\ 1, & k \neq l,\ m_k \text{ and } m_l \text{ have relationship} \end{cases}$$

We name formula (1) as mathematical model of product modular system. The meaning of the model is that it abstracts the product modular system as a module's netty architecture, which takes the modules selection and the definition of their correlative as system input, and takes the product function and its structure as system output. And this makes the modular design easier to understand.

The mathematical model of product modular system can be used to describe the process of partitioning product modules. The process of partitioning product modules can be descript as: product →first class module→second class module→ ⋯ → basic module, we call the modules that is not the top product and can be partitioned again, such as first class module, second module and etc, as middle module, the process can be described as product→middle module→ basic module.

If mid_v^u represents the module v(v=1,2,⋯) in u(u=1,2,⋯) class, the second module in the first class, power equipment, will be showed as mid_2^1. Suppose mid_v^u can be partitioned into s(s=1,2,⋯) modules, and these modules make up of module vector $Ch_v^u M_s = (ch_v^u m_1, ch_v^u m_2, \cdots, ch_v^u m_s)$. According to formula (1), middle module mid_v^u can be described as:

$$mid_v^u = f(Ch_v^u M_s, Ch_v^u W_{s\times s}) \tag{2}$$

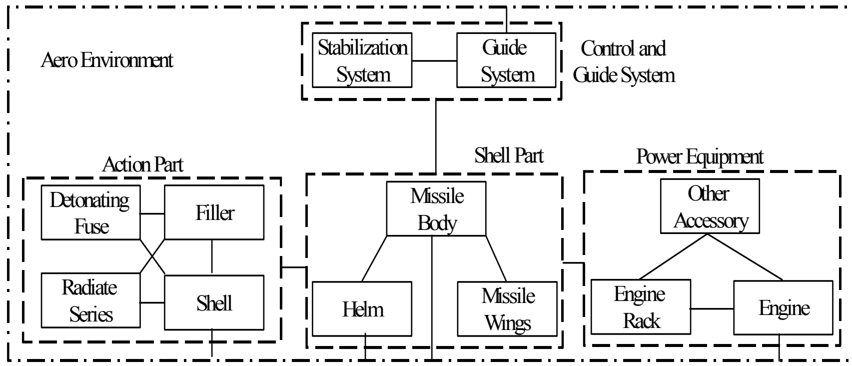
Where $Ch_v^u W_{s\times s} = (\omega_{fg})_{s\times s}$, f,g=1,2,⋯,s, and ω_{fg} has the same way as ω_{kl} to get its value.

Through formula (1) and (2), the process of partitioning product modules be described as a series of module vectors and Boolean matrices of interfaces relationship in modules. Combining with modular tree, these module vectors and Boolean matrices describe the structure and the hiberarchy and netty architecture of product system at the same time.

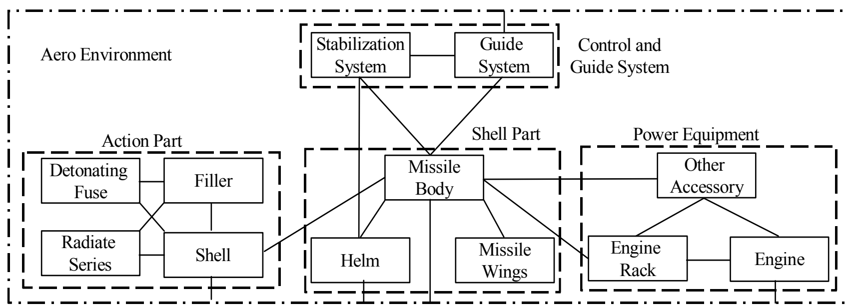
But through the module vectors and Boolean matrices of interfaces relationship in modules we cannot get the Boolean matrix of interfaces relationship in basic modules directly. The process of partitioning of product modules is Top-Down and classified. In each partitioning action we get the son modules of the middle module or product and the relationship among them, for example, action part, power equipment, control and guide system and shell part are partitioned from winged missile product by a partitioning action, and at the same time the relationship among them and Boolean matrix of their interfaces relationship is determined (Figure. 2 (a)). But we cannot get the correlative among the modules belong to different father modules but at the same class, for instance, the correlative between stabilization system in control and guide system and helm in shell part, or engine rack in power equipment and missile body in shell part, etc. This paper presents extending Boolean matrix of modular interface relationship and reverse reasoning strategy of modular interface relationship to solve the problem.

2.2 Extending boolean matrix of modular interface relationship and reverse reasoning strategy of modular interface relationship

Modular interface relationship can be divided into inter interface relationship and outer interface relationship. The inter interface relationship refers to the interface relationship among the modules belong to a same superior module, for instance, the relationship between stabilization system and control and guide system; the outer interface relationship refers to the interface relationship among the modules belong to different superior module or between the module and the environment, for example, the correlative between stabilization system in control and guide system, and helm in shell part, or engine rack in power equipment and missile body in shell part. Especially, we define the relationship between



(a) Modular relationship in partitioning



(b) Real modular relationship

Figure 2 Modular relationship in partitioning and real modular relationship

module and the outer system environment as system interface relationship. This relationship belongs to outer interface relationship.

Boolean matrix of modular interface relationship only describes the inter interface relationship and cannot show the outer interface relationship.

In order to describe the process of the modular partitioning process and the structure of product modular system, we introduce the extending subordinate modular vector

$ExCh_v^u M_{s+1} = (ch_v^u m_1, ch_v^u m_2, \cdots, ch_v^u m_s, om)$,

om represents outer module that don't belong to a same superior module in the same class or be the outer environment. Based on the extending subordinate modular vector, extending Boolean matrix of interface relationship in the subordinate modules of the middle module can be showed as

$$ExCh_v^u W_{(s+1)\times(s+1)} = (\psi_{bc})_{(s+1)\times(s+1)},$$

where $b, c = 1, 2, \cdots, s+1$. When $b, c = 1, 2, \cdots, s$, ψ_{bc} and ω_{fg} have the same way to get there values. They show if module $ch_v^u m_b$ and module $ch_v^u m_c$ have interface relationship, and we call them inter correlative coefficient. When $b = s+1$ or $c = s+1$, ψ_{bc} shows that if module $ch_v^u m_c$ or $ch_v^u m_b$ have the interface relationship with om, we call it outer correlative coefficient. If an interface relationship is exist between b and c, then $\psi_{bc} = \psi_{cb} = 1$, else $\psi_{bc} = \psi_{cb} = 0$.

If $Exmid_v^u$ is used to represent the middle module that its outer interface relationship is defined, the extending mathematical model of middle module can be described as

$$Exmid_v^u = f(ExCh_v^u M_{s+1}, ExCh_v^u W_{(s+1)\times(s+1)}) \quad (3)$$

Accordingly, if we use

$$ExM_{n+1} = (m_1, m_2, \cdots, m_n, om)$$

to represent extending basic modular vector of product modular system, and use

$$ExW_{(n+1)\times(n+1)} = (\psi_{eq})_{(n+1)\times(n+1)}$$

($e, q = 1, 2, \cdots, n+1$)to describe the interface relationship among basic modules or between basic module and system environment, and utilize ExP to represent the product system, then extending mathematical model of product modular system can be described as

$$ExP = f(ExM_{n+1}, ExW_{(n+1)\times(n+1)}) \quad (4)$$

The extending mathematical model of product modular system not only shows the relationship of system basic modules, but also defines the correlative of basic modules and the environment. The model further uncovers the essence of product modular system, and can be used to describe the process of partitioning product modules, which is the coming into being process of the modules and the extending Boolean matrices of their relationship. This paper presents the reverse reasoning strategy of modular interface relationship to get the relationship of basic modules, that is to say, the extending Boolean matrices of interface relationship in basic modules.

Reverse reasoning strategy of modular interface relationship is that using the modular tree and the extending Boolean matrices of modular interface relationship formed in partitioning process, and through interface matching to superpose the extending Boolean matrices of modular interface relationship of every class to the last class by breaking up the interface relationship in middle modules and transforming it to the interface relationship in basic modules, in the end obtaining the extending Boolean matrix of interface relationship in product basic modules. In order to put in force the reverse reasoning strategy of modular interface relationship, and realize the coming into being of the extending Boolean matrix of interface relationship in basic modules automatically, the product modular design system must have the ability:

(1) The outer interface relationship can be transferred and their information is equal. That is to say, the outer interface relationship can be divided and distributed to the outer interface relationship in the subordinate modules, and parameters of the interface will not change.

(2) Descript and definition of modular interface information can be queried and distinguished. So we can confirm if the modular interface relationship is matching according to the qualitative and quantitative information.

The reverse reasoning strategy of modular interface relationship can be described as next five steps:

(1) According to principle of partitioning modules, the product is broken up to suitable classes. Partitioning process of middle modules includes confirming subordinate modules number, describing basic information of subordinate modules, defining interface relationship (including inter interface relationship and outer interface relationship) of subordinate modules and describing interface information of subordinate modules. We get the extending Boolean matrices of interface relationship in the subordinate modules of product and middle modules, and at same time we get the modular tree that shows the hiberarchy of product system. Suppose that the max class of modular tree is z, if array num(u) (u=1,2,⋯,z) represents the module number of the class u, chn(u)(v) represents the subordinate module number of the module v(v=1,2,⋯,num(u)) in class u, then according to formula (3), the middle module of every class can be showed as

$$mid_v^u = f(ExCh_v^u M_{chn(u)(v)+1}, ExCh_v^u W_{(chn(u)(v)+1)\times(chn(u)(v)+1)})$$

where

$$ExCh_v^u M_{chn(u)(v)+1} = (ch_v^u m_1, ch_v^u m_2, \cdots, ch_v^u m_{chn(u)(v)}, om)$$

$$ExCh_v^u W_{(chn(u)(v)+1)\times(chn(u)(v)+1)} = (\psi_{(row,col)})_{(chn(u)(v)+1)\times(ch(u)(v)+1)}$$

$row, col = 1,2,\cdots,chn(u)(v)+1$.Then the subordinate module of class u is the module of module u+1, and $ch_v^u m_j = mid_{vi+j}^{u+1} (vi = \sum_{x=1}^{v-1} chn(u)(x)$, $j = 1,2,\cdots,chn(u)(v))$·

Especially, when a basic module appears in middle classes, it will be taken as the subordinate module of itself.

(2) Suppose that the module number of last class is n, then $num(z) = n$. We arrange the basic modules of the product to the basic vector of the product modular system according to their top-down sequence in modular tree. Combing with the outer module, the basic vector is extended to the extending basic vector $(m_1, m_2, \cdots, m_n, om)$ (where $m_i = mid_i^z$, and $i = 1,2,\cdots,n$).The extending basic vector determines the columns and the rows of the extending Boolean matrix of interface relationship in basic modules. Let

correlative coefficient be zero, that is to say,

$$ExW_{(n+1)\times(n+1)} = (\psi_{eq})_{(n+1)(n+1)} = (0)_{(n+1)(n+1)}.$$

Table 1 is an extending Boolean matrix of interface relationship in winged missile basic modules.

(3) For $mid_v^{z-1}(v = 1,2,\cdots,num(z-1))$, if mid_v^{z-1} is a middle module, then its subordinate modules must be basic modules. Suppose that $row, col = 1,2,\cdots,chn(z-1)(v)$, then $\psi_{(row,col)}$ in $Exch_v^{z-1}W_{(chn(z-1)(v)+1)\times(chn(z-1)(v)+1)}$ and $\psi_{(vi+row,vi+col)}$ in $ExW_{(n+1)\times(n+1)}$ are equal. According to inter correlative coefficient of the extending Boolean matrices of interface relationship in the subordinate modules of middle modules, we may get inter correlative coefficient of the extending Boolean matrix of interface relationship in basic modules, and these inter correlative coefficients are close to the diagonal of the extending Boolean matrix of interface relationship in basic modules, see the part of Figure 1. For example, we can get inter correlative coefficients of basic module missile body, missile wings and helm in shell part through the extending Boolean matrix of interface relationship in the subordinate modules of module shell part.

(4) For module $mid_v^{z-2}(v = 1,2,\cdots, num(z-2))$ in class z-2, if mid_v^{z-2} is a middle module and its subordinate modules are basic modules, then we replace inter correlative coefficients of the basic modules with inter correlative coefficients of the subordinate modules of module mid_v^{z-2} following the step 3. If the subordinate modules of mid_v^{z-2} are middle modules, we suppose that $row, col = 1,2,\cdots,chn(z-2)(v)$ and seeing about the middle module $ch_v^{z-2}m_{row}$ and $ch_v^{z-2}m_{col}$ in $Exch_v^{z-1}W_{(chn(z-2)(v)+1)\times(chn(z-2)(v)+1)}$ where $\psi_{(row,col)} = 1$.

According to the characters of modular design, the inter interface relationship in the father modules is the outer interface relationship in their son modules, so if module $ch_v^{z-2}m_{row}$ and $ch_v^{z-2}m_{col}$ have a inter interface relationship, it means that there are outer interface relationships in their son modules. In this situation, in the extending Boolean matrices of interface relationship in their son modules we should find out the basic modules that their outer correlative coefficients are 1. According to the ability that the interface information can be queried and distinguished, and the character that the interface relationship can be transferred and its quantity is equal, we can find the basic modules that are the subordinate modules and have interface relationship by the type of interface relationship, the main parameters of interface relationship and the condition that the middle modules are partitioned. And we define the correlative coefficients accordingly as 1. For example, the confirming of $\psi_{2,6}$ and $\psi_{6,10}$, and so on in Table 1. When subordinate modules of mid_v^{z-2} are middle modules or basic modules, the management is same as above, and at this time, if basic module and the subordinate modules of middle module have a interface relationship, that is to say, the relationship is the inter interface relationship of basic module and the outer interface relationship of the subordinate module.

(5) Repeat step 4, and transfer the inter interface relationship in every middle modules to outer interface relationship in their subordinate modules, so interface relationships are transferred downwards and finally turn into the interface relationship of basic modules. And the system interface relationship of product also can be transferred to the outer interface relationship in basic modules. See the outer interface relationship of m_5 and m_8 in Table 1. The whole extending Boolean matrix of interface relationship in product basic modules is established.

Reverse reasoning strategy of modular interface relationship realizes establish of the extending Boolean matrix of interface relationship in product basic modules. The matrix shows the relationship in basic modules and in basic modules and the environment. Combining with the modular definition model, we can realize the auto form of the extending Boolean matrix of interface relationship in basic modules.

3 Application of the Model

Through the mathematics of product modular system we can know the interface relationship of every two basic modules. If we use $R_k\,(k=1,2,\cdots,n)$ to express the basic modules aggregate which have relationship with the basic module m_k, then

$$R_k = \{r_l \mid r_l = m_l \times \omega_{lk} \text{ and } r_l \neq 0, l = 1,2,\cdots,n\} \quad (5)$$

Where $r_l = m_l \times \omega_{lk} = \begin{cases} 0, & \omega_{lk} = 0 \\ m_l, & \omega_{lk} = 1 \end{cases}$.

Table 1 The extending Boolean matrix of interface relationship in product basic modules of Figure 1

	m_1	m_2	m_3	m_4	m_5	m_6	m_7	m_8	m_9	m_{10}	m_{11}	m_{12}	Om
M_1	0	1	1	1									
M_2	1	0	1							1			1
M_3	1	1	0										
M_4	1			0									
M_5					0	1	1						1
M_6					1	0	1			1			
M_7					1	1	0			1			
M_8								0	1	1			1
M_9								1	0	1		1	
M_{10}		1				1	1	1	1	0	1	1	1
M_{11}										1	0		1
M_{12}									1	1		0	1
om		1			1			1		1	1	1	0

Note: the space in the Table is 0.

And we can know if basic module m_k has relationship with outer environment through judging the value of $\omega_{(n+1)k}$ and $\omega_{k(n+1)}$. The basic modules aggregate that relates with outer environment is

$$R_{n+1} = \{r_l \mid r_l = m_l \times \omega_{l(n+1)}, \text{and} \quad r_l \neq 0, l = 1,2,\cdots,n\}$$

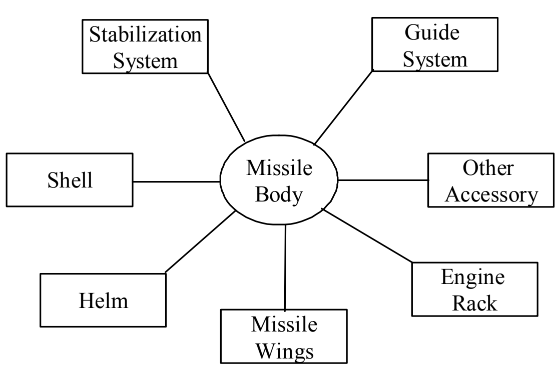


Fig. 3. Relationship figure of Missile Body

For example, R_{10} is a basic modules aggregate that relate to the basic module m_{10}. According to Table 1 and formulation (5), we can get

$$R_{10} = \{m_2, m_6, m_7, m_8, m_9, m_{11}, m_{12}\}.$$

Specific relationship is showed in Figure 3.

According to the relationship in basic modules, the hierarchy and inclusive relationship between basic modules and middle modules, we can judge if there is a relationship between random two modules (middle modules or basic modules). The judgment of the relationship in modules provides a possibility of partitioning the task of modular design and demonstrates which two companies have relationship in the process of product concurrent design.

The application of the mathematic model of product modular system also can be researched and discussed as follow:

(1) Realize the automatization and intelligentization of product modular synthesis using the

technology of neural net. We can take product basic modular vector and the extending Boolean matrix of interface relationship in basic modules as input and take product function and structure as output, using the technology of neural net to realize modular synthesis.

(2) Judge if the modular partition is reasonable by the extending Boolean matrix of interface relationship in basic modules. The degree of the product modularization relates to the interface numbers of product basic modules directly. We can Judge if the modular partition is reasonable by computing interface numbers in the extending Boolean matrix of interface relationship. Denmark scholar Juliana Hsuan has done some research (Hsuan, 1999).

(3) Application of the extending Boolean matrix's variation. The extending Boolean matrix of interface relationship in basic modules describes the common interface relationship in basic modules. We can map the matrix to different departments of the company, such as design, product, assemble, and so on.

4 Conclusion

Orienting the modular design a mathematic model of modular system is presented and is used to realize the mathematic description of modular decomposing "Top-Down" process. Base on the model, mathematic expression of the product interface relationship is discussed. Extending Boolean matrix of modular interface relationship is gotten by reverse reasoning strategy of modular interface relationship. The application based on the mathematic model of modular system and extending Boolean matrix of modular interface relationship is concluded, which provides theory foundation for the researching of modular design software system.

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