# Chaotic Analysis on Precipitation Time Series of Sichuan Middle Part in Upper Region of Yangtze

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**Abstract:** Based on the introduction of main quantitative indexes (correlation dimension $D_2$ and Kolmogorov entropy) in reconstruction phase-space technique, according to precipitation time series of Sichuan middle part in upper regions of Yangtze, relationship between embedding dimension $m$ and correlation dimension $D_2$ is discussed and saturation correlation dimension, minimum embedding dimension and Kolmogorov entropy are calculated, that is, $D_2 = 5.11$, $m = 10$ and $k = 0.338$. Meanwhile, primary component analytic method (PCA) is applied to validate its chaotic character and result shows forecasting length for this precipitation time series should be less than 2.96 years. Thus, chaotic analysis on precipitation time series provides a scientific gist for precipitation forecasting. [Nature and Science, 2004,2(1):74-78].

**Key words:** correlation dimension; Upper regions of Yangtze; Primary component analysis (PCA); Kolmogorov entropy

## 1 Introduction

Mainly focusing on those anomalistic and noperiodic macroscopical spatio-temporal phenomenon, chaotic theory is about research of special muddle behavior in dynamic system evolving, which reveals the underlying regulations of those appearing stochastic and out-of-order behavior. Chaotic theory, as a newly development of nonbalancable Stat – physics, comes up with new methods of researching some special and muddleheaded phenomenon.

Amount of yearly precipitation has direct effect on water resource exploitation and utility and is also closely related to occurrence and evolution of drought and flood. Therefore, precise forecast of annual precipitation provides not only scientific gist for water resource exploitation but also guidance for calamity precaution and salvage. Considering precipitation is uncertain or stochastic, we need form scientific and reasonable predicting models as well as probe system causing precipitation as uncertain and stochastic and determine effective phase-space, providing foundation . for making dynamical forecasting models.

Fractal theory, aiming at researching fractal characteristic of complex system and determining state space of time series, is an efficient tool for explaining complicated dynamical system and is widely used in field of physics, chemistry, biology, medicine, geology, weather, hydrology etc. In this thesis, according to precipitation time series of Sichuan middle part in upper regions of Yangtze, from 1953 to 2002 (Figure 1), from the view of correlation dimension $D_2$ and Kolmogorov entropy, regulation of precipitation formation and evolution is discussed, and then PCA is applied to validate chaotic feature of precipitation time series.

## 2 Chaotic Systems

Out-of-order in chaotic theory refers to those appearing disorder but not simply and normal chaos. For this reason, before there is ripe identifying method in describing features of attractor, we can recognize chaos from the point of correlation dimension $D_2$ and Kolmogorov entropy.

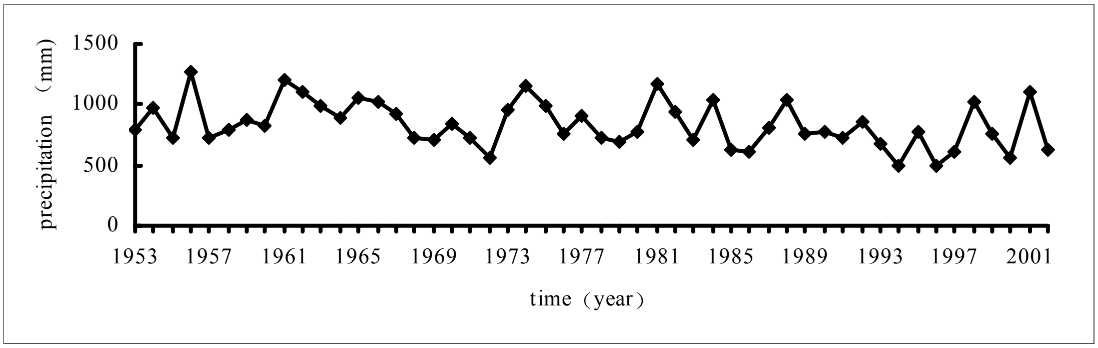


**Figure 1 Precipitation Series in Middle Region Sichuan Province of Yangtze Rive Up Reaches**

### 2.1 Phase-space reconstruct technique

Ruelle proposes to change system phase-space with a new $n$ dimensional phase-space (embedding phase-space) resulting from time series $x_t$ and its continual movement $x_{t+\tau}, x_{t+2\tau}, \cdots, x_{t+(n-1)\tau}$, and points out dimension of reconstruction phase-space $m$ should be more than or equal to two times plus 1 of state space, that is

$$m \geq 2d + 1 \tag{1}$$

Take a scalar time series $x_1, x_2, \cdots, x_n$ in system phase-space as an example. Supposing its dimension $d$ is 1, its dimension of embedding phase-space should be 3. If here $m = 4$, $x_1, x_2, x_3, x_4$ forms the first vector $Y_1$ of a four-dimensional state space and then moving right one step, $x_2, x_3, x_4, x_5$ forms the second vector $Y_2$. Just do in the same way, $Y_1, Y_2, Y_3, \cdots, Y_l$ forms the time series of reconstruction phase-space.

### 2.2 Correlation dimension $D_2$

In time series of $Y_1, Y_2, Y_3, \cdots, Y_k$, supposing $r_{ij}$ is the absolute value of difference between two sectors, namely,

$$r_{ij} = \left| Y_i - Y_j \right| \tag{2}$$

Then $r_0$ which is in the range of maximum and minimum $r_{ij}$ is determined. With the justification of $r_0$, a group of value of $\ln r_0, \ln C(r)$ and correlation dimension can be calculated. The definition of correlation dimension is as follows:

$$D_2 = \lim_{r \to 0} \ln C(r) / \ln (r_0) \tag{3}$$

$$C(r) = \frac{1}{l^2} \sum_i^l \sum_j^l H\left(r_0 - \left|Y_i - Y_j\right|\right) = \frac{1}{l^2} \sum_i^l \sum_j^l H\left(r_0 - r_{ij}\right) \tag{4}$$

$$H(z) = \begin{cases} 1 & z > 0 \\ 0 & z \leq 0 \end{cases} \tag{5}$$

Where: $l$ is the total number of state vectors $Y$, $r_{ij}$ is sphere diameter with the center of $Y_i$ or $Y_j$, $\left|Y_i - Y_j\right|$ is Euclid distance, $H$ is the function of Heaviside step function.

Numbers in Table 1 come from (3),(4),(5), $r_0 = \{300, 400, \cdots, 1000\}$, embedding dimension $m = \{4, 5, \cdots, 13\}$.

Using numbers in Table 1, we can draw a plot of $\ln C(r) \sim \ln r_0$. If line part exits, we say this time series has the feature of fractal, and slope of line is the correlation dimension (correlation dimension of the attractor). Figure 2 shows the relation between $\ln C(r)$ and $\ln r_0$.

From Figure 2, we see in chart of $\ln C(r) \sim \ln r_0$, with different embedding dimension $m$, part of linear correlation does exit, that is, precipitation time series of Sichuan middle part in upper regions of Yangtze has fractal feature. Slope of line in each curve is the correlation dimension corresponding $m$. Figure 3 shows the relation between correlation dimension $D_2$ and different embedding dimension $m$.

In Figure 3, we see when $m = 10$, correlation dimension tends to be stable, namely saturate. So when $D_2 = 5.11$, we say minimum embedding dimension $m = 10$ represents effective freedom rate of dynamical system. That is, when embedding phase-space of precipitation time series is 10, correlation dimension of the attractor is 5.11. From this aspect, we can draw a conclusion that precipitation time series of middle part of Sichuan in upper regions of Yangtze has chaotic feature. In order to further organize chaotic feature of this time series, PCA is introduced.

**Table 1 Data Calculation Table of $r_0$ ~ C(r) ~ m**

| C(r) r0 / n | 300 | 400 | 500 | 600 | 700 | 800 | 900 | 1000 |
|---|---|---|---|---|---|---|---|---|
| 4 | 0.1290 | 0.2884 | 0.4903 | 0.7103 | 0.8841 | 0.9674 | 0.9946 | 1.0000 |
| 5 | 0.0766 | 0.1777 | 0.3526 | 0.5709 | 0.7949 | 0.9272 | 0.9839 | 0.9981 |
| 6 | 0.0499 | 0.1062 | 0.2415 | 0.4331 | 0.6948 | 0.8647 | 0.9644 | 0.9921 |
| 7 | 0.0351 | 0.0651 | 0.1581 | 0.3140 | 0.5733 | 0.7893 | 0.9298 | 0.9824 |
| 8 | 0.0330 | 0.0471 | 0.1011 | 0.2353 | 0.4451 | 0.7112 | 0.8821 | 0.9654 |
| 9 | 0.0317 | 0.0385 | 0.0646 | 0.1508 | 0.3333 | 0.6145 | 0.8175 | 0.9342 |
| 10 | 0.0291 | 0.0351 | 0.0506 | 0.1041 | 0.2374 | 0.5253 | 0.7537 | 0.9084 |
| 11 | 0.0275 | 0.0325 | 0.0425 | 0.0762 | 0.1725 | 0.4275 | 0.6963 | 0.8738 |
| 12 | 0.0256 | 0.0309 | 0.0401 | 0.0572 | 0.1256 | 0.3123 | 0.6292 | 0.8435 |

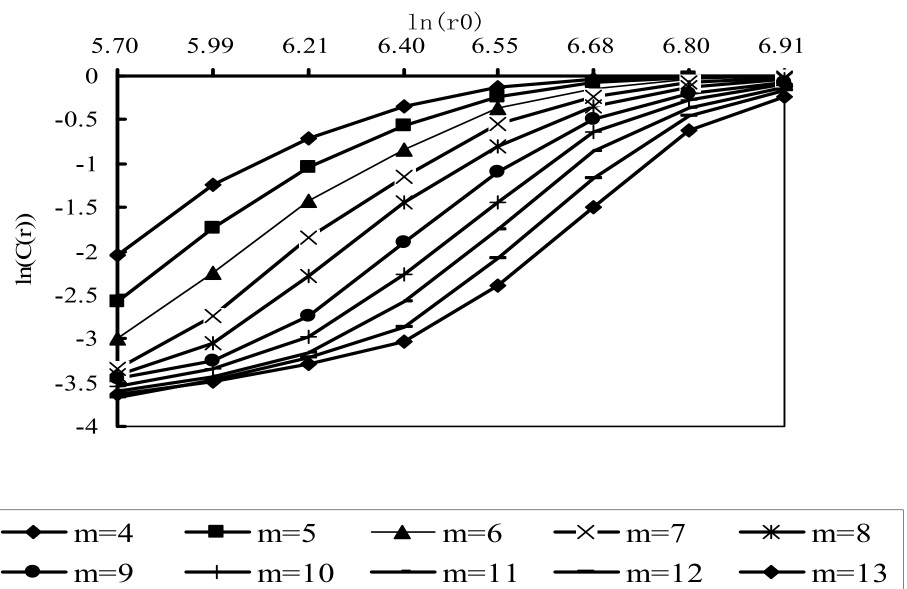


**Figure 2 Relation between ln ( $r_0$ ) and lnC ( r )**

### 2.3 Primary component analysis (Pca)

PCA is a newly proposed method in organizing noise and chaos. The step of this method is as follows:

Supposing a scalar time series is $x_1, x_2, \cdots, x_n$, after reconstructing phase-space (embedding dimension is $m$, and delay time is $\tau$), matrix $Y_{l\times m}\ (l = n-(d-1))$ is formed:

$$Y_{l\times m} = \frac{1}{l^{1/2}}\begin{bmatrix} x_1 & x_2 & \cdots & x_m \\ x_2 & x_3 & \cdots & x_{m+1} \\ \vdots & \vdots & \vdots & \vdots \\ x_l & x_{l+1} & \cdots & x_n \end{bmatrix} = \frac{1}{l^{1/2}}\begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_l \end{bmatrix}$$

Calculate covariance matrix $A_{d\times m} = \frac{1}{l} Y_{l\times m}^T Y_{l\times m}$

and its eigenvalue $\lambda_i$ $(i = 1,2,3,\cdots,m)$ and eigenvector $U_i$ $(i = 1,2,3,\cdots,m)$, then order them $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_m$ in descending sequence. Eigenvalue and eigenvector is called primary sector. Sum of all eigenvalue $\gamma$ is $\gamma = \sum_{i=1}^{d} \lambda_i$. Chart of $i$ and $\ln(\lambda_i / \gamma)$ is called primary spectrum. Primary spectrum of noise, which is parallel to $x$ axis, is quite different from that of chaotic serial, which is a line across fixed dot with negative slope.

Figure 4 is the chart of primary spectrum. According to feature of spectrum of chaotic time series, we can further determine that precipitation time series has chaotic feature.

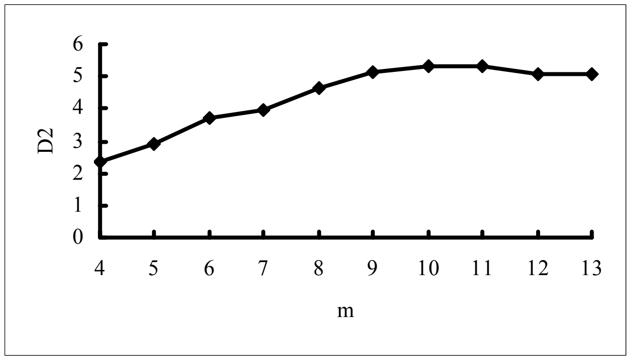


**Figure 3 Relation between $m$ and $D_2$**

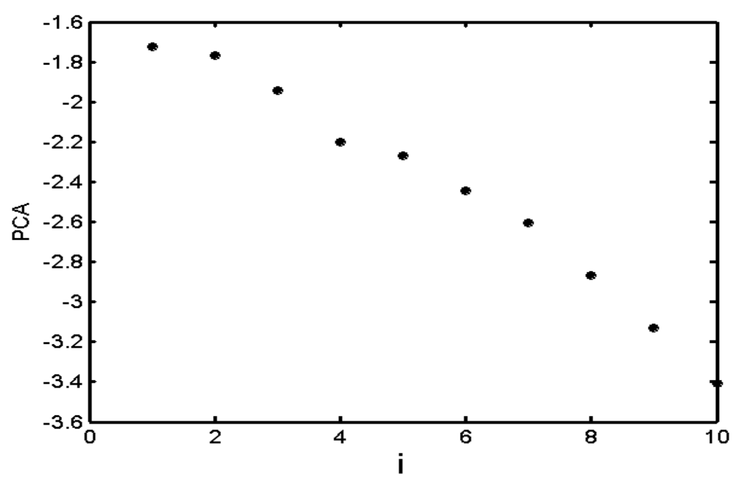


**Figure 4 Relation between i and PCA**

### 2.4 Kolmogorov entropy

Another important index of chaotic feature is Kolmogorov entropy, which provides upper and lower range of average amount of information in unit time. Generally, for a sequential system, $K = 0$; for a stochastic system, $K = \infty$. When $0 < K < \infty$, system is a chaotic system, and the bigger $K$ is, the more serious the degree of chaos is. Formula proposed by Grassberger-Procaccia algorithm is:

$$K_2 = \frac{1}{\tau} \ln \frac{C_m(r)^2}{C_{m+1}(r)^2} \tag{4}$$

Where: $\tau$ is delay time, $C_m(r)$ is the value of $C(r)$ when embedding dimension of phase-space is $m$, $C_{m+1}(r)$ is the value of $C(r)$ when embedding dimension of phase-space is $m+1$.

Choice of $\tau$ and $m$ is key to calculation of dimension, index and entropy. In application, we need to consider dimension of embedding phase-space as well as $\tau$ which has better simulating effect.

In theory, when $m \to \infty$, $K_2 \to K$. In fact, when $m$ is somewhat value, $K_2$ tends to be stable and this stable value can be used as estimating value of $K$.

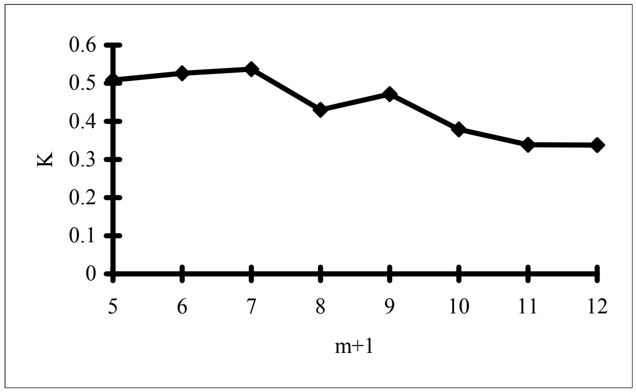


**Figure 5 Relations between m+1 and K**

Figure 5 shows the relation between $k$ and $(m+1)$. From Figure 5, we know with the increase of $m$, $k$ tends to be stable and when $(m+1)=11$, Kolmogorov entropy comes to saturation, that is, $K = 0.338\ (>0)$. This data also indicates the chaotic feature of precipitation time series of Sichuan middle part in upper regions of Yangtze. $1/K$ shows predictable length of this system is 2.96 years.

## 3 Conclusion

Correlation dimension $D_2 = 5.11$ and Kolmogorov entropy $k = 0.338$ are achieved by reconstructing phase-space. Primary component analysis further validates the chaotic feature of precipitation time series of Sichuan middle part in upper regions of Yangtze and the reciprocal of Kolmogorov entropy tells us predicting length of precipitation time series should be 2 to 3 years instead of long-term prediction, which provides scientific gist for determining length of predicting period.


### Acknowledgement

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