

# Application of Energy Approach to Estimating Scour Depth

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**Abstract:** Many permanent spillway tunnels of high earth-rock dams are rebuilt from their diversion tunnels. Scour problem is focused because of their high water head and velocity. Energy Approach is a new method to Estimating Scour and is used here to estimate the depth of scour downstream of 1<sup>#</sup> spillway tunnel of Zipingpu Project. The method is illustrated working fine by comparing its results with data from model test and empirical formula, and its advantages and disadvantages are analyzed at last. [Nature and Science, 2004,2(2):77-82]

**Key words:** energy approach; spillway tunnel; bed rock scour; ski-jump energy dissipation

## 1 Introduction

Zipingpu Hydraulic project lies on upstream of Minjiang River of China, and is the forth project in the Minjiang River cascade development program. The diversion tunnels of the project used in construction stage is designed to rebuild into permanent spillway tunnels with ski-jump energy dissipation works. Because of the low elevation of the diversion tunnels, the water head of the permanent spillway tunnels and the water speed through them are both high. Therefore the erosion control downstream of the tunnels is very difficult (Chen, 1993). Scour depth is an important index to estimate erosion, and many researches are made in this field and some are fruitful. But all the present methods have little effect in obtaining accurate and reliable scour depth according to the problem's complexity. In this article, energy approach is applied to estimate scour depth downstream of the 1<sup>#</sup> spillway tunnel, and compared with the results gotten by empirical formula and model test, it shows that energy approach works fine.

## 2 Principle of Energy Approach

In 1985 Canadian K.J.W. Spurr presented a new approach using an energy scour index (ESI-defined below) to estimating scour downstream of a large dam according to the relationship between the mean

surplus jet energy and the scour depth of the reference and study site plunge pool. The principle of the method is that the scour depth  $d_s(t)$  at any instant resulting from the action of a submerged jet impinging on a bedrock is assumed as a function of the jet energy  $E_a$  available at the surface of the bedrock and the rock's capacity  $E_{TH}$  to absorb or deflect the erosive forces, such that:

$$d_s(t) = f(E_a - E_{TH} - E_x)$$

Where  $E_x$  is the jet energy deflected by the bedrock at any instant.

The ESI is defined as the ration of the mean energy lost by the jet (or absorbed by the eroded portion of the bedrock) during the scour process at one site in relation to another, multiplied by the effect that the differences in the degree of plunge pool confinement are likely to have on the 11ping.docjet energy.

$$ESI = \frac{\int_0^{t_1} (I_s - I_r)_1 dt}{\int_0^{t_2} (I_s - I_r)_2 dt} \times F$$

where:  $t_0$  is the initial time and  $t_e$  is the time when equilibrium is approached.

$I_s$  is the mean stream power /unit area possessed by the submerged jet acting at the surface of a bedrock, and is function of a characteristic velocity acting near the bedrock surface  $U_x$  and the hydraulic shear force imparted to the rock, such that

$$I_s = f(\tau \cdot \bar{U}_x).$$

$I_r$  is the bedrock threshold capacity (mean

power/unit area) to absorb or deflect the erosive forces, beyond which it will degrade.

$1/1.25 \leq F \leq 1.25$  is an estimate of the percentage change in  $d_e$  resulting from the relative difference in the

$$ESI = \frac{Q_1 H_1 \rho / A_1 - d_{s1} \sigma_{c1} / t_{e1}}{Q_2 H_2 \rho / A_2 - d_{s1} \sigma_{c1} / t_{e2}} \times \frac{T_{e1}}{T_{e2}} \times F$$

degree of confinement between the plunge pools.

After simplification, the following formula is gotten.

where:  $Q$  is the total discharge;  $H$  is the effective potential head between the reservoir and the plunge pool tailwater levels;  $\rho$  is the density;  $d_s$  is scour depth;  $\sigma'_c$  is the effective uniaxial compressive strength of the bedrock;  $T_e$  is spill duration.

$A$  is the horizontal jet impaction area at initial bed level and may be estimated from considerations of the mean limiting entrainment velocities  $V_p$  acting at the plunge pool periphery in relation to the mean velocity  $V$  exiting the pool:

$$A_s = Q / V \approx Q / \beta V_p$$

where:  $Q$  is the total discharge and  $\beta$  is a typical empirical velocity distribution coefficient such that  $1 \leq \beta \leq 2.0$  for rough river courses.

Where in equation 2, variables of the reference site plunge pool have subscript 1 while the ones of the study site plunge pool have subscript 2.

The procedure of estimating scour follows:

(1) Choose a built work that has similar geology and spillway characters with the study site as the reference site;

(2) Choose an empirical formula that has been proved effective by the discharge and scour relationship of the reference site;

(3) Use the chosen formula on the study site and calculate the uncorrected scour depth  $d'_s$ ;

(4) Correct the results gotten from step 3 by ESI according to the differences of geology and spillway characters between the reference and study site and calculate the scour depth  $d_s$ .

### 3 Scour Depth Calculation

#### 3.1 Choice of the reference site and the empirical formula

The key point of applying ESI approach is the choice of the reference site. When used in the depth

estimation of Zipingpu's 1<sup>#</sup> spillway tunnel, the 9<sup>#</sup>, 10<sup>#</sup> sections of Dengman dam and 24<sup>#</sup> section of Huanren Dam is chosen as the reference sites (Dong, 1992; Institute, 1974). For both scours of the chosen dams have approached or near approached the equilibrium depth and have the spill duration records, which are 189h and 195h respectively. And the empirical formulas put forward by Guo Zizhong and Chen Chunting are used here, for comparison computation of 9 formulas on 14 prototype dams proved by Liu (2000) the two mentioned above are better.

#### 3.2 Parameter choice and simplification of formula ESI

##### 3.2.1 Calculation of H and A.

1<sup>#</sup> spillway tunnel of Zipingpu Project is long and the frictional loss of head can't be ignored; and the diameter and bodily form change along the tunnel, so the local head loss also can't be ignored. So H can't simply be replaced by the head difference of upstream and downstream. Here H is calculated according to the test data from the reference (Hydroelectric, 2001). A is calculated as mentioned above.

##### 3.2.2 Calculate $t_s$ uncorrected by empirical formula

The hydraulic characteristics are listed in Table 1 and calculation results are in Table 2.

##### 3.2.3 Choice of $\sigma'_c$

The intact uni-axial compressive strength ( $\sigma$ ) of Fengman, Huanren and Zipingpu is 260.5, 104 and 100  $\text{Mp}_a$  respectively (Hydraulic, 2000), and the class of erosion resistance of predominant rock of the study site plunge pool is III (Hydraulic, 2000; Cui, 1985). So let  $\sigma'_c = 3 \text{ Mp}_a$ , and for Fengman and Huanren, let

$$ESI = \frac{Q_1 H_1 \rho / A_1}{Q_2 H_2 \rho / A_2} \times \frac{T_{e1}}{T_{e2}} \times F \sigma'_c = 5 \quad (3)$$

$\text{Mp}_a$  and  $\sigma'_c = 3 \text{ Mp}_a$  respectively. Because of high water head and huge discharge, the term  $\sigma'_c d_s / t_e$  is much more smaller than  $QH \rho / A$ , so it can be ignored when calculating ESI. So the formula can be further simplified to:

##### 3.2.4 Simplification of $T_e$

The term  $T_e$  has greater influence on ESI than others, but its value is difficult to decide exactly. In reference 8,  $T_{e1}$  takes the value of spill duration on prototype and  $T_{e2}$  must be estimated from experience. According to the conditions of Zipingpu Project discussed above and referring to some prototypes, the value of  $T_{e2}$  is chosen as 200h. In another way, the

**Table 1 Hydraulic characteristics of 1<sup>#</sup> spillway tunnel of zipingpu project**

Group	Discharge Q (m <sup>3</sup> /s)	Water level upstream Z <sub>u</sub> (m)	Water level Downstream Z <sub>d</sub> (m)	H=Z <sub>u</sub> -Z <sub>d</sub> (m)	Tailwater Depth h <sub>t</sub> (m)	Discharge per unit width q(m <sup>3</sup> /(s.m))
1	1700	883.10	746.48	136.62	7.48	74.14
2	1640	877.00	746.44	130.56	7.44	71.52
3	1580	871.20	746.35	124.85	7.35	68.91
4	1470	860.00	746.19	113.81	7.19	64.11
5	1355	848.00	746.00	102.00	7.00	59.09
6	1230	836.00	745.74	90.26	6.74	53.64

**Table 2 t<sub>s</sub> Uncorrected calculations**

Formula	1	2	3	4	5	6
$t=1.1q^{0.50}H^{0.25}$	32.16	31.22	30.29	28.51	26.60	24.52
$t=1.1q^{0.58}H^{0.13}$	26.32	25.62	24.92	23.60	22.17	20.61
$QH\rho/A$	2306536	2337737	2391034	2274670	2138824	1992716

Note: For reference site Fengman Dam,  $T_e=189h$ ,  $QH\rho/A=3480000$ ; for reference site Huanren Dam,  $T_e=195h$ ,  $QH\rho/A=4374000$ .

influence of  $T_{e2}$  can be ignored by changing the ratio of the mean energy of the jet during the scour process to the ratio of the jet energy per unit time. Thus a new

$$ESI_1 = \frac{Q_1 H_1 \rho / A_1}{Q_2 H_2 \rho / A_2} \times F \quad (4)$$

energy scour index  $ESI_1$  is introduced and its formula is:

### 3.2.5 Choice of F

The parameter F reflects the relative difference in the degree of confinement to the progress of scour by the bedrock between the plunge pools. For the 9<sup>#</sup>, 10 sect of Fengman dam, the intact uni-axial compressive strength  $\sigma$  is high but a fault goes through, so the confinement from both the sides is big but which along the fault is small. As for the Zipingpu Project, considering the influence of slope protection on the right bank, the scour of the reference site is comparatively deeper, so F is chosen between 1.0 and 1.25, and 1.12 is selected at last. For 24<sup>#</sup> section of Huanren, the intact uni-axial compressive strength  $\sigma$  is quite the same to Zipingpu's, and the eroded portion of the bedrock is mainly consist of weakly

andesite-tuff, and the remained is andesite-tuff block in big size, so the confinement is great and  $1.0/1.25 \leq F \leq 1.0$ . 1.0 is chosen in the end (Dong, 1990). But the formation of the scour is such a complex progress that a definite depth is impossible to obtain from calculation, so 1.0, 1.12, 1.25 and 1.0, 1/1.1, 1.10 are given to F respectively to evaluate a range of the scour depth. From formula 3 and 4, calculations of  $ESI$  and  $ESI_1$  are obtained, as shown in Table 3.

### 3.2.6 Modify $d_s$ to $t_s$

Dong Junrui thought that  $d_s \cdot \sigma_c' / t_e$  in formula 2 is not enough to represent the energy loss by the jet. In fact the cushion in the scour absorbs a lot of energy (Liu, 2000). So when using  $ESI$ ,  $ESI_1$  to modify the scour depth calculated from empirical formula, the influence of the cushion should be considered. This can be carried out by changing  $d_s$  to  $t_s=(d_s+h_t)$ . So:

$$t_s = t_s' / ESI \quad (5-a)$$

$$t_{s1} = t_s' / ESI_1 \quad (5-b)$$

where  $t_s'$  is the calculations from empirical scour formula having not been corrected by  $ESI$ .

**Table 3** Calculations of ESI and ESI<sub>1</sub>

Reference site	Values of ESI and ESI <sub>1</sub>	1	2	3	4	5	6	
Fengman	ESI	1.00	1.43	1.41	1.38	1.45	1.54	1.65
		1.12	1.60	1.58	1.54	1.62	1.72	1.85
		1.25	1.78	1.76	1.72	1.81	1.92	2.06
	ESI <sub>1</sub>	1.00	1.51	1.49	1.46	1.53	1.63	1.75
		1.12	1.69	1.67	1.63	1.71	1.82	1.96
		1.25	1.89	1.86	1.82	1.91	2.03	2.18
Huanren	ESI	1.00	1.85	1.82	1.78	1.87	1.99	2.14
		1/1.1	1.68	1.41	1.38	1.45	1.54	1.65
		1.10	2.03	1.76	1.72	1.81	1.92	2.06
	ESI <sub>1</sub>	1.00	1.90	1.87	1.83	1.92	2.05	2.19
		1/1.1	1.72	1.70	1.66	1.75	1.86	2.00
		1.10	2.09	2.06	2.01	2.12	2.25	2.41

**3.3 Estimation of scour of 1<sup>#</sup> spillway tunnel of Zipingpu Project**

According to the data of the two reference sites of Fengman and Huanren and the hydraulic characteristics of 1<sup>#</sup> spillway tunnel, estimation is carried out by formula 3, 4 and 5, and the results are in Table 4.

In Table 4,  $t_s$  is obtained from (5-a) and  $d_s=t_s-h_i$ ; and  $t_{s1}$  and  $d_{s1}$  is calculated in the same way through (5-b). Considering that  $T_e$  has great influence on scour calculation and is difficult to choose an exact value,  $d_{s1}$  is selected as the scour depth. And the mean value of results from the two reference sites is taken as

the equilibrium depth, as shown in Table 5. Depths from some empirical formulas are also shown in Table 5 for compare. The empirical formulas vary greatly in results because of their different fundamentals, but they provide a scour depth range. And the results from energy approach are all enveloped in the range.

From Table 5 we find the rule that all the scours under the six hydraulic conditions develop along the water head and discharge per unit width rise.

In Table 6, scour from energy approach is shallower than results from empirical formulas, but deeper compared with test data.

**Table 4** Values of  $d_s$  and  $d_{s1}$

Reference site	Values of ESI and ESI <sub>1</sub>	1	2	3	4	5	6	
Fengman	$d_s$	1.00	15.08	14.75	14.67	12.53	10.30	8.12
		1.12	12.66	12.38	12.31	10.42	8.44	6.53
		1.25	10.57	10.32	10.27	8.59	6.84	5.15
	$d_{s1}$	1.00	13.84	13.53	13.46	11.45	9.35	7.30
		1.12	11.55	11.29	11.23	9.45	7.59	5.80
		1.25	9.57	9.34	9.30	7.72	6.08	4.49
Huanren	$d_s$	1.00	6.75	6.60	6.62	5.40	4.12	2.89
		1/1.1	8.18	10.77	10.77	9.13	7.42	5.75
		1.10	5.46	7.13	7.14	5.87	4.54	3.25
	$d_{s1}$	1.00	6.40	6.25	6.27	5.08	3.84	2.65
		1/1.1	7.79	7.62	7.63	6.31	4.93	3.59
		1.10	5.14	5.01	5.03	3.97	2.86	1.80

**Table 5 Scour cepts of 1# spillway tunnel of zipingpu project**

Scour Depth (m)	Upstream Water Level (m)					
	883.10	877.00	871.20	860.00	848.00	836.00
Energy Approach	9.05	8.84	8.82	7.33	5.77	4.27
$T=Kq^{0.50}H^{0.25}$	23.68	22.78	21.94	20.32	18.60	16.78
$T=Kq^{0.58}H^{0.13}$	17.84	17.18	16.57	15.41	14.17	12.87
$T=1.20q^{0.75}H^{0.125}/k$	17.27	16.44	15.57	14.33	13.00	11.50
$T=1.18q^{0.51}H^{0.235}$	17.59	16.81	15.90	14.90	13.79	12.44
$T=1.05q/Vc$	4.49	4.08	3.71	3.03	2.34	1.65

**3.4 Discussion**

(1) Energy approach is essentially an engineering analogy method, so the similarity of discharge type is required when choosing reference sites. In this article, both the reference sites of Fengman and Huanren dam adopt spillway face, while Zipingpu Project adopts spillway tunnel. And another difference between the reference and study site in this article is the magnitude of discharge. So it is easy to understand that the frictional loss of head through spillway tunnel is bigger than through spillway face, and the effective potential head is smaller under the same initial water head. So it is necessary to adopt effective potential head instead of initial water head to eliminate the influence caused by the difference of discharge type.

(2) It needs some experience in choosing the

reference site, spill duration and confinement coefficient F. So in spite of some improvements are adopted in calculation, and the results also shows superiority over the traditional empirical formula, there are still influences on the approach.

(3) The influences of asymmetry of nappe and aeration to scour are not considered in this paper and A is calculated by empirical formula. So the result of energy approach is unavoidably empirical.

(4) The consideration of the effect of cushion makes the method more reasonable. But what degree of effect the cushion acts in the energy absorb needs further research. Li (1994) points out that the effect of cushion is not so good as expected. As for Zipingpu project, the cushion is shallow and the effect can be forecasted not ideal.

**Table 6 Elevation contrast of the deepest points**

Description		1	2	3	4	5	6
Energy Approach	Maximum	733.86	733.99	733.97	735.03	736.14	737.20
	Minimum	725.16	725.47	725.54	727.55	729.65	731.70
	Mean	729.95	730.16	730.18	731.67	733.23	734.73
Empirical Formula	Maximum	734.51	734.92	735.29	735.97	736.66	737.35
	Minimum	715.32	716.22	717.06	718.68	720.40	722.22
	Mean	722.83	723.54	724.26	725.40	726.62	727.95
Test Data		731.96	732.56	734.06	734.06	732.86	734.36

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