Applying Non-stable Time Series Model to Forecast the Groundwater Dynamic Variation in the Well-Irrigated Rice Area in Sanjiang Plain

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Abstract: Groundwater shortage has become a very serious problem in Sanjiang Plain. More and more so-called "funnels" and "hanging pumps" situations occurred due to water shortages. The overall trend of groundwater level has decreased continually over the years. In this study, the Chuang Ye Farm in Jian San Jiang Department was taken as an example to develop a non-stable time series random model by applying the random analysis method. The model was used to simulate the dynamic variation and forecast the trend in the future. This study aims to provide a better understanding for the sustainable development of agriculture and use of groundwater resources. [Nature and Science. 2004;2(3):55-61].

Key words: non-stable time series model; well-irrigated rice; groundwater; Sanjiang Plain

1 Introduction

Sanjiang Plain is an important base of food production in China. In recent decades, great progress has been made through developing well-irrigated rice and adopting rice to control floods. The area of rice had increased to about 10 million acres by the end of 1998, 80% of which is well-irrigated rice. However, the expansion of rice planting areas without an optimized irrigation plan has led to the overuse and wasteful use of groundwater. Because of overuse of the ground water and some existing water-saving measurements become ineffective, groundwater shortage situation is worse than ever and in some areas, it becomes what is called "hanging pumps" and "funnel". In the spring of 1996, more than 600 "hanging pumps" did not work properly in Jian San Jiang Administration Bureau alone. The groundwater shortage and lack of understanding and techniques on effective and sustainable use of water resources have limited the potential of future development in Sanjiang Plain. Therefore, it is necessary to conduct a research to better understand the groundwater dynamic variety law and to provide scientific guidance for assessing the development scale of well-irrigated rice in Sanjiang Plain.

2 Developing the Random Model

The dynamic variation of the groundwater table resulted from the precipitation, water use and other random factors. Because of seasonal precipitation, the groundwater table varies periodically. Variation of the groundwater table has the trend of periodical descent because water use is increasing year after year. The mathematical expression of the groundwater level dynamic change in the area of well-irrigated rice is as follows (Li, 1999).

$$H(t) = h(t) + v(t) + x(t) \qquad (t = 1,2,3,.....n) \tag{1}$$

$$f(t) = H(t) - h(t) \tag{2}$$

Where $H(t)$ is the groundwater level (m), $h(t)$ is the varied trend item (m) arose by the overuse of groundwater, $v(t)$ is periodical change item (m) arose by seasonal precipitation, $x(t)$ is the random item and $f(t)$ is surplus value of the groundwater level.

Thus, long-term observation sequences belong to non-stable time series. It is necessary to analyze the series' components individually.

2.1 The Varied Trend Item (Ding, 1988; Yang, 1996; Ding, 1998)

The varied trend of the groundwater level annual time series can be determined by the stepwise regression analysis method. Firstly, give a common polynomial function:

$$h(t)=b_0+b_1t+b_2t^2+b_3t^3+b_4t^4+b_5t^{-1}+b_6t^{-2}+b_7t^{\frac{1}{2}}+b_8t^{-\frac{1}{2}}+b_9e^{-t}+b_{10}lnt$$

Then determine the mathematics expression of trend item through adopting stepwise regression method and selecting in computer, or append the trend line to original data directly in the software of MS Excel 2000. If water level fits the one of the followed formulae, it is the required trend variety item.

(1) $h(t)=at+b$

(2) $h(t)=a\ln t+b$

(3) $h(t)=ae^{bt}$

Where a and b are indefinite coefficients and t is time serial number.

In practice, the moving average method, orthogonal polynomial method and spline function may be also adopted to select trend item and gain the trend item equation. To analyze periodical item of the trend item is deducted from the original data.

2.2 Periodical Variation Item

The common methods of selecting periodical item $v(t)$ include variance analysis, correlation analysis, harmonic analysis and periodical figure analysis (Fu, 2000; Li, 1999; Ding, 1988; Ding, 1998).

Since the groundwater dynamic variation takes place yearly or monthly as period obviously, this paper adopts harmonic analysis method. The aim of harmonic analysis is to separate some simple harmonic waves with different amplitude and phase from the irregular curve of groundwater dynamic variation for researching its statistical law and character. Because Fourier series may fit the physical phenomena with periodicity and separate the periodical variation item and random item in the surplus value of groundwater level, the periodical variation item of groundwater level can be ascertained by Fourier series harmonic analysis. The formula of Fourier series is:

$$\begin{cases}\hat{a}_0=\dfrac{1}{N}\sum\limits_{t=0}^{N-1}y_t\\ \hat{a}_k=\dfrac{2}{N}\sum\limits_{t=0}^{N-1}y_t\cos\omega_k t=\dfrac{2}{N}\sum\limits_{t=0}^{N-1}y_t\cos\dfrac{2\pi k}{N}t\\ \hat{b}_k=\dfrac{2}{N}\sum\limits_{t=0}^{N-1}y_t\sin\omega_k t=\dfrac{2}{N}\sum\limits_{t=0}^{N-1}y_t\sin\dfrac{2\pi k}{N}t\end{cases}$$

$k=1,2,\ldots,p$

The frequency multiplication limit is $N/2$ and the number of largest wave p equals to $[N/2]$. Where take $p=N/2$ when N is even number and $p=\frac{N-1}{2}$ when N is odd number. Same as the regression analysis, regression variance of original series $\{y_t\}$ can be proved. It is:

$$S_y^2=\frac{1}{N}\sum_{t=0}^{N-1}(\hat{y}_t-\overline{y}_t)^2=\frac{1}{N}\sum_{t=0}^{N-1}\left[\sum_{k=1}^{p}(\hat{a}_k\cos\omega_k t+\hat{b}_k\sin\omega_k t)\right]^2=\sum_{k=1}^{p}\frac{1}{2}(\hat{a}_k^2+\hat{b}_k^2)=\sum_{k=1}^{p}\frac{1}{2}\hat{C}_k^2=\sum_{k=1}^{p}S_k^2$$

Thus, the total variance $S_y^2=\frac{1}{N}\sum\limits_{t=0}^{N-1}(\hat{y}_t-\overline{y}_t)^2$ of series $\{y_t\}$ may be expressed by the summation of each harmonic variance and surplus variance. Therefore, the statistical measurement F can be carried out to inspect the significance of the k th harmonic, that

$$F=\frac{\frac{1}{2}C_k^2/2}{(S_y^2-\frac{1}{2}C_k^2)\Big/(N-2-1)}\sim F(2,N-3)$$

Follows F distribution with of freedom degree of $(2,N-3)$. We may do significant test each harmonic one by one through using F_α inspection with the mentioned data. The periodical item of the series can be expressed as the superposition of each significant harmonic. Consequently, ascertain the model of periodical item function is as follows.

$$v(t)=a_0+\sum_{k=1}^{m}(a_k\cos\omega_k t+b_k\sin\omega_k t)$$

Where: m = number of significant harmonic.

Thus, there is only random item $x(t)$ when trend item $h(t)$ and periodical item $v(t)$ are deducted from the original series of groundwater $H(t)$.

$$x(t)=H(t)-h(t)-v(t)$$

2.3 Random Item

If random component $x(t)$ is stable, $x(t)$ consists of stable dependent component $D(t)$ and stable independent random component $\varepsilon(t)$ (pure random component), that is: $x(t)=\varepsilon(t)+D(t)$.

For the stable random component $x(t)$, we may use linear stable random model to express its statistic character. Establish $AR(p)$ model and its expression is:

$$x(t)=\mu+\varphi_1[x(t-1)-\mu]+\varphi_2[x(t-2)-\mu]+\cdots+\varphi_p[x(t-p)-\mu]+\varepsilon(t)$$

where μ is the average of the random series, and $\varepsilon(t)$ is the independent random variable. The average of $\varepsilon(t)$ is zero. The variance is σ_ε^2. φ_1, φ_2 ,.....φ_p are weight coefficients of auto regression, which is called auto regression coefficient commonly. Because of the existing of some relations between the variance σ_ε^2 of independent random variable $\varepsilon(t)$ and the variance σ^2 in the series $x(t)$, there are parameters (μ, σ and φ_1, φ_2 ,.....φ_p) in common auto regression model. Where, μ means the level of the series $x(t)$, σ means the variation degree of $x(t)$ around the mean value, and φ_1, φ_2 ,.....φ_p depend degree of the series on time.

The $AR(p)$ model is established as followed:

Step 1: to select the model's type.

Through analyzing and treating the random series, the auto correlation figure can be drawn according to the followed formula. The formula is:

$$r_k=\frac{\sum_{t=1}^{n-k}x_tx_{t+k}-\frac{1}{n-k}(\sum_{t=1}^{n-k}x_t)(\sum_{t=1}^{n-k}x_{t+k})}{\left[\sum_{t=1}^{n-k}x_k^2-\frac{1}{n-k}(\sum_{t=1}^{n-k}x_t)^2\right]^{\frac{1}{2}}\left[\sum_{t=1}^{n-k}x_{t+k}^2-\frac{1}{n-k}(\sum_{t=1}^{n-k}x_{t+k})^2\right]^{\frac{1}{2}}}$$

Where time delay k equals to 0,1,2,...m. When n is larger than *50* ($n>50$), m may be less than n/4 and m equals to n/10 usually. Draw the allowable range that confidence level of auto-correlation coefficient is 95% in the auto-correlation figure. If α equals to 0.05, allowable range of auto-correlation coefficient r_k is as:

$$r_k(\alpha=0.05)=\frac{-1\pm1.96\sqrt{n-k-1}}{n-k}$$

If p-step correlation coefficient is prominent to the independent sequence, the original random sequence is a mean square contingency sequence and the $AR(p)$ model can be adopted.

Step 2: normal conversion (Ding, 1988; Zheng, 1999).

Do normal test for the random series x_t. The mean value $\bar{x}$ and variance s^2 of original random series are $\bar{x}=\frac{1}{n}\sum_{j=1}^{n}x_j$ and $s^2=\frac{1}{n-1}\sum_{j=1}^{n}(x_j-\bar{x})^2$. The variation coefficient is $C_v=\frac{s}{\bar{x}}$ and skewness coefficient is $C_{sx}=\frac{1}{n-3}\sum_{i=1}^{n}(x_i-\bar{x})^3/s^{3/2}$. If the series is skewness distribution, it is necessary to doing normal conversion.

Step 3: identification of the model form (Ding, 1988; Ding, 1998; Du, 1991).

The identification of the model form is to ascertain the steps p. The main method is to do statistical analysis for partial correlation coefficient. The coefficient φ_k can be calculated by Yule-Walker

$$h(t)=b_0+b_1t+b_2t^2+b_3t^3+b_4t^4+b_5t^{-1}+b_6t^{-2}+b_7t^{\frac{1}{2}}+b_8t^{-\frac{1}{2}}+b_9e^{-t}+b_{10}\ln t$$

equation estimation method through solving the followed matrix. It is as the following:

$$\begin{bmatrix}\varphi_1\\ \varphi_2\\ \cdots\\ \varphi_p\end{bmatrix}=\begin{bmatrix}1 & \rho_1 & \rho_2 & \cdots & \rho_{p-1}\\ \rho_1 & 1 & \rho_1 & \cdots & \rho_{p-2}\\ \cdots & \cdots & \cdots & \cdots & \cdots\\ \rho_{p-1} & \rho_{p-2} & \rho_{p-3} & \cdots & 1\end{bmatrix}^{-1}\begin{bmatrix}\rho_1\\ \rho_2\\ \cdots\\ \rho_p\end{bmatrix}$$

The allowable range of partial correlation coefficient is $p\left\{\left|\varphi_{k,k}\right|<\frac{1.96}{\sqrt{n}}\right\}=95.0\%$. When the value of φ_k is in the allowable range, the value of k acts as the steps p ($p=k$).

Step 4: parameter estimation

$$\sigma_\varepsilon^2=\sigma^2(1-\varphi_1\rho_1-\varphi_2\rho_2-\cdots-\varphi_p\rho_p)$$

Where $\varphi_{k,k}$ is gained during solving the partial correlation coefficient. Therefore, the $AR(p)$ model can be ascertained.

Step 5: Farther identification of the model.

When the selected steps are p, we may identify the random series by adopting AIC criterion in order to ensure the model has the fewest parameters. The criterion of AIC is $AIC(p,q)=n\ln(\bar{\sigma}_\varepsilon^2)+2(p+q)$. $AIC(p), AIC(p-1), AIC(p+1)$ should be calculated respectively. If $AIC(p)$ is the smallest of them, the $AR(p)$ model is the best. Otherwise, select the smallest AIC value and compare them further.

Step 6: model inspection.

The main inspection is whether the residual error item $\varepsilon(t)$ is independent or not.

$$\varepsilon(t)=x(t)-\mu-\varphi_1[x(t-1)-\mu]-\varphi_2[x(t-2)-\mu]-\cdots-\varphi_p[x(t-p)-\mu]$$

Calculate $\varepsilon(1),\varepsilon(2),\varepsilon(3),...\varepsilon(n)$ and auto correlation coefficients ($r_1(\varepsilon),r_2(\varepsilon),r_3(\varepsilon),...r_k(\varepsilon)$). The calculation formula of statistical measurement is: $Q = n\sum_{k=1}^{m} r_k^2(\varepsilon)$. Therefore, Q gradually follows χ^2 distribution with the freedom degree $(m-p-q)$. If Q is less than χ^2 (0.05), $\varepsilon(t)$ is independent each other. The normal test of the residual error $\varepsilon(t)$ is the same as the former step.

The former steps are basic steps for establishing $AR(p)$ model.

2.4 Establish Non-stable Time Series Model

Through surplus the former trend item $h(t)$, periodical item $v(t)$ and random item $x(t)$, the non-stable time series random model of the groundwater can be obtained as:

$$v(t) = \hat{a}_0 + \sum_{k=1}^{p} (\hat{a}_k \cos \omega_k t + \hat{b}_k \sin \omega_k t)$$

$$x(t) = \mu + \varphi_1[x(t-1)-\mu] + \varphi_2[x(t-2)-\mu] + \cdots + \varphi_p[x(t-p)-\mu] + \varepsilon(t)$$

$$H(t) = h(t) + v(t) + x(t) \qquad (t = 1,2,3,\ldots\ldots n)$$

3 Modeling Example

3.1 Introduce Basic Conditions

Chuang Ye farm occupies the area of 533.3 km^2 in Jian San Jiang Administration Bureau, Sanjiang Plain. The main crop is rice and the areas of the farm had increased largely from 2000 acres in 1985 to 365000 acres in 2000. Annual groundwater use had increased from 1.2 million m^3 in 1985 to 164.25 million m^3 in 2000. The investigation data in 1996 indicated that the exploitable groundwater was 49.26 million m^3/a in the farm and use of the groundwater had reached to 75 million m^3/a. The groundwater is used excessively. If it is necessary to satisfy the well-irrigated rice areas increasing year after year, this need exploit the groundwater more extensively in the condition that irrigation norm varies very little. The groundwater use in 2000 has reached to 164.25 million m^3 and the phenomena of funnel and "hanging pump" have appeared largely. This has treated to the agriculture production and domestic water. Therefore, it is necessary to simulate the groundwater dynamic variation law and forecast developable trend according to the observation data of groundwater level from 1985 to 2000 in Chuang Ye farm in order to provide guidance for the decision-makers.

Table 1. The Data Table of the Groundwater Depth in Sanjiang Plain (1985～2000)

	Jan.	Feb.	Mar.	Apr.	May.	Jun.	Jul.	Aug.	Sept.	Oct.	Nov.	Dec.	Mean
1985	2.97	3.02	3.29	3.35	5.60	5.41	4.82	3.61	2.92	2.03	1.95	1.94	3.41
1986	3.01	3.18	3.45	3.51	5.82	5.58	4.95	3.95	3.01	2.21	2.13	2.13	3.58
1987	3.2	3.31	3.56	3.72	6.01	5.73	5.2	4.11	3.21	2.33	2.3	2.3	3.75
1988	3.4	3.42	3.8	3.88	6.21	5.88	5.41	4.31	3.35	2.48	2.4	2.41	3.91
1989	3.45	3.51	3.62	3.91	6.31	6.02	5.52	4.37	3.39	2.52	2.48	2.45	3.96
1990	3.84	3.84	3.95	4.23	6.59	6.41	5.83	4.76	3.61	2.61	2.60	2.55	4.24
1991	4.15	4.17	4.25	4.41	7.05	6.93	6.31	5.20	3.81	2.95	2.85	2.84	4.58
1992	4.25	4.30	4.33	4.43	7.23	7.00	6.65	5.48	3.90	3.61	3.50	3.47	4.85
1993	4.5	4.53	4.7	4.99	7.68	7.45	7.30	5.73	4.55	3.86	3.65	3.65	5.22
1994	4.75	4.78	4.81	4.91	7.97	7.81	7.80	6.14	4.79	4.01	3.90	3.90	5.46
1995	4.60	4.63	4.80	5.00	8.16	8.01	7.92	6.37	5.40	4.76	4.27	4.25	5.68
1996	4.60	4.65	4.83	5.00	8.47	8.00	7.90	6.47	5.52	4.93	4.60	4.60	5.80
1997	5.01	5.11	5.1	5.29	8.67	8.21	8.06	6.69	5.73	5.18	4.95	4.95	6.08
1998	5.03	5.33	5.63	6.10	11.07	10.93	9.10	8.02	7.52	5.33	5.19	5.19	7.04
1999	5.22	5.22	5.34	6.17	11.59	11.85	13.06	10.58	8.62	7.50	6.57	6.57	8.19
2000	6.49	6.39	6.40	6.55	10.25	13.76	12.75	10.92	8.95	7.86	7.01	7.00	8.69

From Table 1, the groundwater embedded depth has taken on ascending trend since Chuang Ye farm developed the well-irrigated rice planting in 1985. Annual mean embedded depth of the groundwater increased 2.39 meter among 1985 and 1996 because the well-irrigated rice area is about 0.12 million acres and the groundwater use is relatively small. However, the groundwater level has decreased rapidly in recent four years because the rice area increase has broken the dynamic balance of groundwater and could not supply the lateral influent of ambient river water. Because of the effect of seasonal precipitation, the groundwater table has periodical variation during the year. Therefore, the type of data model belongs to non-stable time series random model. It can be expressed by adding the above items as:

$H(t) = h(t) + v(t) + x(t) \qquad (t = 1,2,3,.....n)$

Where, the symbolic meanings are the same as the above and under-mentioned symbols.

3.2 Establish the Model of Trend Item $h(t)$

Selection of the appropriate trend item based on the model in the specific condition which takes the significant level α =0.05 by fitting has been established. The model is as:

$h(t) = 0.0002t^2 - 0.0034t + 0.3956 \qquad$ r = 0.8952

The curve of selected trend item shows in Figure 1

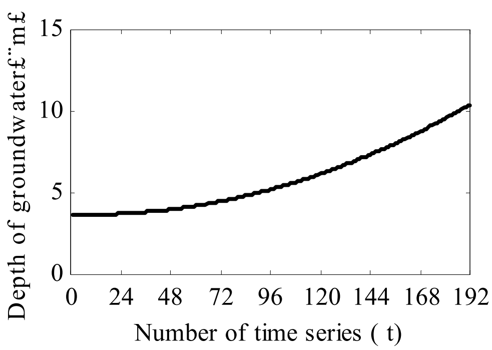

Figure 1. The Curve of Tendency Item

3.3 Establish the Fit Model of the Periodical Item $v(t)$

According to the above formulae, the limit of frequency multiplication (the maximal number of wave) has been taken as $p = N/2$, where N is the number of samples. If N equals to 192 and p equals to 96, Fourier series is showed as in Table 6.

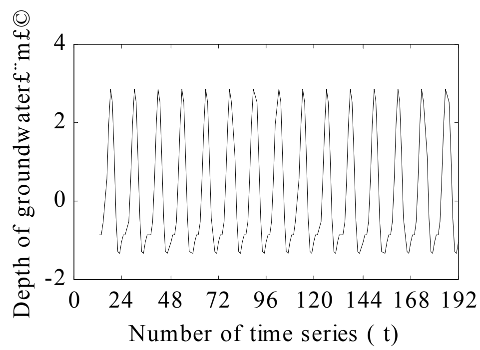

Figure 2. The Curve of Period Item

The total variance S_y^2 of series $v(t)$ is 2.9873. Calculate the variance of each harmonic. F_α equals to 3.05 in the condition of significant level $\alpha = 0.05$. After inspecting each harmonic variance, there are some harmonics to reach the significant level. They are No.1, No.16 and No.32. Their variance is 8.56%, 63.4% and 9.31%, and their F statistical measurement equals to 8.8433, 163.6796 and 9.7032 respectively. Because the values are larger than $F_\alpha = 3.05$, establish periodical model to fit periodical series through selecting No.1, No.16 and No.32 harmonic. It is:

$$v(t) = a_0 + a_1\cos\frac{2\pi t}{N} + b_1\sin\frac{2\pi t}{N} + a_{16}\cos\frac{2\times 16\pi t}{N} + b_{16}\sin\frac{2\times 16\pi t}{N} + a_{32}\cos\frac{2\times 32\pi t}{N} + b_{32}\sin\frac{2\times 32\pi t}{N}$$

$$= -0.5677 - 0.2319\cos\frac{2\pi t}{192} + 0.6764\sin\frac{2\pi t}{192} - 1.9462\cos\frac{32\pi t}{192} - 0.0069\sin\frac{32\pi t}{192} + 0.6748\cos\frac{64\pi t}{192} + 0.3177\sin\frac{64\pi t}{192}$$

The curve of periodical item function is shown in Figure 2.

If the trend item $h(t)$ and periodical trend item $v(t)$ are deductible from the original series $H(t)$, the random item $x(t)$ is as: $x(t) = H(t) - h(t) - v(t)$.

3.4 Establish Random Model x_t

3.4.1 Normal Inspection of Random Series x_t

In calculating, the manual value $\bar{x}$ of the random series x_t equals to 0, whose variance s^2 is 0.5625 and whose skewness coefficient C_{sx} is 0.0239. Because the skewness coefficient tends to 0, the series may be taken account as a standard distribution and do not need normal conversion.

3.4.2 Auto Correlation Analysis of the Random Series

Applying the above formulae, the auto-correlation coefficient may be obtained when $k = N/2 = 96$. The auto correlation figure has been shown in Figure 3. From Figure 3 the step 1 and step 12 of the random series is prominent to the independent series. Therefore, annual variation of the groundwater has dependent character and is a mean square contingency series. Adopt $AR(p)$ model according to the above reasons.

3.4.3 Partial Correlation Analysis of Random Series

The task of partial correlation analysis is to ascertain the model steps. The partial correlation figure shows in Figure 4. It shows that $\varphi_{k,k}$ drops into the allowable range of 95% when $k \geq 12$. Thus we can ascertain that steps of the $AR(p)$ model are $p = 12$.

Select k=1，4，5，9，12 and their $\varphi_{k,k}$ is out of the allowable rang of 95%. The correlation component $AR(p)$ model of the random series is as followed:

$$x(t)=D(t)+\varepsilon(t)=\varphi_1 x(t-1)+\varphi_4 x(t-4)+\varphi_5 x(t-5)+\varphi_9 x(t-9)+\varphi_{12}x(t-12)+\varepsilon(t)$$
$$=0.4371x(t-1)-0.1565x(t-4)-0.1796x(t-5)-0.2047x(t-9)+0.6091x(t-12)+\varepsilon(t)$$

3.4.4 Farther Identification of $AR(p)$ Model

Farther identification is that the steps of model are fit or not by using AIC criterion. The result of calculation is as followed:

$AIC(11)=-258.8934$

$AIC(12)=-409.5111$

$AIC(13)=-395.6289$

The model with 12 steps is best one according to AIC criterion.

3.4.5 $AR(p)$ Model Inspection

The main inspection is that the residential item ε_t is independent or not. Through calculating the statistical measurement $Q=n\sum_{k=1}^{m} r_k^2(\varepsilon)=16.0968$, $m=27, n=192$, gain $\chi^2_{0.05}=40.113$ by investigating the table of χ^2 when the significant level α equals to 0.05. Because $Q=16.0968<\chi^2_{0.05}=40.113$, the presumption that ε_t depend each other can be received.

The normal test of independent random series $\varepsilon(t)$ is as followed $C_s(\varepsilon)=0.0179\to 0$. Thus pass the normal inspection. Because variance $\sigma_\varepsilon^2=0.2207$, $\sigma_\varepsilon=0.4698$ and mean value is 0, $\varepsilon(t)\sim(0,0.2207)$ obeys normal distribution. The random item model is:

$$x(t)=D(t)+\varepsilon(t)=\varphi_1 x(t-1)+\varphi_4 x(t-4)+\varphi_5 x(t-5)+\varphi_9 x(t-9)+\varphi_{12}x(t-12)+\varepsilon(t)$$
$$=0.4371x(t-1)-0.1565x(t-4)-0.1796x(t-5)-0.2047x(t-9)+0.6091x(t-12)+\varepsilon(t)\quad \varepsilon(t)\sim(0,0.2207)$$

3.5 Model combination

After folding each trend item model, the non-stable time series random model about the dynamic variation of the groundwater embedded depth can be carried out as:

$$h(t)=0.0002t^2-0.0034t+0.3956$$
$$v(t)=-0.5677-0.2319\cos\frac{2\pi t}{192}+0.6764\sin\frac{2\pi t}{192}-1.9462\cos\frac{32\pi t}{192}-0.0069\sin\frac{32\pi t}{192}+0.6748\cos\frac{64\pi t}{192}+0.3177\sin\frac{64\pi t}{192}$$
$$x(t)=0.4371x(t-1)-0.1565x(t-4)-0.1796x(t-5)-0.2047x(t-9)+0.6091x(t-12)+\varepsilon(t)\quad \varepsilon(t)\sim(0,0.2207)$$
$$H(t)=h(t)+v(t)+x(t)\qquad (t=1,2,3,......n)$$

Table 2. The Data of Forecast Groundwater Depth in Chuang Ye Farm in Sanjiang Plain (2001 ~ 2003)

	1	2	3	4	5	6	7	8	9	10	11	12	Mean
2001	7.7790	7.6623	7.4138	8.0388	10.7718	13.8892	14.3394	13.4114	11.7809	10.0815	8.7644	8.3037	10.19
2002	8.5626	8.4424	8.2990	9.1161	11.3228	13.9963	15.2516	15.2722	14.4037	12.9368	11.3916	10.2419	11.6031
2003	9.5539	8.9517	8.7462	9.6365	11.6317	14.0312	15.7047	16.5313	16.6389	16.0139	14.7786	13.1547	12.95

4 Fitness and Forecast of the Model

Applying the model to fit the monthly mean precipitation from 1985 to 2000 in Chang Ye farm, the calculation has been carried out with a good fitness as shown in Figure 5. The forecast is the embedded depth of the groundwater month by month in next 3 years. The forecast curve is shown in Figure 6 and the forecast value is in Table 2.

From the previous figure, the results calculated by the non-stable random model fit the original groundwater embedded depth series very well. The relative error of the model is $\xi_t=\left|\frac{\hat{H}(t)-H(t)}{H(t)}\times 100\%\right|$.

Through calculating, the mean relative error of the model is 6.49%. It can be found from Table 2. The groundwater level will descend continually in next 3 years in the region and the mean annual draw down rang is about 1.5 meter if the present development trend is assumed.

About Groundwater Depth

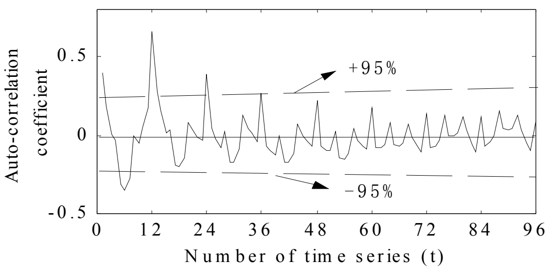

Figure 3. The Auto-correlation Figure of the Random Serial

About Groundwater Depth

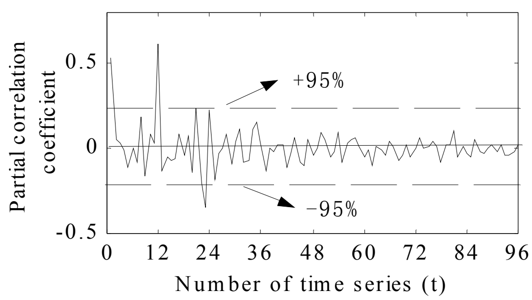

Figure 4. The Partial Correlation Figure of the Random Serial

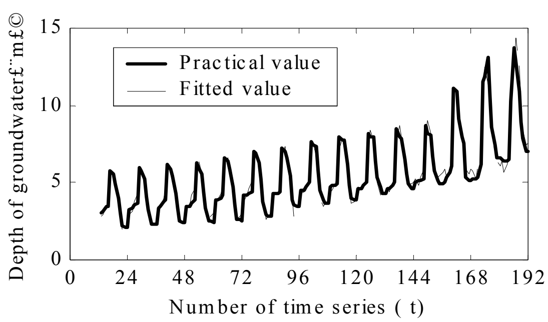

Figure 5. The Fitted Curve of the Random Mod

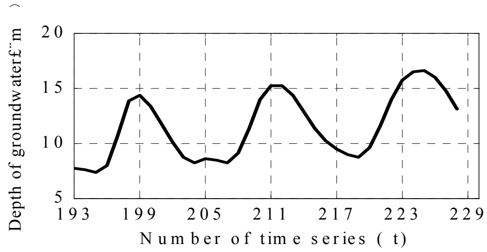

Figure 6. The Forecasting Curve of the Groundwater Depth Dynamic varied in Chuang Ye Farm (2001-2003)

5 Conclusion

Applying surplus and fitting the regression analysis, Fourier series harmonic analysis and auto regression model, the authors have established the groundwater embedded depth forecast model with high fitting precision and good forecast result in Chuang Ye Farm, Sanjiang Plain. Meanwhile, the authors provided some insights from this research for future sustainable utilization of the groundwater resource in the region and developed quantitative indexes and scientific analysis in theory for the well-irrigated rice production in Chuang Ye Farm and Sanjiang Plain.

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