Control of the Behavior Dynamics for Motion Planning of Mobile Robots

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Abstract: A new approach to the motion planning problems of mobile robots in uncertain dynamic environments based on the behavior dynamics is proposed. The fundamental behavior of a mobile robot in motion planning, which is regarded as a dynamic process of the interaction between the robot and its local environment, is modeled and controlled for the motion planning purpose. The behavior dynamics is the law in a dynamic process that the involved objects must obey, and it is controlled by the robot's dynamics. Based on the control of the behavior dynamics, the dynamic motion-planning problem of the mobile robots is transformed into a control problem of the integrated planning-and-control system. No restrictions are assumed on the shape and trajectories of obstacles. Collision avoidance between multiple mobile robots can also be realized. The stability of the integrated planning-and-control system can be easily guaranteed. Simulations illustrate our results. [Nature and Science. 2004;2(4):57-64].

Key Words: Behavior dynamics; motion planning; mobile robots; uncertain dynamic environment

1. Introduction

In all the applications of mobile robots, a good motion planning method is very important to accomplish tasks efficiently and stably [1-2]. However, whenever it comes to dealing with an environment that is totally or partially unknown or even dynamically changing, such a dynamic motion-planning problem is intractable [3], since the mobile robot is required to decide its motion behavior on line using only sensors' limited information. Various methods have been proposed for this purpose, such as (1) Configuration-time space based methods [4-7,30], (2) Planning in space and time independently [8-11], (3) Artificial potential fields based methods [12-18], (4) Behavior based methods [19-20], (5) Intelligent computing based methods [20-24]. Some new methods are also proposed recently, *e.g.*, cooperative collision avoidance and navigation [20,25,26], velocity obstacles method [27], collision cone approach [28], etc. Other sensor-based navigation frameworks can also refer to [13,29]. Significant improvements on the motion-planning problems of a robot have been obtained in the past decades. However, many of the existing methods are only kinematic planning, or rely on some knowledge of the global environment, or require some constraints on the shape or velocity of obstacles, etc. Moreover, behavior decision-making usually is to simply sum up all the impacts from different obstacles with different weights in conventional behavior-based methods [19]. This may lead to counteraction of different reactive behaviors, and consequently result in unexpected motion behavior or performance of the mobile robot.

In motion planning problems of mobile robots, motion behaviors of the mobile robot can be classified into two fundamental behaviors: Collision-Avoidance behavior, and Going-to-the-Goal behavior (in short, CA-behavior and GG-behavior, respectively). How these behaviors are realized and performed in motion planning, how the goodness of the motion planning is. In most of the existing motion-planning methods, motion behaviors are generally regarded as static and discrete behavioral reactions to the environments, instead of dynamic processes. These methods cannot effectively map the local changing environment into dynamic behaviors of the robot. The reactive behaviors, together with the robot dynamics, cannot be integrated into one uniform planning-and-control system to be designed to achieve some performances of the whole system. For these reasons, the conventional behavior-based methods may offer poor performance for the robotic system of large mass or high velocity.

In this paper, the fundamental behaviors of a mobile robot is modeled as dynamic processes of the interactions between the robot and its local environments, then the desired dynamic motion behaviors can be obtained based on the control of these dynamic processes, which are called behavior dynamics. The control input of the behavior dynamics is right the desired acceleration of the mobile robot. The behavior dynamics and the robot's dynamics are integrated into one uniform planning-and-control system, and thus a new method using only sensors' information for the motion planning of mobile robots in uncertain dynamic environments is proposed. The behavior dynamics is sensor-based, and no knowledge of the shape and velocity of the obstacles are needed. The dynamic constraints of the mobile robot are considered. Collision avoidance between multiple mobile robots can also be realized. The stability of the whole planning-and-control system can be guaranteed. To the best of our knowledge, this behavior dynamics-based approach is novel. Simulations are given to illustrate our method.

Notations: A black bold symbol denotes a vector, *e.g.* a vector $\boldsymbol{V}$, and then V denotes its norm. $x = \arg(\cdot)$ means to choose an x satisfying $(\cdot)$. For any vector $\boldsymbol{A}$, let $\boldsymbol{e}(\boldsymbol{A}) = \boldsymbol{A}/\|\boldsymbol{A}\|$. A point p in a plane is written as $\boldsymbol{p}=(p_x, p_y)$, or $\boldsymbol{p} = [p_x, p_y]^T$, where p_x and p_y are the corresponding components on each coordinate. Let $\boldsymbol{e_X} = [1,0]^T$.

2. Modeling of the fundamental behaviors in motion planning

In this section, behavior dynamics is defined and modeled. Without loss of generality, we regard a mobile robot as a point mass, and restrict our study to the 2-D planar case. Similar method can be extended to more than 2-D cases.

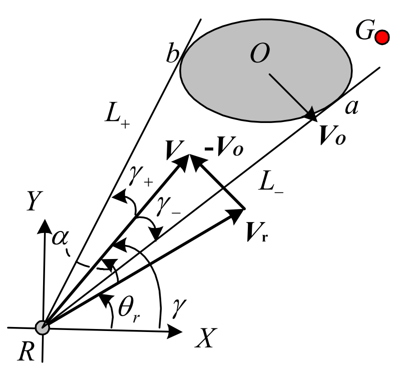

Figure 1. The robot meets an obstacle

At time t, without speciality, refer to Figure 1. The mobile robot $\boldsymbol{R}$ is at the original point of the global coordinates $\{X,Y\}$ with velocity $\boldsymbol{V_r}$ and meets an obstacle at point $\boldsymbol{O}$ with velocity $\boldsymbol{V_O}$,. Let point $\boldsymbol{G}=(x_d,y_d)$ be the goal of the robot, and $\boldsymbol{V}=\boldsymbol{V_r}-\boldsymbol{V_O}$, be the relative velocity of the mobile robot with respect to the obstacle. The angle from vector $\boldsymbol{V_r}$ and $\boldsymbol{V_O}$ to vector $\boldsymbol{V}$ are denoted by α and β, which are also written as $\alpha = \angle(\boldsymbol{V},\boldsymbol{V_r})$ and $\beta = \angle(\boldsymbol{V},\boldsymbol{V_O})$, respectively. This paper assumes that, all the angles in this paper belongs to $[-\pi,\pi]$, and an angle is positive if it is formed by rotating a vector anti-clockwise, else it is negative. In Figure 1, γ,θ_r,θ_O are the angles from X-coordinate to the vectors $\boldsymbol{V}$, $\boldsymbol{V_r}$, and $\boldsymbol{V_O}$, respectively. Obviously, $\gamma = \theta_r + \alpha = \theta_O + \beta$. From point R, there are two lines that are tangent to the boundary of the obstacle O and denoted by L_+ and L_-. The angles from vector $\boldsymbol{V}$ to L_+ and L_- are denoted by γ_+ and γ_-, respectively. For any vector $\boldsymbol{V} \neq 0$, it cannot only be written as $\boldsymbol{V} = [V_x,V_y]^T$ using its corresponding components on each coordinate, also be denoted by $[V,\gamma]^T$, where $V = \|\boldsymbol{V}\|$, $\gamma = \angle(\boldsymbol{V},\boldsymbol{e_X})$. For the two forms, we have

$$V_x = V\cos\gamma, \text{and } V_y = V\sin\gamma. \tag{1}$$

Using the variables given above, the collision condition can be written as

$$\gamma_+ \cdot \gamma_- \leq 0 \tag{2}$$

Once inequality (2) holds, the mobile robot is heading a collision. That is, if inequality (2) holds, the relative velocity vector $\boldsymbol{V}$ should be controlled to rotate a desired angle γ_+ or γ_- in order to avoid a collision. This is the dynamic process of a collision-avoidance behavior. The model of this dynamic process is developed in the following.

From (1), it is easy to obtain the following relationship:

$$\dot{\boldsymbol{V}} = \begin{bmatrix}\dot{V}_x\\ \dot{V}_y\end{bmatrix} = \boldsymbol{J}(V,\gamma)\cdot\begin{bmatrix}\dot{V}\\ \dot{\gamma}\end{bmatrix} \text{ or}$$

$$\begin{bmatrix}\dot{V}\\ \dot{\gamma}\end{bmatrix} = \boldsymbol{J}(V,\gamma)^{-1}\cdot\begin{bmatrix}\dot{V}_x\\ \dot{V}_y\end{bmatrix} \tag{3}$$

where $\boldsymbol{J}(V,\gamma) = \begin{bmatrix}\cos\gamma & -V\sin\gamma\\ \sin\gamma & V\cos\gamma\end{bmatrix}$.

Recalling that $\boldsymbol{V}=\boldsymbol{V_r}-\boldsymbol{V_O}$, and from (3) we have

$$\begin{bmatrix}\dot{V}\\ \dot{\gamma}\end{bmatrix} = \boldsymbol{J}(V,\gamma)^{-1}\cdot\begin{bmatrix}\dot{V}_x\\ \dot{V}_y\end{bmatrix} = \boldsymbol{J}(V,\gamma)^{-1}\cdot(\begin{bmatrix}\dot{V}_{rx}\\ \dot{V}_{ry}\end{bmatrix} - \begin{bmatrix}\dot{V}_{Ox}\\ \dot{V}_{Oy}\end{bmatrix}) \tag{4}$$

Utilizing (3a), Equation (4) yields

$$\begin{bmatrix} \dot{V} \\ \dot{\gamma} \end{bmatrix} = \boldsymbol{J}(V,\gamma)^{-1} \cdot \left(\boldsymbol{J}(V_r,\theta_r) \cdot \begin{bmatrix} \dot{V}_r \\ \dot{\theta}_r \end{bmatrix} - \boldsymbol{J}(V_O,\theta_O) \cdot \begin{bmatrix} \dot{V}_O \\ \dot{\theta}_O \end{bmatrix} \right) \quad (5)$$

Recalling that $\gamma = \theta_r + \alpha = \theta_O + \beta$, and the following relationships

$$\left. \begin{aligned} \sin(\omega+\sigma) &= \sin\omega\cdot\cos\sigma + \cos\omega\cdot\sin\sigma \\ \cos(\omega+\sigma) &= \cos\omega\cdot\cos\sigma - \sin\omega\cdot\sin\sigma \end{aligned} \right\} \quad \forall \omega,\sigma \text{ with}$$

some calculations, Equation (5) further yields

$$\begin{bmatrix} \dot{V} \\ \dot{\gamma} \end{bmatrix} = \begin{bmatrix} \cos\alpha & V_r \sin\alpha \\ -\frac{1}{V}\sin\alpha & \frac{V_r}{V}\cos\alpha \end{bmatrix} \cdot \begin{bmatrix} \dot{V}_r \\ \dot{\theta}_r \end{bmatrix} - \begin{bmatrix} \cos\beta & V_O \sin\beta \\ -\frac{1}{V}\sin\beta & \frac{V_O}{V}\cos\beta \end{bmatrix} \cdot \begin{bmatrix} \dot{V}_O \\ \dot{\theta}_O \end{bmatrix} \quad (6)$$

Equation (6) is the Behavior Dynamics of a Collision-Avoidance behavior, in short as CA-dynamics. It is the law that the involved robot and obstacle must obey in a collision- avoidance process. Equation (6) is developed with assumption that $\boldsymbol{V} \neq 0$. It is noted that $\boldsymbol{V}=0$ can not hold for all the time in order to avoid an obstacle, thus this paper only considers the case $\boldsymbol{V} \neq 0$. In each planning period, $\boldsymbol{V_O}$ can be assumed to be constant. Hence, we have $\dot{V}_O = 0$ and $\dot{\theta}_O = 0$. Otherwise, the last term of equation (6) can be estimated and thus its effect can be cancelled using some methods. Then (6) can be rewritten as follows:

$$\begin{bmatrix} \dot{V} \\ \dot{\gamma} \end{bmatrix} = \begin{bmatrix} \cos\alpha & V_r \sin\alpha \\ -\frac{1}{V}\sin\alpha & \frac{V_r}{V}\cos\alpha \end{bmatrix} \cdot \begin{bmatrix} \dot{V}_r \\ \dot{\theta}_r \end{bmatrix} \quad (7a)$$

As for the CA-dynamics in (7a), the control input is $[\dot{V}_r, \dot{\theta}_r]^T$, it is easy to be transformed into $[\dot{V}_{rx}, \dot{V}_{ry}]^T$, which is the acceleration of the mobile robot in the global coordinates. It should be noted that α, V and γ can all be detected or calculated in the local coordinates on the mobile robot, therefore, there are only sensor information needed in Equation (7a). This facilitates the motion planning for mobile robots in dynamic uncertain environments. Once Inequality (1) holds, the control problem of (7a) is: *Given the desired* $[V_d, \gamma_d]^T$ *for a collision-avoidance behavior, the task is to find a control law*

$$\begin{cases} \dot{V}_r = u_V(V_d, V, \gamma_d, \gamma, \alpha) \\ \dot{\theta}_r = u_\theta(V_d, V, \gamma_d, \gamma, \alpha) \end{cases} \quad (7b)$$

such that system (7) is stable, i.e.,

$$\lim_{t\to T_1} V(t) = V_d, \quad \lim_{t\to T_1} \gamma(t) = \gamma_d \qquad (\exists T_1 > 0).$$

Though $[V_d, \gamma_d]^T$ is time varying, it can be regarded as constant in each planning period. For a collision-avoidance behavior, such that $\gamma_+ \cdot \gamma_- < 0$ does not hold, should only γ_d be set to be γ_+ or γ_- for the above control problem whatever V is. That is, γ is the key variable to be controlled in system (7) to avoid an obstacle, and V_d can be set to be any positive value. For this reason, as for the CA-dynamics we need only consider a simplified system as follows:

$$\dot{\gamma} = \begin{bmatrix} -\frac{1}{V}\sin\alpha & \frac{V_r}{V}\cos\alpha \end{bmatrix} \cdot \begin{bmatrix} \dot{V}_r \\ \dot{\theta}_r \end{bmatrix} \quad (8)$$

That is, only the steering angle of $\boldsymbol{V}$ is considered.

If the goal of the mobile robot is regarded as a special obstacle, then the CA-dynamics can also be regarded as the dynamics of GG-behavior. In this case, $\boldsymbol{V}=\boldsymbol{V_r}$, $\alpha = 0$. Then Equation (7a) is now written as

$$\begin{cases} \dot{V} = \dot{V}_r \\ \dot{\gamma} = \dot{\theta}_r \end{cases} \quad (9)$$

Obviously, (9) is included in system (7a), and $[V_d, \gamma_d]^T$ in (7b) for GG-behavior is to be defined.

Another behavior frequently generated in a collision-avoidance process or navigation of the mobile robot is the Wall Following (WF) behavior, which is to follow the boundary of an obstacle till avoiding it. The dynamics of WF-behavior is the same to (9), and the desired $[V_d, \gamma_d]^T$ in (7b) for WF-behavior is also to be defined. If there exists collision risk, then CA-behavior is carried out. Once there is no collision risk but the obstacle is still near enough to the mobile and between the mobile robot and its goal, then WF-behavior is generated in this case.

From above all, it is noted that all the fundamental behaviors in motion planning problem can be described by the same dynamics model (7) with different desired $[V_d, \gamma_d]^T$.

3. Control of the behavior dynamics and the robot dynamics

In order to illustrate our idea, only a simple case for the robot dynamics is considered in this paper. For other cases, similar results can be developed. Hence, for simplicity, the robot dynamics considered in this paper is described by

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{f}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) = \boldsymbol{\tau} \quad (10)$$

Using the computed torque control

$$\boldsymbol{\tau} = \boldsymbol{M}(\boldsymbol{q})\boldsymbol{u} + \boldsymbol{f}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) \quad (11)$$

then (10) can be rewritten as

$$\ddot{\boldsymbol{q}} = \boldsymbol{u} \quad (12)$$

It is now needed to plan the desired states $[\ddot{\boldsymbol{q}}_d^T, \dot{\boldsymbol{q}}_d^T, \boldsymbol{q}_d^T]^T$ for (12). Utilizing (7b), we have

$$\ddot{\boldsymbol{q}} = \begin{bmatrix} \dot{V}_x \\ \dot{V}_y \end{bmatrix} = \boldsymbol{J}(V_r, \theta_r) \cdot \begin{bmatrix} \dot{V}_r \\ \dot{\theta}_r \end{bmatrix} = \boldsymbol{J}(V_r, \theta_r) \cdot \begin{bmatrix} u_V(V_d, V, \gamma_d, \gamma, \alpha) \\ u_\theta(V_d, V, \gamma_d, \gamma, \alpha) \end{bmatrix} \tag{13}$$

where, $V_r = \|\dot{\boldsymbol{q}}\|, \theta_r = \angle(\dot{\boldsymbol{q}}, \boldsymbol{e}_X)$. (13) is the desired $\ddot{\boldsymbol{q}}_d$ for (12). This implies that it needs only to control the acceleration of the mobile robot for a desired behavior. Now, the integrated planning-and-control system based on behavior dynamics can be described as:

$$\begin{cases} \dot{\boldsymbol{v}} = \boldsymbol{B}(V, V_r, \alpha, \theta_r) \cdot \ddot{\boldsymbol{q}} & (14a) \\ \ddot{\boldsymbol{q}} = \boldsymbol{u} & (14b) \end{cases}$$

which further yields

$$\dot{\boldsymbol{v}} = \boldsymbol{B}(V, V_r, \alpha, \theta_r) \cdot \boldsymbol{u} \tag{15a}$$

where, $\boldsymbol{v} = \begin{bmatrix} V \\ \gamma \end{bmatrix}$,

$$\mathbf{B}(V, V_r, \alpha, \theta_r) = \begin{bmatrix} \cos\alpha & V_r \sin\alpha \\ -\frac{1}{V}\sin\alpha & \frac{V_r}{V}\cos\alpha \end{bmatrix} \cdot \mathbf{J}(V_r, \theta_r)^{-1}$$

From (14), control of the behavior dynamics is in fact a control problem in the acceleration space of the mobile robot. The control problem for (15a) is: *Given a desired* $\boldsymbol{v}_d = [V_d, \gamma_d]^T$ *for a desired behavior, to find the control law* $\boldsymbol{u}$, *such that* $\lim_{t\to\infty} \boldsymbol{v}(t) = \boldsymbol{v}_d$. For this problem, the following result is obvious.

Theorem 1. Given a desired $\boldsymbol{v}_d$ for (15a), choose any a simple linear feedback control law $\boldsymbol{u}$ as follows:

$$\boldsymbol{u} = -\boldsymbol{K}_1 \boldsymbol{e}_v \tag{15b}$$

Then (15a) is asymptotically stable, *i.e.*, $\lim_{t\to\infty} \boldsymbol{v}(t) = \boldsymbol{v}_d$. Where, $\boldsymbol{e}_v = \boldsymbol{v} - \boldsymbol{v}_d, \boldsymbol{B}\boldsymbol{K}_1 > 0$.

It should be noted that (15b) is the desired $\ddot{\boldsymbol{q}}_d$ for the mobile robot. To realize a desired behavior, is to control the robot's dynamics to follow a desired acceleration. For this reason, we can let $\dot{\boldsymbol{q}}_d(t) = \dot{\boldsymbol{q}}(t_0) + \int_{t_0}^t \ddot{\boldsymbol{q}}_d(s) ds$, and $\boldsymbol{q}_d(t) = \boldsymbol{q}(t_0) + \int_{t_0}^t \dot{\boldsymbol{q}}_d(s) ds$.

Then for the robotic system, it is easy to design a tracking control law to guarantee the realization of the desired behavior.

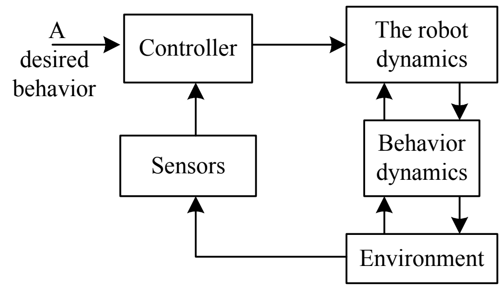

Figure 2. A structure of the whole system

From aforementioned discussions, stability of a desired behavior and the whole planning-and-control system is easy to be guaranteed. The motion-planning problem is now transformed into a control problem of the behavior dynamics. A structure of the whole system can be referred to Figure 2. By using behavior dynamics, the planning task is now to give the desired $\boldsymbol{v}_d$ at each planning time. Various methods can be adopted to plan the desired $\boldsymbol{v}_d$, a simple method is provided in the following section.

4. Realization of the desired behaviors

In this section, we design the desired $\boldsymbol{v}_d$ using the local knowledge of the environment in order to realize the desired behaviors for the mobile robot according to the control laws in Theorem 1.

As for the GG -behavior, let $[V_d, \gamma_d]^T$ in (7b) be:

$$\begin{cases} V_d = \min(0.7 \cdot V_{\max}, \sqrt{\|\boldsymbol{G} - \boldsymbol{R}\|}) \\ \gamma_d = \angle(\boldsymbol{e}(\boldsymbol{G} - \boldsymbol{R}), \boldsymbol{V}_r) \end{cases} \tag{16}$$

where, V_{max} is a positive constant, $\boldsymbol{G}=(x_d, y_d)^T$ and R are the goal and current position of the mobile robot, $\boldsymbol{e}(\boldsymbol{G}-\boldsymbol{R})$ is the desired direction for the robot to move in the global coordinates, which is assumed to be known. And the desired $[V_d, \gamma_d]^T$ in (7b) for WF-behavior is given as

$$\begin{cases} V_d = \min(0.7 \cdot V_{\max}, \sqrt{\|\boldsymbol{G} - \boldsymbol{R}\|}) \\ \gamma_d = \arg \min_{\gamma \in \{\gamma_+, \gamma_-\}} (|\gamma|) \end{cases} \tag{17}$$

where $\{\gamma_+, \gamma_-\}$ corresponds to the obstacle to be followed.

Since the relative velocity is the robot's velocity for the GG/WF-behavior, we have

$$\dot{\boldsymbol{q}}_d = \begin{bmatrix} V_d \cos\gamma_d \\ V_d \sin\gamma_d \end{bmatrix}, \text{ and } \ddot{\boldsymbol{q}}_d \text{ is (17b), } \boldsymbol{q}_d = [x_d, y_d]^T. \tag{18}$$

where $V_d = \min(0.7 \cdot V_{\max}, \sqrt{\|\boldsymbol{G}-\boldsymbol{R}\|})$.

As for the CA-behavior, we only consider the control problem of the simplified CA-dynamics (8). If $\gamma_+ \cdot \gamma_- \le 0$, we let

$$\gamma_d = \arg \min_{\gamma \in \{\gamma_+, \gamma_-\}} (|\gamma|) \tag{19}$$

The $\ddot{\boldsymbol{q}}_d$, $\dot{\boldsymbol{q}}_d$ and $\boldsymbol{q}_d$ are still (18).

Considering the dynamic constraints of the mobile robot, that is, the maximum velocity and acceleration are assumed to be V_{max} and a_{max}, respectively. Let $P(R, t)$ be the observable region of the mobile robot, in which an obstacle can be effectively detected by the sensors. For an obstacle O_i, let ∂D_i be its boundary. If there exists a point $P \in \partial D_i \cap P(R,t)$, such that $|\angle(\boldsymbol{e}(\boldsymbol{V}_i), \boldsymbol{P}-\boldsymbol{R})| = 0 \text{ or } \pi$, then let $\boldsymbol{P}_{ci} = \arg\left(|\angle(\boldsymbol{e}(\boldsymbol{V}_i), \boldsymbol{P}-\boldsymbol{R})| \underset{\boldsymbol{P} \in \partial D_i \cap P(\boldsymbol{R},t)}{=} 0 \text{ or } \pi\right)$, otherwise let $\boldsymbol{P}_{ci} = \infty$. Let $\boldsymbol{V}_i = \boldsymbol{V}_r - \boldsymbol{V}_{Oi}$, and $V_i = \|\boldsymbol{V}_i\|$. Let $l_i = (\boldsymbol{P}_{ci} - \boldsymbol{R}) \cdot \boldsymbol{e}(\boldsymbol{V}_i)$ be the collision distance (C-distance) of the mobile robot and an obstacle O_i. If the mobile robot is heading to an obstacle O_i and the distance l_i is too near, only going in the contrary direction with the maximum acceleration can the robot guarantee the safety. It is easy to prove that the minimum distance for this case is $l_0 = \frac{V_i^2}{2(a_{\max} + \dot{\boldsymbol{V}}_{O_i} \boldsymbol{e}(\boldsymbol{V}_i))}$. If the obstacle is static or its velocity is changing slowly, *i.e.*, $\dot{\boldsymbol{V}}_{O_i} \approx 0$, then $l_0 = \frac{V_r^2}{2a_{\max}}$. Once the robot is approaching to this distance, the steering angle γ of the relative velocity $\boldsymbol{V}_i$ should be modified to the direction opposite to the current direction. For this purpose, we have the following results.

For an obstacle O_i, we redesign the γ_d in (19) to be

$$\begin{cases} \gamma_{ds} = \arg \min\limits_{\gamma \in \{\gamma_+, \gamma_-\}} (|\gamma|) \\ \gamma_d = \gamma_{ds} + \dfrac{\pi - |\gamma_{ds}|}{1 + k_s \cdot pos(l_i - l_0)} \cdot sign(\gamma_{ds}) \end{cases} \tag{20}$$

where, $\forall x$, $pos(x)=\max(0,x)$, $sign(x) = \begin{cases} 1 & x \ge 0 \\ -1 & x < 0 \end{cases}$, $k_s>0$. Note that $\gamma_d \to \pm\pi$ whenever $l_i \to l_0$, and $\gamma_d \to \pm\gamma_{ds}$ if $l_i = \infty$.

Note that (20) provides the γ_d only for one obstacle case. Considering the multiple obstacles case, we need give the optimal γ_d for the robot to go according to all the steering angles of the relative velocity $\boldsymbol{V}_i$ corresponding to different obstacles.

We have mentioned that there are two steering angles, *i.e.* γ_{i+} and γ_{i-}, corresponding to an obstacle O_i. And in order to avoid the obstacle, the relative velocity $\boldsymbol{V}_i$ should be adjusted such that $\gamma_{i-} \le \gamma_i \le \gamma_{i+}$ not holds. Let $D_i = \{\gamma \mid \gamma_{i-} \le \gamma \le \gamma_{i+}\}$, it is called dangerous region with respect to the obstacle O_i. Considering the idea used in (20), D_i can be rewritten as

$$D_i = \{\gamma \mid \gamma_{i-} + \delta(\gamma_{i-}) \le \gamma \le \gamma_{i+} + \delta(\gamma_{i-})\} \tag{21}$$

where, $\delta(\gamma) = \frac{\pi - |\gamma|}{1 + k_s \cdot pos(l_i - l_0)} \cdot sign(\gamma)$. Then for all the obstacles in the observable region $P(R, t)$, the dangerous region is

$$D = \bigcup_i D_i \tag{22}$$

Then from (22), the decision-making space of γ_d for the mobile robot is

$$U = [-\pi, \pi] \setminus (\bigcup_i D_i) \tag{23}$$

The desired γ_d for the robot to avoid the obstacles should be chosen from U. However, a constraint for this decision-making corresponding to the dynamic constraints of the mobile robot should be satisfied. From (15b) and note that $\|\ddot{\boldsymbol{q}}_d\| \le a_{\max}$, we have $\|\boldsymbol{K}_1 \gamma_d\| \le a_{\max}$. Hence, we have $\gamma_d \le \|\boldsymbol{K}_1\|^{-1} a_{\max}$.

With consideration of the GG-behavior, decision of γ_d can now be described as an optimization problem as follows (**P**lanning **P**roblem for γ, in short **PP**-γ):

To find γ^* in U (23) such that Min $J(\gamma)$

And satisfying $|\gamma| \le \|\boldsymbol{K}_1\|^{-1} a_{\max}$.

Let

$$J(\gamma) = k_a \gamma^2 + k_b (\gamma - \gamma_{dG})^2 + k_c sat(\|\boldsymbol{K}_1\|^{-1} a_{\max}) \tag{24}$$

where $sat(\gamma, \Gamma) = \begin{cases} 0 & |\gamma| \le \Gamma \\ (\gamma - \Gamma)^2 & \text{else} \end{cases}$, and $k_a, k_b, k_c > 0$, $k_a + k_b = 1$, k_c should be enough large, γ_{dG} is the desired steering angle γ_d for GG-behavior. Under the evaluation function of (24), the decision-making of PP-γ can be formulated to be

$$\gamma_d = \arg \min_{\gamma \in U} (J(\gamma)) \tag{25}$$

In (25), the CA-behavior, GG-behavior and the dynamic constraints of the mobile robot are all considered. Different k_a and k_b lead to a different tradeoff between the CA-behavior and GG-behavior. In order to try to guarantee the safety of the mobile robot and no local minima are encountered, we can let $k_a << k_b$. Note that the following equation is hold:

$$\arg\min_{\gamma}(J(\gamma))$$

$$= \arg\min_{\gamma}\{k_a\gamma^2 + k_b(\gamma - \gamma_{dG})^2 + k_c sat(\|\mathbf{K_1}\|^{-1} a_{\max})\}$$

$$= \arg\min_{\gamma}\{(\gamma - k_b\gamma_{dG})^2 + k_c sat(\|\mathbf{K_1}\|^{-1} a_{\max})\}$$

Hence, (24) can also be substituted by

$$J(\gamma) = (\gamma - k_b\gamma_{dG})^2 + k_c sat(\|\mathbf{K_1}\|^{-1} a_{\max}) \qquad (26)$$

5. Simulations

In this section, simulations are provided to illustrate our method. Any dynamic model of an omni-directional mobile robot can be used in the simulations. Parameters of the mobile robots used in simulations are: $a_{\max}$=0.5 m/s^2, $V_{\max}$=0.5 m/s, $\omega_{\max}$=0.1 rad/s, the radius of the robot is r=0.3 m. The effective detecting radius of the sensors is 1.5 m. Velocity of the moving obstacles, if any, is 0.35m/s. T_1 is chosen to be the planning period, and let T_1=0.1s. Moreover, let k_a=0.9, k_b=0.1, k_c=5 (in (24) and (26)), k_s=100 (in (20)).

Specially, we assume that, only the distance between the mobile robot and a static obstacle is less than 1 m, then the CA-behavior with respect to this static obstacle is adopted. Additionally, in order to remove the oscillation on the trajectory and guarantee the safety when the mobile robot goes along the boundary of an obstacle, the following strategy is used:

(1) If $|\gamma| \leq \delta$, let $\gamma = 0$, where δ is a small positive number.

Simulations are given under different situations in order to illustrate our method (Figure 3-4). In Figure 3, the environment is static with "SIA"-shape obstacles. The result shows that the mobile robots can coordinate to avoid collision with each other in this environment when they meet, and they can also navigate the U-shape obstacles without being trapped in local minima. Note that wall-following-like behavior is automatically generated in this case. Figure 4 shows a more complex environment, in which there are not only static obstacles but also two moving obstacles. The robots can effectively avoid collision with the unknown moving obstacles.

From the simulation, it can be seen that, (1) The trajectories planned for the mobile robot is smooth, and there are no local minima encountered Shapes of obstacles can be arbitrary, and no knowledge is required about the boundary or velocity of obstacles. (2) All the variables needed in the decision-making of **PP**-γ are in the local coordinates, and only local knowledge of the environment is needed. (3) Our method can be adaptable to dynamic environments, it has fast response to moving obstacles. (4) Our method can make different mobile robots coordinate to avoid collision with each other without "dead-lock".

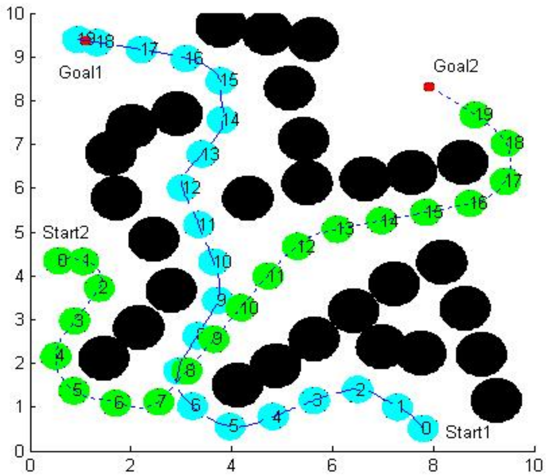

Figure 3. Two mobile robots are moving in an uncertain static environment with "SIA"-shape obstacles.

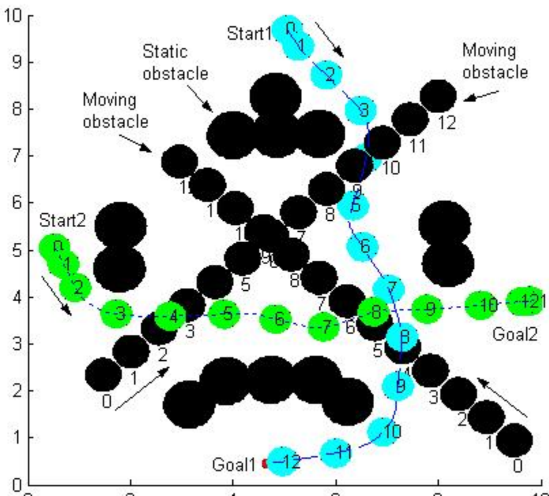

Figure 4. Two mobile robots are moving in a uncertain complex environment with static and moving obstacles.

6. Conclusions

The fundamental behaviors in motion planning of mobile robots are regarded as dynamic processes, and thus are modeled and controlled for motion planning purpose. Control of the behavior dynamics is shown to be an acceleration control problem of the mobile robot. The behavior dynamics and the robot's dynamics are formulated into an integrated planning-and-control system. The motion-planning problem is thus transformed into a sensor-based control problem. The stability of a desired behavior can be guaranteed. By controlling and modeling of the behavior dynamics, a unique insight to the motion-planning problem is provided. Our method can effectively consider the robot dynamics into the motion-planning problem, use only relative coordinates and local knowledge of the environment, and respond quickly to obstacles of arbitrary shape. It should be noted that, behavior dynamics may also be used in some other motion control problems of robots such as formation control, cooperation of multi-robots, tele-operation, etc. Further study will focus on these problems.

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