Comprehensive Evaluating Model Based on Coefficient of entropy and Grey relating Degree to Underground Water Quality

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Abstract: According to independence and non-compatibility of indexes in the evaluating of water quality, this research builds up a new model based on coefficient of entropy to evaluate water quality, which combines entropy value theory and grey relating degree together. Entropy value method is adopted to calculate coefficient of weight and this avoid problems of distribution difficulty through using experts' experience. A new method is exercised to evaluate water quality in this paper, the outcome from application shows applicability to water quality evaluating. [Nature and Science. 2004;2(4):91-94].

Key Words: grey relating degree; water quality; entropy; evaluating

1. Introduction

Evaluating water quality is an important basic work to administration and programming of water environment. There are some evaluating methods, such as integrated index method, attribute recognition method, fuzzy mathematic analysis, multi-criteria comprehensive evaluating method and ANN etc. These methods possess respective specialties. Because of independence and non-compatibility of evaluating indexes, much useful information had been lost, even errors are gained when carrying through an evaluating water quality with those methods. For instance, integrated index method has a stronger subjectivity. It is very difficult to judge which grade it belongs to when the index of water body is on the boundary between two adjacent grades; multi-criteria comprehensive evaluating method is short of reliable way to figure out the coefficient of weight. The quantity and regularity of the sample aren't restricted to grey relating degree method, furthermore with less calculation and more clear thought, it can avoid unsuitable complexion of quantitative and qualitative analysis (Hu, 2000). Simple count mean deviation, AHP method and standard deviation are usual adopted to the calculation of the coefficient of weight in the evaluating of grey relating degree, these methods can impact the outcome of calculation to a certain extent. According to the differentiation degree of indexes, the notion of information entropy is introduced to the evaluating. It can make outcome more objective and more compatible with the actual situation, and provide us with a new method to comprehensive evaluating water quality.

2. Grey relating degree model based the coefficient of entropy

2.1 Grey relating analysis

2.1.1 Ascertain analysis sequence

Considering the aggregation of decision region as A : A ={scheme 1, scheme 2, ⋯, scheme m }= $\{A_1, A_2, \cdots, A_m\}$ and the aggregation of evaluating indexes as V : $V = \{V_1, V_2, \cdots, V_n\}$. Marking attribute value of scheme A_i to index V_j as X_{ij} ($i = 1,2,,\cdots,m; j = 1,2,\cdots,n$).

2.1.2 Structure decision-making matrix

For the sake of eliminating the distinctness degree resulted from different dimension before evaluating, it is necessary to process non-quantized dimension. Generally speaking, the index is divided into the type of benefit and the type of cost, the index so-called type of benefit means better and better as the index turn bigger, and the index so-called type of cost means better and better as it turn smaller. Given the index of comparative optimum scheme A_0 as X_{0j}, at the same time, meeting the conditions as follows, when index V_j belongs to the type of benefit, $X_{0j} = \max(X_{1j}, X_{2j}, \cdots, X_{nj})$, when index V_j belongs to the type of cost, $X_{0j} = \min(X_{1j}, X_{2j}, \cdots, X_{nj})$. Name X_{ij} :

$$X'_{ij} = X_{ij} / X_{0j} \; (i = 0,1,\cdots,m; j = 1,2,\cdots,n) \qquad (1)$$

Non-dimension matrix can be gained by non-quantizing dimension disposal:

$$X'_{0j} = [1,1,\cdots,1]$$

$$X'_{ij} = [X'_{i1}, X'_{i2}, \cdots, X'_{in}] \qquad (2)$$

At this time, consider the increased matrix including the comparative optimum decision scheme $X = (X_{ij})_{(m+1)\times n}, (i = 0,1,\cdots,m; j = 1,2,\cdots,n)$ as the decision-making matrix of A to V.

2.1.3 Calculate sequence of absolute difference and minimum difference, maximum difference of two poles

The sequence of absolute difference:

$$\Delta_{ij} = \left| x'_{ij} - x'_{0j} \right| \qquad (3)$$

The minimum difference of two poles: $\Delta(\max) = \max\max(\Delta_{ij})$

$(i = 0,1,\cdots,m; j = 1,2,\cdots,n)$

The maximum difference of two poles: $\Delta(\min) = \min\min(\Delta_{ij})$

$(i = 0,1,\cdots,m; j = 1,2,\cdots,n)$

2.1.4 Calculate the coefficient of relating degree

The coefficient of relating degree of ith scheme jth index (shows the degree of relation between the compared sequence and the criterion sequence) ξ_{ij} can be defined as:

$$\xi_{ij} = \frac{\Delta(\min) + \rho \cdot \Delta(\max)}{\Delta_{ij} + \rho \cdot \Delta(\max)} \qquad (4)$$

Where, ρ is coefficient of distinguish, and its' value vary in the extent (0，1), generally speaking, its' value is given to 0.5 (Qiao, 2004; Men. 2003).

2.2 Calculate the coefficient of weight by way of entropy value method

Entropy value method is a way through which indexes' coefficient of weight can be gained with judgement matrix composed by value of indexes (Qiou, 2001; Yan, 2003), the method can avoid the subjectivity of indexes' coefficient of weight as much as possible, and make the outcome more suitable to fact. Its' steps is as follows:

(1) Structure judgment matrix including m schemes and n indexes:

$R = (x_{ij})_{mn}$ （$i = 1,2,\cdots,m; j = 1,2,\cdots,n$）

(2) Carry through normalization disposal, and get normalized judgement matrix B:

$$b_{ij} = \frac{x_{ij} - x_{\min}}{x_{\max} - x_{\min}} \qquad （5）$$

Where, $x_{\max}$ 、$x_{\min}$ mean the most satisfactory value or the most unsatisfactory value under the same index and different schemes.

(3) According to the define of entropy, there are m schemes and n indexes, the entropy of indexes can be defined as:

$$H_j = -\frac{1}{\ln n}(\sum_{j=1}^{n} f_{ij} \ln f_{ij}) \quad i = 1,2,\cdots,m; j = 1,2,\cdots,n \qquad （6）$$

$$f_{ij} = \frac{b_{ij}}{\sum_{j=1}^{n} b_{ij}}$$

In order to make $\ln f_{ij}$ a meaning, you need to assume that when $f_{ij} = 0$, $f_{ij} \ln f_{ij} = 0$, but when $f_{ij} = 1$, $\ln f_{ij}$ =0 do not suit the actual condition of, and against the meanings of entropy, so modify f_{ij} as:

$$f_{ij} = \frac{1 + b_{ij}}{\sum_{j=1}^{n}(1 + b_{ij})} \qquad （7）$$

(4) Calculate the weight of entropy of indexes W:

$$\omega_j = \frac{1 - H_i}{n - \sum_{i=1}^{m} H_i}, \quad W = (\omega_j)_{1\times n}, \quad \sum_{j=1}^{n} \omega_j = 1 \qquad （8）$$

2.3 Calculate the relating degree

The relating degree E_i is defined to approach degree between the evaluated schemes and the criterion scheme, the bigger its value, the more adjacent to the optimum, whereas the further from the optimum. So you can make an order on the basis of the relating degree, and classify scheme in the light of criterion value. The matrix E_i can be expressed as:

$$E_i = \begin{bmatrix} & A_1 & A_2 & \cdots & A_m \\ E_i & E_1 & E_2 & \cdots & E_m \end{bmatrix} \qquad （9）$$

Where, $E_i = \sum_{j=1}^{n} \omega_j \xi_{ij} \quad (j = 1,2,\cdots,n)$

3. Case study

The case is from chemistry zone in Handan city (Shu, 1998). There are 7 survey spots in all, and every spot includes rigidity of water body (CaO), SO_4^{2-}, Cl^-, F and organic P, according to the criterion, water body can be classified as three grades, □ is not-polluted water body, □ is polluted water body, □ is severely polluted water body. The data of factual measure and criterion are presented in Table 1.

Table 1. Value of evaluating indexes of factual measure and criterion

Survey spot	Rigidity	SO_4^{2-}	Cl^-	F	Organic P
1	1289.06	192.05	1856.05	1.20	0.050
2	1804.00	153.21	2573.50	1.35	0.171
3	1176.34	277.80	2094.07	1.35	0.384
4	689.44	142.20	782.32	0.73	0.019
5	1422.32	1120.05	726.88	0.99	0.028
6	1381.55	217.02	1694.78	0.67	0.051
7	397.66	279.02	216.04	0.63	0.022
□ grade	250.00	250.00	250.00	1.00	0.050
□ grade	400.00	500.00	350.00	1.50	0.100
□ grade	1700.00	1000.00	1250.00	3.00	0.500

3.1 Structure the decision matrix

Make data in Table 1 dimensionless, in this case, you can know that all the indexes are the type of benefit, so can use equation (1), then structure the increased matrix X with the data and the optimum scheme:

$$X = \begin{bmatrix} & V1 & V2 & V3 & V4 & V5 \\ A0 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 \\ A1 & 0.7146 & 0.1715 & 0.7212 & 0.4000 & 0.1000 \\ A2 & 1.0000 & 0.1368 & 1.0000 & 0.4500 & 0.3420 \\ A3 & 0.6521 & 0.2480 & 0.8137 & 0.4500 & 0.7680 \\ A4 & 0.3872 & 0.1270 & 0.3040 & 0.2433 & 0.0380 \\ A5 & 0.7884 & 1.0000 & 0.2824 & 0.3300 & 0.0560 \\ A6 & 0.7658 & 0.1938 & 0.6586 & 0.2233 & 0.1020 \\ A7 & 0.2204 & 0.2491 & 0.0839 & 0.2100 & 0.0440 \\ B1 & 0.1386 & 0.2232 & 0.0971 & 0.3333 & 0.1000 \\ B2 & 0.2217 & 0.4464 & 0.1360 & 0.5000 & 0.2000 \\ B3 & 0.9424 & 0.8928 & 0.4857 & 1.0000 & 1.0000 \end{bmatrix}$$

3.2 Structure the matrix of coefficient of relating

Structure the sequence of absolute difference on the basis of equation (3), and calculate the minimum difference and the maximum difference of two pole of every scheme, then work out the coefficient of relating by equation (4) and structure the judgment matrix of coefficient of relating ξ_{ij}:

$$\xi_{ij} = \begin{bmatrix} & V1 & V2 & V3 & V4 & V5 \\ A0 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 \\ A1 & 0.6014 & 0.3425 & 0.6216 & 0.3970 & 0.3483 \\ A2 & 1.0000 & 0.3333 & 1.0000 & 0.4180 & 0.4223 \\ A3 & 0.5532 & 0.3647 & 0.7109 & 0.4180 & 0.6746 \\ A4 & 0.4127 & 0.3308 & 0.3969 & 0.3430 & 0.3333 \\ A5 & 0.6706 & 1.0000 & 0.3896 & 0.3709 & 0.3375 \\ A6 & 0.6478 & 0.3487 & 0.5729 & 0.3371 & 0.3488 \\ A7 & 0.3559 & 0.3560 & 0.3333 & 0.3333 & 0.3347 \\ B1 & 0.3333 & 0.3572 & 0.3366 & 0.3721 & 0.3483 \\ B2 & 0.3563 & 0.4381 & 0.3465 & 0.4413 & 0.3755 \\ B3 & 0.8820 & 0.8011 & 0.4711 & 1.0000 & 1.0000 \end{bmatrix}$$

3.3 Calculate coefficient of weight by way of entropy value method

Structure normalized judgment matrix B by using equation (5):

$$B = \begin{bmatrix} 0.6338 & 1.0000 & 0.5537 & 0.2139 & 0.7286 & 0.6996 & 0.0000 \\ 0.0510 & 0.0113 & 0.1387 & 0.0000 & 1.0000 & 0.0765 & 0.1399 \\ 0.6957 & 1.0000 & 0.7966 & 0.2402 & 0.2167 & 0.6273 & 0.0000 \\ 0.7917 & 1.0000 & 1.0000 & 0.1389 & 0.5000 & 0.0556 & 0.0000 \\ 0.0849 & 0.4164 & 1.0000 & 0.0000 & 0.0247 & 0.0877 & 0.0082 \end{bmatrix}$$

By way of equation （6）, （7） and （8） you can get:

$$H_j = (1.0349, 1.0421, 1.0337, 1.0272, 1.0396)^T \quad j = (1,2,\cdots,5)$$

$$\omega_j = (0.1965, 0.2371, 0.1900, 0.1533, 0.2232)^T \quad j = (1,2,\cdots,5)$$

3.4 Calculate the relating degree of every scheme

By way of equation (9), the matrix E_i can be presented:

$$E_i = \begin{bmatrix} & spot\,1 & spot\,2 & spot\,3 & spot\,4 & spot\,5 & spot\,6 & spot\,7 & grade\ I & grade\ II & grade\ III \\ E_i & 0.4561 & 0.6239 & 0.5449 & 0.3619 & 0.5751 & 0.4484 & 0.3456 & 0.3489 & 0.3912 & 0.8292 \end{bmatrix}$$

3.5 Analysis of the outcome

In the light of the relating degree that be calculated, the polluted degree of groundwater body in chemistry zone of Handan city can be make a order as: spot 2, spot 5, spot 3, spot 1, spot 6, spot 4 and spot 7. Compared with the relating degree of criterion, you can get the result: the water body of spot 2, spot 5, spot 3, spot 1 and spot 6 belong to the extent of severely polluted water body; spot 4 belongs to the polluted and spot 7 belongs to the non-polluted. The outcome so you say is consistent to that of the literature [6] and using comprehensive evaluating method (Table 2), what the calculation gain shows it is feasible and reasonable to apply this way to evaluating groundwater quality.

Table 2. Comparison of evaluating outcomes

Methods	spot1	spot2	spot3	spot4	spot5	spot6	spot7
Attribute recognition method	β	β	β	α	β	β	–
comprehensive evaluating method	β	β	β	α	β	β	α
Grey relating degree method	β	β	β	α	β	β	–

4. Conclusion

Needing to consider completely attribute of many indexes in evaluating water quality, the method and criterion of evaluating water quality is not very mature up to the present. Applying relating degree evaluating model based on the coefficient of entropy to the comprehensive evaluating water quality can effectively account for the grade and the sort of water body, and provide us with a new way to evaluate reasonably water quality. This model considers more objective information entirely and scientifically, with clear thought and convenient calculation, and comes out to be one more effective method. In addition, this method can be applied to project decision-making, bidding and evaluating sustainable utilization of regional water resources and so on.

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