Two Probability Theories in Physics

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Abstract: It is proved that in a double slit diffraction experiment, superposition principle of probability amplitude means (the probability amplitude of the event) that an electron passes through a certain slit and arrives somewhere on the screen under the condition that two slits open simultaneously, is equal to the sum of two probability amplitudes of the same event under the condition that two slits open in turn. From this thesis it is concluded that the probabilities do not obey superposition principle, which is just the reason that classical probability theory is inapplicable for the very experiment. Kolmogorov's probability theory is based on two foundations: frequency definition of probabilities and Boolean algebra of event operations. In micro processes, the former holds true while the latter, especially its multiplicative permutation law is violated. So long as we do not deal with event operations, joint probabilities holds good in micro processes. But, the event operation formulae provided are applied and the wrong conclusions present probably. The famous Bell's inequality is a wrong conclusion. [Nature and Science. 2005;3(2):82-91].

Key words: double slit diffraction experiment; Stern-Gerlach experiment; probability amplitudes; Boolean algebra; classical probability theory

Introduction

It is well known that there are two kinds of probabilities in physics. Classical probabilities are applicable for macrophysics while quantum probabilities for microphysics. Hereon, the signification and the scope of application of these two probabilities will be reexamined.

1 Mystery in the Double Slit Diffraction

Famous American physicist R. P. Feynman[1] said that: The double slit diffraction experiments show the whole mystery in quantum mechanics. So far as we know, this mystery can boil down to one conclusion: Classical probability theory is inapplicable to the micro processes. However, it remains an open question that which proposition in classical probability theory is invalid on this occasion and why it is invalid. This question will be examined hereinafter.

To clear the mystery Feynman said, let us examine the following electron double slit diffraction processes.

Firstly, opening the two slits simultaneously, consider that an electron, say e, emitted from the source and has arrived on the screen. Let the E denote that the e passes through the first slit and the F that through the second slit. Then, both E and F are stochastic events. Because that the e has arrived on the screen, it must pass one and only one of the two slits, namely, we have

$$E + F = U \text{ (necessity event)},$$
$$E \cdot F = \varnothing \text{ (impossible event)}. \qquad (1)$$

Secondly, let the sign Ω denote a small area on the screen, and the X denote the event that the e falls on the Ω, hence, $E \cdot X$ ($F \cdot X$) denotes that the e passes through the first (second) slit and falls on the Ω. According to the Boolean algebra of proposition calculation, from Eq, (1) it is concluded that:

$$(E \cdot X) \cdot (F \cdot X) = \varnothing, \qquad E \cdot X + F \cdot X = X.$$

Thirdly, by probability frequency definition, the above formulae give that:

$$\Pr(X) = \Pr(E \cdot X) + \Pr(F \cdot X). \qquad (2)$$

That is a special form of the probability additional formula.

Fourth, by the probability multiplication formula, we have

$$\Pr(E \cdot X) = \Pr(E) \cdot \Pr(X|E);$$
$$\Pr(F \cdot X) = \Pr(F) \cdot \Pr(X|F).$$

Substitute these two formulae into Eq. (2), we have

$$\Pr(X) = \Pr(E) \cdot \Pr(X|E) + \Pr(F) \cdot \Pr(X|F). \qquad (3)$$

That is "total probability formula".

To concision, the above proof for Eq. (3) will be written by P-proof in the following. No matter how to examine repeatedly the four steps in this proof, it seems that there is with no chink.

According to the definition, if only open the first slit, the probability that the e fall on the Ω is $Pr(X|E)$; similarly, if only open the second slit, that probability is $Pr(X|F)$. Also, if both slits are open simultaneously, the probability of the very event is $Pr(X)$. According to the total probability formula, $Pr(X)$ is equal to the mean value for $Pr(X|E)$ and $Pr(X|F)$ in accordance with the ratio of $Pr(E)$ and $Pr(F)$. Particularly, if $Pr(E) = Pr(F) = 1/2$, $Pr(X)$ is the arithmetic mean of $Pr(X|E)$ and $Pr(X|F)$. So, the probability that the e falls on the Ω under the condition that the double slit open simultaneously is equal to the arithmetic mean of the two probabilities of the same event under that the double slit open in turn. As a result, the diffraction pattern obtained under the condition that two slits open simultaneously is the overlapping of two diffraction patterns obtained under the condition that two slits open in turn. But the experimental facts have refuted this conclusion: the diffraction patterns under the above two conditions are widely different. It seems certain that this fact indicates: The total probability formula is inapplicable for the double slit diffraction processes.

Now, a contradiction appears. On the one hand, according to the unassailable theory consequence, the total probability formula must be applicable for any a process; on the other hand, the experiment facts indicate the very formula cannot use for the double slit diffraction process. That is just the mystery in quantum mechanics what Feynman said.

2 Double Diffraction and Interpretations for Quantum Mechanics

People are forced to rack their brains for explaining the proposition (a); as a result, many unusual interpretations for quantum mechanics are established.

The first one is the Copenhagen interpretation, of which the main point is as follows: To derive Eq. (3), it needs to affirm Eq. (1). This formula means that the e either passes the first slit or passes the second slit. Copenhagen school demur at this step, the reason is that in the first experiment, we cannot determine which slit the e passed. For Copenhagen school, this fact means that the e neither passes the first slit nor passed the second slit. However, provided e moves along an orbit, it must passes one slit. So, Copenhagen school comes to the conclusion that the electron movement is not orbit movement. And go a step further; they assert that the concepts like "orbit" are "classical concepts" which are inapplicable for micro processes. As such, the Copenhagen interpretation makes a denial of the first step of the P-proof. Because that the first step is unacceptable, the whole proof is invalid naturally.

Quantum logic interpretation (one kind of it) predicates: in proposition calculation the distributive law

$$(E+F)\cdot X = E\cdot X + F\cdot X$$

is inapplicable for this process, and thereby from $E+F = U$ we cannot obtain $E\cdot X + F\cdot X = X$. Hence, this interpretation validates the first step of the P-proof, but negates the second step and so are the after steps.

Both the above two interpretations affirm that the total probability formula and thereby classical probability theory is applicable for micro process, Copenhagen interpretation traces back this promise to classical concept, while quantum logic to classical logic.

Another kind of interpretations only negates classical probability theory itself. For instance, France physicist G. Lochak[2,3] considers that classical probability theory only applicable for "hidden variables", but due to a certain reason, it cannot be used for the mean value of the measurement outcomes. So, Lochak accepts the first and the second step of the P-proof, but refuses the third step.

L. Accardi[4] , who celebrated for establishing quantum probability interpretation, raised a new point: all the preceding three steps of the P-proof hold true so that the addition formula in classical probability theory is applicable for any processes. The trouble is with the fourth step, namely, with the probability multiplication formula, which Accardi calls "Bayes axiom". He said: all of the paradoxes in quantum mechanics result from the improperly using this axiom.

The existence of the above interpretations indicates that: 1. in the P-proof, there is a promise, which is regarded as perfectly justified but is applicable for micro process really. 2. it is still an open question what such a promise is.

3 A Hidden Promise

As we see, to explain the (a) various interpretations for quantum mechanics has been advanced. Each of them abandoned a specific promise of P-proof. Concretely speaking, four promises are given up respectively: the classical concept about orbit; the contribution law in proposition calculation; the probability addition formula and the probability multiplication formula.

English philosopher Karl Popper put forward a new point about double slit diffraction experiment as follows[5]: Every change of the experiment devices, for instance, closing a slit, will make an impact on the distribution of various possibilities. This point made the pivotal step for revealing the above mystery.

In the P-proof, all the four promises above mentioned are necessary, but Popper opened out another promise that is overlooked all the way: In the proving Eq. (2), the $Pr(E\cdot X)$ and $Pr(F\cdot X)$ are the probabilities under the condition that two slits are open simultaneously, but when Eq. (3) is used for double diffraction process, they are regarded as the probabilities under the condition that two slits are open in turn. Therefore, when people obtain the (a), it is tacitly approved that the $Pr(E\cdot X)$ (as well as the $Pr(F\cdot X)$) takes the same value under the above two conditions. However, this promise is not an unalterable principle. We must regard it as the fifth promise of the P-proof.

To formulize this promise, let the sign C denote the condition that two slits are open simultaneously, and the D that open in turn. Then, according to the preceding three steps of the proof, we can only obtain

$$\Pr(X|C) = \Pr(E\cdot X|C) + \Pr(F\cdot X|C), \qquad (4)$$

but the addition formula inapplicable for double slit diffraction process is

$$\Pr(X|C) = \Pr(E\cdot X|D) + \Pr(F\cdot X|D). \qquad (5)$$

Now, let us distinguish Eq. (5) from Eq. (4).

Eq. (4) can be interpreted as follows: Consider a process, two slits are opened for a time interval T, there are N electrons falling on the screen and n one falling on the Ω. Among those n electrons, n_1 one passed through the first slit and n_2 one passed through the second slit. Due to the number N is large enough, the probability in the left side of Eq. (4) is n / N; and those in the right side of Eq. (4) are respectively n_1 / N and n_2 / N, so that Eq. (4) indicates

$n = n_1 + n_2$, which is a self-evident outcome.

Also, Eq. (5) can be interpreted as follows: Consider another process, under otherwise identical conditions, at the beginning only the first slit is opened for a time interval T, there are m_1 electrons falling on the Ω; and later on only the second slit is opened for an equal time interval and m_2 electrons falls on the Ω. Consequently, the probabilities in the right side of Eq. (5) are m_1 / N and m_2 / N respectively, so that Eq. (5) indicates: $n = m_1 + m_2$, *(6)*, which is just the fifth promise of P-proof.

Now, let us explain the meaning of this promise, when two slits open for the time interval T, the electrons passing through the first slit form an electron beam, say C_1; and another electron beam, say C_2, is formed by the electrons passing through the second slit. In the first process above mentioned, the C_1 and the C_2 arrive the screen together, while in the second process, the C_1 and the C_2 arrive the screen one after the other. As a result, the fifth promise can be expressed as follows: The electron number falling on the Ω under the condition that the C_1 and the C_2 arrive the screen simultaneously, is equal to two electron numbers falling on the Ω under the condition that the two beams arrive the screen in turn. Expressed by the probabilistic form, it becomes that: In the double slit experiment, the probability of the event that an electron falls on the Ω under the condition that two slits open simultaneously, is equal to two probabilities of the same event under the condition that two slits open in turn.

4 Superposition Principle

According to electrostatics, if there are two point charges, and field intensity at observe point is $\boldsymbol{E}_1$ if only the first change exists, and is $\boldsymbol{E}_2$ if only the second change exists, then that is $\boldsymbol{E}_1 + \boldsymbol{E}_2$ if both two charges exist. This fact is called after superposition principle about electrostatic field. From this instance, the general definition for superposition principle can be described as follows:

Definition 1: Assume that the presence of something will produce a certain effect. If in a process, the effect produced by the presence of two such things is equal to the sum of the effects of the separated presence of each thing, then we say in the very process, the effects of the things obey superposition principle.

Compared with this definition, it is seen that the proposition (b) indicates that the probabilities obey superposition principle. But, because that the (b) is not an experimental fact, it cannot be named as a principle, so that we call it after "probability superposition assumption" herein. Hence, the conclusion we obtained from the double slit experiment is that the probability superposition assumption is invalid.

To manifest this conclusion in a more universal form, two terms will be introduced as follows: Firstly, the conditions such as the C and the D in called by "Popper conditions". Secondly, following quantum mechanics, the events like "the e passing through the first slit falls on the Ω" are called as "transition events". As such, the above conclusion is generalized as follows:

In a transition process, for Popper conditions, the probabilities do not obey superposition principle.

Adopting the above symbols, Eq. (6) is equivalent to:

$$n_1 = m_1, \qquad n_2 = m_2. \qquad (7)$$

Rewriting the $Pr(A|C)$ as $Pr_C(A)$ and the $Pr(A|D)$ as $Pr_D(A)$, as a result of the probability frequency definition, the total probability formula is

$$\Pr_C(X) = \Pr(E)\cdot\Pr_C(X|E) + \Pr(F)\cdot\Pr_C(X|F), \quad (8)$$

but the formula negated by facts is

$$\Pr_C(X) = \Pr(E)\cdot\Pr_D(X|E) + \Pr(F)\cdot\Pr_D(X|F), \quad (9)$$

which is a formula possessing the classical probability theory characters. The transition from Eq. (8) to Eq. (9) requires the following relations:

$$\Pr_C(X|E) = \Pr_D(X|E), \quad \Pr_C(X|F) = \Pr_D(X|F). \quad (10)$$

These equations are just that is the probability expression for Eq. (7), which signify the fifth promise.

It seems that all of the preceding four promises are unassailable; so as to no matter giving up any one of them, we will result in certainly some inconceivable outcomes. But the fifth promise does not so; it is far not perfectly justified. Merely due to carelessness, it is accepted. As viewed from the other angle, the discovery of this promise is by means of careful analysis instead of by a certain bold new idea.

Eq. (10) can be expressed as follows: Under the known condition that the e passes through a certain slit, the probability that it falls on the Ω is independent of the condition whether or not the other slit is open. Hereon, the condition such as the other slit is open is another form of Popper condition. So that, the above proposition can be generalized as that: Under the change of Popper conditions, the transition probabilities hold constant. This is another form for the fifth promise in P-proof. Give up this promise, classical concept such as orbit movement; classical logic such as the distributive law of proposition calculation; classical probability theory laws such as addition formula and multiplication formula, all can be saved from abandon.

The (c) is applicable to various processes; especially, it is applicable to the Stern-Gerlach experiment that we will examine in the following.

5 Stern-Gerlach Experiment

When an electron beam passes through a non-uniform magnetic field with orientation ***n***, it will split apart into two sub-beams. In one of them, which we call the beam magnetized along the ***n***, the projections in the ***n*** direction of the electron spins will be $\sigma_n = 1$ (measured by h/4π); in the other, we call the beam magnetized in the −***n***, $\sigma_n = -1$.

Assume that there is a beam having N electrons magnetized along the ***n***, after passing through a magnetic field orientated by ***m***, there are M electrons magnetized along the ***m***, then we say the probability that a single electron in the very beam transits from the state $\sigma_n = 1$ to the $\sigma_m = 1$ is M/N (if the N is large enough), which is written as a conditional probability $Pr(\sigma_m = 1 \mid \sigma_n = 1)$, and is called a "transition probability".

Feynman described a kind of perfect Stern-Gerlach devices (abbreviated by "devices" below), which possess the following properties: One of such devices has a character direction, after entering into it a beam will split to two sub-beams, the one is magnetized along the character direction and the other along the reverse direction. Besides, two devices can be linked up such that all electrons escaping from the preceding device can enter into the behind one. In each device, there is a baffle that may absorb one of the two sub-beams. In the following it is stipulated that a device is written as G_n if its character direction is ***n***.

To compare the Stern-Gerlach experiment with the double slit diffraction experiment, we consider a special process, which we call "character process", as follows: Assume that a beam, say R, passes through a device G_a and splits apart into two sub-beam, the one is absorbed in it and the other, say A, escapes from it. Then, the A, which has N electrons and is magnetized along the ***a***, enters into another device G_c, and splits apart into two sub-beam C_1 and C_2. The C_1 has N_1 electrons and is magnetized along the ***c*** and the C_2, N_2 electrons and magnetized along the $-\boldsymbol{c}$. Afterwards, C_1 and C_2 depart from the G_c and enter into the third device G_b together. In the G_b, the entered electrons realign and become two sub-beams, one of them, say B, magnetized along ***b***, leave the G_b and the other is absorbed in it.

In this process, after entering into the G_c, an electron may belong to C_1 as well as to C_2, which are two passages through the G_c. We call the preceding one after the first passage and the other the second passage. Assume that in the A, there are M_1 electrons passing through the first passage of the G_c and finally departing from G_b; and M_2 electrons through the second passage and departing from the G_b, and thereby there are $M_1 + M_2$ electrons in the A finally belonging in the B.

Let e denote an electron in the A, then, according to the frequency definition, under the condition that the N is large enough, the probability that the e passes the first passage is $\Pr(\sigma_c = 1 \mid \sigma_a = 1) = N_1/N$; when the condition that the e passes the first passage is known, the probability that it departs from G_b is

$\Pr(\sigma_b = 1 \mid \sigma_c = 1) = M_1/N_1$,

So, the probability that the e passes through the first passage of the G_c and finally departs from the G_b is

$p_1 = \Pr(\sigma_c = 1 \mid \sigma_a = 1) \cdot \Pr(\sigma_b = 1 \mid \sigma_c = 1) = M_1/N$. *(11)*

Similarly, the probability that the e through the second passage of the G_c and finally departs from the G_b is

$p_2 = \Pr(\sigma_c = -1 \mid \sigma_a = 1) \cdot \Pr(\sigma_b = 1 \mid \sigma_c = -1) = M_2/N$. *(12)*

According to definition, the probability that the e finally departs from the G_b is: $p = (M_1 + M_2)/N$.

Eq. (11), Eq. (12) and the above formula give

$p = p_1 + p_2$. *(13)*

On the other hand, according to the experiment facts, if the e enters into the G_b directly (namely, it does not pass through the G_c), the probability that it departs from the G_b is also the p. So, $p = \Pr(\sigma_b = 1 \mid \sigma_a = 1)$. *(14)*

Substitute Eq. (11), Eq. (12) and Eq. (14) into Eq. (13), we obtain that:

$\Pr(\sigma_b = 1 \mid \sigma_a = 1) = \sum_z \Pr(\sigma_c = z \mid \sigma_a = 1) \cdot \Pr(\sigma_b = 1 \mid \sigma_c = z)$, where $z \in \{1, -1\}$.

Generally speaking, for any $x, y, z \in \{1, -1\}$,

$\Pr(\sigma_b = y \mid \sigma_a = x) = \sum_z \Pr(\sigma_b = y \mid \sigma_c = z) \cdot \Pr(\sigma_c = z \mid \sigma_a = x)$. *(15)*

It should be noted that this formula is only significant for the character process. Generally speaking, in micro world, a formula consisting of probability expressions are only significant for a given process, because that probability superposition assumption is valid.

6 Comparison Between two Total Probability Formulae

To compare the Stern-Gerlach experiments with the double slit diffraction experiments, let S denote the condition that the e is falls on the screen, this is the precondition condition for the later process, hence for this process the total probability formula

$\Pr(X) = \Pr(E) \cdot \Pr(X|E) + \Pr(F) \cdot \Pr(X|F)$,

is rewritten as

$\Pr(X|S) = \Pr(E|S) \cdot \Pr(X|E) + \Pr(F|S) \cdot \Pr(X|F)$.

Herein, the S is the collection of the electrons, which pass through one of the double slit. This is a beam corresponding with the beam A in the Stern-Gerlach experiment, namely, the beam $\sigma_a = 1$. Also, the E also denotes a beam, of which the electrons pass through the first slit. It is corresponding with the beam C_1 in the Stern-Gerlach experiment, and the X, a beam with electrons fall on the Ω, is corresponding with the beam B. It is thus seen that Eq. (15) is corresponding to the total probability formula for the transition probability $Pr(\sigma_m = 1 \mid \sigma_n = 1)$.

According to definition, all of p_1, p_2 and p in Eq. (11) are probabilities under the condition that two passages open simultaneously. The p is the ratio of the number of the electrons departing from the G_b to that entering into the G_c, which is measurable; but the p_1 and the p_2 is not. To measure them it is need to consider another experiment. Provided that after the A entering into G_c and splitting to C_1 and C_2, absorbing the C_2 by the baffle in the G_c, we can measure the p_1. Similarly, we can also measure the p_2. However, the p_1 and p_2 obtained in such a way are the probabilities under the condition that two slits open in turn. The values of them may be different from those in Eq. (11) and Eq. (12), because those are obtained from the distinct Popper conditions.

Such being the case, in Eq. (15), which probabilities are dependent on Popper conditions? So-called Popper conditions herein means under otherwise identical conditions let the G_c open two passages simultaneously or in turn. So, the p in Eq. (14) is independent of Popper conditions. The p_1 given by Eq. (11) has two factors, in them the $Pr(\sigma_c = 1 \mid \sigma_a = 1)$ is Popper condition free because that the splitting of the A in the G_c is before than the C_1 or C_2 is absorbed. The unique probability herein dependent on Popper conditions is the factor $Pr(\sigma_b = 1 \mid \sigma_c = 1)$. Similarly, all the probabilities in the form $Pr(\sigma_b = y \mid \sigma_c = z)$ depend on Popper conditions. Therefore, such expressions have twofold meaning. In the following, we will distinguish them by symbols.

Using the symbols Pr_C and Pr_D to distinguish the probabilities under conditions two passages open simultaneously and that in turn, as a formula obtained by frequency definition, Eq. (15) ought to be expressed as that:

$$\Pr(\sigma_b = y \mid \sigma_a = x) = \sum_z \Pr_C(\sigma_b = y \mid \sigma_c = z) \cdot \Pr(\sigma_c = z \mid \sigma_a = x). \qquad (15a)$$

but if the p_1 and p_2 is regarded as the measurement value, it becomes

$$\Pr(\sigma_b = y \mid \sigma_a = x) = \sum_z \Pr_D(\sigma_b = y \mid \sigma_c = z) \cdot \Pr(\sigma_c = z \mid \sigma_a = x). \qquad (15b)$$

This formula manifests the superposition assumption of probabilities in the very process; it is a relation between the measurement values. However, this relation is invalid.

7 Probability Amplitudes

As we see, by means of the probability frequency definition we obtain Eq. (15); but, because that the probability superposition assumption is invalid, the frequency definition is incompetent for finding the relation between the measurable probabilities in Stern-Gerlach experiment. To this end, we must find a quantity in micro processes, which obey superposition principle and is able to calculate the probabilities. Fortunately, such a quantity has been found, it is called "probability amplitude".

Feynman said that the concept of probability amplitude is the core of quantum mechanics. Actually, the importance of probability amplitude just rests with that it obeys the superposition principle about the Popper conditions. For the double slit diffraction experiment or the like of it, the superposition principle for probability amplitude can be expressed as follows:

Assume that there are two passages to transit from A state to B state, the probability amplitude of the event that a single electron from the A arrives at the B when two passages open simultaneously is equal to the sum of two probability amplitudes of the same event when two passages open in turn.

Also, quantum mechanics gives that:

The corresponding relation between the transition probability $Pr(B|A)$ and its amplitude, which is written as $\langle B|A\rangle$, is $\Pr(B|A) = |\langle B|A\rangle|^2$.

Similar to probabilities, the amplitudes also satisfy addition formula and multiplication formula. By means of addition formula, superposition principle can be rewritten as follows: Under different Popper conditions, the probability amplitudes for a given transition event is the same.

According to quantum mechanics, for arbitrary given unit vectors $\boldsymbol{a}$, $\boldsymbol{b}$ and $\gamma = \angle(\boldsymbol{a}, \boldsymbol{b})$, the probability amplitude $\langle \sigma_b = y \mid \sigma_a = x\rangle$ takes the values as follow:

$$\langle \sigma_b = 1 \mid \sigma_a = 1\rangle = \langle \sigma_b = -1 \mid \sigma_a = -1\rangle = \cos\frac{\gamma}{2};$$

$$\langle \sigma_b = -1 \mid \sigma_a = 1\rangle = \langle \sigma_b = 1 \mid \sigma_a = -1\rangle = i\sin\frac{\gamma}{2}. \qquad (16)$$

Now, Eq. (16) is applied to rewriting Eq. (15b). Consider the process that the beam A entering into the G_b directly. From Eq. (16) and the (B), it is concluded that:

$$\Pr(\sigma_b = 1 \mid \sigma_a = 1) = \Pr(\sigma_b = -1 \mid \sigma_a = -1) = \cos^2\frac{\gamma}{2},$$

$$\Pr(\sigma_b = 1 \mid \sigma_a = -1) = \Pr(\sigma_b = -1 \mid \sigma_a = 1) = \sin^2\frac{\gamma}{2}. \qquad (17)$$

Next, think about the process that C_1 and C_2 enter into the G_b in turn. To simplify the question, we assume that ***a***, ***b***, ***c*** are in one plan and $\angle(\boldsymbol{a}, \boldsymbol{b}) = \angle(\boldsymbol{b}, \boldsymbol{c}) + \angle(\boldsymbol{a}, \boldsymbol{c})$.

Let

$\angle(\mathbf{b}, \mathbf{c}) = \alpha$, $\angle(\mathbf{a}, \mathbf{c}) = \beta$, $\angle(\mathbf{a}, \mathbf{b}) = \gamma = \alpha + \beta$.

By Eq. (17), the probability of the event that the e, a single electron in A, obtain the measurement value $\sigma_c = 1$ in G_c is $\cos^2(\beta/2)$, and when this condition has been known, the probability that the e obtain the measurement value $\sigma_b = 1$ in G_b is $\cos^2(\alpha/2)$. As a result, when only the first passage open, the e passes the G_c and finally obtains measurement value $\sigma_b = 1$ is

$q_1 = \cos^2\frac{\alpha}{2}\cos^2\frac{\beta}{2}$.

Similarly, when only the second passage open, it passes the G_c and finally obtains $\sigma_b = 1$ is

$q_2 = \sin^2\frac{\alpha}{2}\sin^2\frac{\beta}{2}$.

When C_1 and C_2 enter into the G_b simultaneously, the probability superposition assumption gives the probability that the e passes through the G_c and the G_b successively and finally obtains $\sigma_b = 1$ is $p = q_1 + q_2$. *(18)*

However, since that the probability superposition assumption is invalid, Eq. (18) is wrong. Fortunately, we can apply Eq. (16) to calculate the *p* in the very process. As such, the amplitude that the e obtains $\sigma_c = 1$ in the G_c is $\cos(\beta/2)$; also, under the above known condition the amplitude that it obtain the measurement value $\sigma_b = 1$ is $\cos(\alpha/2)$. By the (C), the amplitude that the e along the first passage and finally obtains $\sigma_b = 1$ is $\cos(\alpha/2)\cos(\beta/2)$. Similarly, along the second passage the amplitude of the same event is $-\sin(\alpha/2)\sin(\beta/2)$. Applying the (C) once again, we obtain the amplitude corresponding to p as follows: $\cos\frac{\alpha}{2}\cos\frac{\beta}{2} - \sin\frac{\alpha}{2}\sin\frac{\beta}{2} = \cos\frac{\alpha+\beta}{2}$.

Now, the (B) gives $p = \cos^2\frac{\alpha+\beta}{2}$.

The above formulae give

$p = q_1 + q_2 - \frac{1}{2}\sin\alpha\sin\beta$.

Comparing with Eq. (18), it is seen that the $-\frac{1}{2}\sin\alpha\sin\beta$ in the above equality is the "intersection term" that make the probability superposition assumption invalid. Let the sign J denote this intersection term, Eq. (15b) is rewritten as

$$\Pr(\sigma_b = y \mid \sigma_a = x) = \sum_z \Pr_D(\sigma_b = y \mid \sigma_c = z) \cdot \Pr(\sigma_c = z \mid \sigma_a = x) + J.$$

In the other field, the energy density equation of the electrostatic field for two point charges is

$u = u_1 + u_2 + u_{12}$.

In which the u_{12} is the "intersection term" that makes that the energy density of the electrostatic field did not obey the superposition principle. It seems that such comparability indicates that between micro process and macro process there is no impassable chasm.

As to p_1 and p_2, they are immeasurable and thereby cannot be obtained by the calculation of amplitudes.

8 Boolean Algebra and Event Operations

It is well known that the probability theory is founded by A. N. Kolmogorov. In the beginning of this theory, in which there is no superposition assumption of probabilities. On the other hand, when people say that the classical probability theory is inapplicable for micro process, this assumption is regarded as a component of it actually. Considering this practice, we stipulate that the Kolmogorov probability theory is named for the system containing the whole probability operation laws, while the classical probability theory for the conjunction of Kolmogorov probability theory and superposition assumption of probabilities.

It has been seen that due to the existence of the probability superposition assumption probabilities the classical probability theory is not always applicable for micro process. Now, a question appears naturally: whether or not the Kolmogorov probability theory applicable for micro processes completely?

If examining closely, we can see that the Kolmogorov probability theory is based on two foundations: one is frequency definition for probabilities (to derive the multiplication formula from probability frequency definition requires a slight revision for such definition); the other is Boolean algebra for event operations. Frequency definition, which is connatural for probabilities, is universally accepted that it is applicable for micro processes. The question is whether Boolean algebra for event operations applicable for micro processes.

Event operations and proposition calculations are applications of Boolean algebra in different domains. As we see, it is unacceptable to modify the Boolean algebra as the laws of proposition calculations. However, from this reason it cannot be concluded that Boolean algebra as the laws of event calculations is also unchangeable.

In Kolmogorov probability theory, if A and B are two events, the occurrence of the product event $A \cdot B$ indicates that "the A occurs and the B occurs". So, the occurrence of the $B \cdot A$ indicates that "the B occurs and the A occurs". Though the above two product events are different in expression form, the meanings are the same actually. As a result, the commutable law

$$A \cdot B = B \cdot A \qquad (19)$$

holds true. In other words, in the expression $A \cdot B$ the positions of the A and the B are commutable.

However, if we prescribe that the occurrence of the $A \cdot B$ means that the event "the A occurs before and the B occurs behind", then the occurrence of the $B \cdot A$ means that the event "the B occurs before and the A occurs behind". Such, when commuting the positions of the A and the B in the expression $A \cdot B$, not only the expression form but also the real meaning is changed, and Eq. (19) is thus invalid.

In the micro processes, this situation has been fallen across. For example, in the double slit diffraction process, let the sign E denote that "the e passes the first slit" and the X that "the e falls on the small area Ω on the screen", then the product event $E \cdot X$ denotes that "the e passes the first slit and then falls on the Ω". In fact, the $E \cdot X$ posses such content, so that the multiplication operation herein does not obey the commutable law.

It is thus seen that we must be especially careful in the matter relating to micro processes. For instance, according to frequency definition, the multiplication formula

$$\Pr(A \cdot B) = \Pr(A) \cdot \Pr(B \mid A) \qquad (20)$$

as a Kolmogorov probability theory law is valid for micro processes.

Because that Eq. (20) is an identity, which holds true for the commuting of the independent variables, so that from Eq. (20) we can obtain

$$\Pr(B \cdot A) = \Pr(B) \cdot \Pr(A \mid B),$$

so far as the events in it are meaningful. But due to the commutable law is invalid, from Eq. (20) we cannot obtain $\Pr(A \cdot B) = \Pr(B) \cdot \Pr(A \mid B)$.

As a result, if in the Stern-Gerlach experiment we have, by means of multiplication formula, gotten

$$\Pr(\sigma_a = x,\ \sigma_b = y) = \Pr(\sigma_a = x) \cdot \Pr(\sigma_b = y \mid \sigma_a = x), \qquad (21)$$

then, because that the commutable formula

$$\Pr(\sigma_a = x,\ \sigma_b = y) = \Pr(\sigma_b = y,\ \sigma_a = x)$$

is invalid generally, from Eq. (21) it is impossible to obtain

$$\Pr(\sigma_b = y,\ \sigma_a = x) = \Pr(\sigma_a = x) \cdot \Pr(\sigma_b = y \mid \sigma_a = x).$$

In Boolean algebra, some more complex formula, for example, $(A \cdot B) \cdot C = (A \cdot C) \cdot B$, is invalid in the micro processes because that the commutable law has been used.

9 Joint Probabilities

A probability for product of several events is called after "joint probability". It is well known that the very concept is useless in quantum mechanics, but this fact does not mean that the application of joint probabilities herein will certainly go wrong. According to frequency definition, it is possible to introduce joint probabilities without conflicts. But we must keep in mind that the joint probabilities herein may not obey the laws of Boolean algebra for event operation. Otherwise, the mistakes will appear. As an example, we consider a formula playing an important part in so-called Bell's theorem.

Twice applying the multiplication formula, we have

$$\Pr(A \cdot B \cdot C) = \Pr(A) \cdot \Pr(B \mid A) \cdot \Pr(C \mid A \cdot B). \qquad (22)$$

If the $A \cdot B \cdot C$ is regarded as the product of three events occurring successively, this formula is applicable for micro processes. On the other hand, we know that the electrons "lack of memory", which means that in the character process the expression $Pr(\sigma_b = y \mid \sigma_c = z,\ \sigma_a = x)$ can be abbreviated to $Pr(\sigma_b = y \mid \sigma_c = z)$. Therefore, for joint probability $Pr(\sigma_a = x,\ \sigma_b = y,\ \sigma_c = z)$, Eq. (22) gives

$$\Pr(\sigma_a = x,\ \sigma_b = y,\ \sigma_c = z) = \Pr(\sigma_a = x) \cdot \Pr(\sigma_c = z \mid \sigma_a = x) \cdot \Pr(\sigma_b = y \mid \sigma_c = z). \qquad (23)$$

By means of Eq. (23), Eq. (15) becomes

$$\Pr(\sigma_a = x,\ \sigma_b = y) = \sum_z \Pr(\sigma_a = x,\ \sigma_b = y,\ \sigma_c = z)$$

.(24)

This formula can be understood by the addition formula for Stern-Gerlach process.

In Eq. (24) commuting the $\sigma_b = y$ and $\sigma_c = z$, we have

$$Pr(\sigma_a = x, \sigma_c = z) = \sum_y Pr(\sigma_a = x, \sigma_c = y, \sigma_b = z)$$

.(25)

The Kolmogorov probability theory gives

$$Pr(\sigma_a = x, \sigma_b = y, \sigma_c = z) = Pr(\sigma_a = x, \sigma_c = y, \sigma_b = z)$$

(26)

by which Eq. (25) becomes

$$Pr(\sigma_a = x, \sigma_c = z) = \sum_y Pr(\sigma_a = x, \sigma_b = y, \sigma_c = z).$$

In a similar way, we can obtain

$$Pr(\sigma_b = y, \sigma_c = z) = \sum_x Pr(\sigma_a = x, \sigma_b = y, \sigma_c = z).$$

Writing $Pr(\sigma_a = x, \sigma_b = y, \sigma_c = z)$ as $F(x, y, z)$, from Eq. (24) and the above two formulae, it is concluded that:

For any given directions ***a***, ***b***, ***c*** and $x, y, z \in \{1, -1\}$, there exists function $F(x, y, z) \geqslant 0$, such that:

$$Pr(\sigma_a = x, \sigma_b = y) = \sum_z F(x, y, z);$$

$$Pr(\sigma_a = x, \sigma_c = z) = \sum_y F(x, y, z);$$

$$Pr(\sigma_b = y, \sigma_c = z) = \sum_x F(x, y, z).$$

This is a proposition resulting from Kolmogorov probability theory, is it valid? Let us check it step by step.

Firstly, in the proof for the (d), Eq. (22) has been used. By frequency definition we know that Eq. (22) is hold true for the character process.

Secondly, by multiplication formula, from Eq. (22) we get Eq. (24). Due to multiplication formula is valid in micro process, the Eq. (24) is also hold true.

Thirdly, in the both sides of Eq. (24), commuting $\sigma_b = y$ and $\sigma_c = z$, Eq. (25) is obtained, this step is reasonable for identity, so that Eq. (25) is also valid.

Fourth, the deducing the (d) from Eq. (25), the Kolmogorov probability theory formula Eq. (26) is used, this step is unreasonable however. The left side of Eq. (26) is the probability that in a character process that the e passes through the G_c before and through the G_b behind, while the right side of Eq. (26) is the probability that under otherwise equal conditions, commuting G_c and G_b, namely, the e passes through the G_b before and through the G_c behind. Clearly, the latter is different from the farmer naturally. So, Eq. (26) is certainly invalid.

It is thus concluded that the proposition (d) is wrong.

It should be noted that: Firstly, Eq. (26) is independent of Popper conditions, so that the invalidity of Eq. (26) has nothing to do with probability superposition assumption. Secondly, the Kolmogorov probability theory is based on the frequency definition of probabilities and Boolean algebra of event operations. Eq. (26) is invalid because of Boolean algebra of event operations instead of frequency definition of probabilities.

10 Micro Processes and Probability Operations

Because that probability amplitudes obey superposition principle about Popper conditions, provided that apply probability amplitudes to calculating probabilities, it is naturally to be concluded that the probabilities do not obey superposition principle about Popper conditions. Besides, probability amplitude involves two states, one is the state before the transition and the other is behind the transition. These two state are unsymmetrical, it is also naturally to be concluded that the multiplication operation of events do not obey the commutable law. As a result, the application of probability amplitude naturally removes the two factors those are inapplicable for micro processes. So, even if one is ignorant of the character of the probability operations in micro processes, provided holding the techniques of operating the probability amplitude, he is able to gallop across the micro field. This instance is especially lucky for quantum physicists. But there is a small deficiency: when the problem not only involves the techniques but also relates to the substance of probability calculations, they are hard to avoid suffering setback. The muddle brought about by Bell's inequality, which will be examined in another paper, is a fact illustrative of the very point.

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