On the Parabolic Curve of Primary Mirrors

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Abstract: In order for a parabolic mirror to work, light has to reflect off of every point on it and be directed in a straight line to the focus. With this in mind, we know a light ray traveling parallel to the y-axis and reflecting off of the point (x, f) has to be directed to the focus in a straight line parallel to the x-axis. In order for a vertical light ray to reflect in this way, the point (x, f) on the mirror has to have a slope of 45 degrees or, in other words, a slope of one. This means that the derivative of the parabolic function at the point (x, f) has to equal 1. [Nature and Science. 2006;4(1):25-26].

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In order to make a reflecting telescope such as a Newtonian Reflector, the primary mirror is often ground into the shape of a parabola. The reason for this is that when light enters the telescope tube, it is reflected off of the parabolic mirror and, due to the unique parabolic shape, is focused at a single point (the focus) which is a certain distance away from the center of the mirror (the focal length). The purpose of this paper is to try to describe which parabolic curve is suitable to fit a known focal length. The general solution has been known for a long time, but I was able to use a kind of guess and check method to find a similar solution on my own. This is one way in which the solution can be derived. We will assume the parabola is centered at the origin, so we know the general form of the parabola will be similar to $y = ax^2$ where *a* is some particular coefficient depending on the given focal length.

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So, knowing that the parabolic function will take the form $y = ax^2$, and that the derivative of this function evaluated at the point (x, f) must equal 1, we can figure out the general equation for any parabolic mirror with a particular focal length.

Take the case where f, the focal length, equals 4. In order to find the x-coordinate for (x, f) we need to solve $y = ax^2$ for x.

$$4 = ax^2$$
$$(4/a) = x^2$$

$$(2/\sqrt{a}) = x$$

We know that at the point $((2/\sqrt{a}), 4)$ the slope of the parabolic curve must equal 1. To find an equation for the slope of the tangent line to the parabolic curve at any point we take the derivative of $y = ax^2$.

$$y = ax^2$$
$$y' = 2ax$$

We know $x = 2/\sqrt{a}$ and that the slope at this point has to be 1. Plugging in $y' = 1$ and $x = 2/\sqrt{a}$ we get:

$$1 = (4a)/\sqrt{a}$$

Rearranging we get:

$$\sqrt{a} = 4a$$
$$a = 16a^2$$
$$1 = 16a$$
$$1/16 = a$$

Plugging *a* back into our original parabolic equation we get:

$$y = (1/16)x^2$$

which will produce a focal length of 4.

We have just found the parabolic equation for a mirror that produces a focal length of 4. However, we can generalize this equation for *any* focal length. Take *f* to be the focal length, but this time, instead of assigning a value to *f*, we will leave it like it is.

$$f = ax^2$$
$$f/a = x^2$$
$$\sqrt{f}/\sqrt{a} = x$$

$$f' = 2ax$$

1 = (2a*sqrt(f))/sqrt(a)
sqrt(a) = 2a*sqrt(f)
a = 4fa^2
1 = 4fa
1/(4f) = a

Therefore, by plugging *a* back into the parabolic equation we get the general solution:

y = 1/(4f)x^2

where *y* is the curve that a parabolic mirror takes with any focal length *f*.