Matched Quick Switching Variable Sampling System with Quick Switching Attribute Sampling System

Sriramachandran G.V [1], Palanivel .M [2]

[1]Department of Mathematics, Dr.Mahalingam College of Engineering and Technology, Pollachi, Tamil Nadu, 642003, India
[2]Department of Statistics, Government Arts College, Udumalpet. Tamil Nadu 642126, India
gvsriramachandran@yahoo.com, palanivelm@yahoo.com

Abstract: This paper presents a methodology for finding a Quick Switching Single Sampling variables inspection system matching a given Quick Switching attribute sampling inspection system. Here matching implies the same Acceptance Quality Level (AQL) $(p_{0.95}, 0.95)$ and Limiting Quality Level (LQL) $(p_{0.10,} 0.10)$ of the Operating Characteristic (OC) curves for both the systems. [Nature and Science 2009;7(12):33-39]. (ISSN: 1545-0740).

Key words: Operating Characteristic Curve, Acceptance Sampling, Acceptance Quality Level and Limiting Quality Level

1. Introduction

Hamaker (1979) has given a method of constructing single sampling variables inspection plans such that the OC curve of a given single sampling attributes inspection plan and the OC of the variables plan have the same indifference quality level (p_0) and the same relative slope (h_0) of the OC curve at p_0. Bender (1975) has given a table for single sampling variables inspection plans matched at the points $(p_{0.95}, 0.95)$ and $(p_{0.10,} 0.10)$ with attributes inspection plans given in Table II-A of MIL-STD-105D (1963). Bender (1975) achieves this matching by means of an iterative computer program involving non central t-distribution.

Romboski (1969) has studied a new system, comprising of normal and tightened plans, called Quick switching system (n; c2,c1) [(n,c_2) and (n,c_1) are normal and tightened single sampling plans respectively with c_2>c_1] proposed by Dodge (1967). Taylor (1996) investigated how to evaluate and select Quick Switching Systems. Soundarajan.V and Palanivel.M (1997) and Soundarajan.V and Palanivel.M (2000) has investigated on Quick Switching Variables Simple Sampling (QSVSS) Systems.

2. Operating Procedure

Step 1: From a lot, take a random sample of size n and count the number of defectives, d

i) If d $\le c_2$ accept the lot and repeat the step 1 for the next lot.

ii) If $d > c_2$ reject the lot and go to step 2.

Step 2: From the next lot, take a random sample of size n and count the number of defectives, d

i) If $d \le c_1$ accept the lot and go to step 1repeat the step 1 for the next lot.

ii) If $d > c_1$ reject the lot and repeat the step 2.

Based on this procedure, a quick switching single sampling variables inspection system can be operated as follows:

Step 1: Draw a sample of size n_σ from the lot, inspect and record the measurement of the quality characteristic for each unit of the sample. Compute the sample mean $\bar{x}$

If $\bar{x} + k_{N\sigma}\sigma \le U$ (where U is the upper specification limit) accept the lot and repeat step 1 otherwise, reject the lot and follow step 2.

Step 2: Draw a sample of size n_σ from the next lot, inspect and record the measurement of the quality characteristic for each unit of the sample. Compute the sample mean $\bar{x}$,

If $\bar{x} + k_{T\sigma}\sigma \le U$ accept the lot and go to step 1 otherwise, reject the lot and repeat step 2.

3. Preliminaries

According to Romboski (1969) the OC function of Quick Switching Single Sampling Attributes System (n;c2,c1) is given by

$$P_a(p) = \frac{P(d \le c_1)}{1 - P(d \le c_2) + P(d \le c_1)} \quad (1)$$

By assuming the Poisson model, one obtain

$$P(d \le c_1) = \sum_{i=0}^{c_1} \frac{\exp(-np)(np)^i}{i!}$$

$$P(d \le c_2) = \sum_{i=0}^{c_2} \frac{\exp(-np)(np)^i}{i!}$$

and $P_a(p_1) = 0.95$ & $P_a(p_2) = 0.10$ (2)

For a quick switching single sampling variables inspection system QSVSS $(n_\sigma; k_{N\sigma,} k_{T\sigma})$ [$(n_\sigma, k_{N\sigma})$ and $(n_\sigma, k_{T\sigma})$ are the normal and tightened single sampling plans respectively, with $k_{T\sigma} > k_{N\sigma}$] with known standard deviation (σ), the fraction non-conforming in a given lot will be P=F(-v)

with $v = \dfrac{(u-\mu)}{\sigma}$ (3)

and the OC function of the system has been given by

$$P_a(p) = \frac{P(Z_u \ge k_{T\sigma})}{1 - P(Z_u \ge k_{N\sigma}) + P(Z_U \ge k_{T\sigma})} \quad (4)$$

where

$$P(Z_u \ge k_{N\sigma}) = \phi(w_1)\ , \quad P(Z_U \ge k_{T\sigma}) = \phi(w_2)$$

with

$w_1 = (v - k_{N\sigma})\sqrt{n_\sigma}$ and $w_2 = (v - k_{T\sigma})\sqrt{n_\sigma}$

and $L(p_1) = 0.95$ & $L(p_2) = 0.10$ (5)

4. Matched QSS Variable System with Attribute System

The procedure for obtaining a Quick switching Variable Sampling Inspection System $(n_\sigma; k_{N\sigma,} k_{T\sigma})$ matching a given attributes system QSS $(n;c_2,c_1)$ of Soundarajan.V and Arumainayagam S.D. (1990) is as follows:

For the QSS $(n;\ c_2,c_1)$ with p_1=0.014, p_2=0.05, α =0.05 and β =0.10, from Table 1 the following values are determined. $\dfrac{p_2}{p_1} = \dfrac{0.05}{0.014} = 3.571$

The nearest value of 3.571 in the table is 3.5815, which has associated with it value of $np_1 = 1.2104$ $c_1 = 1$ and $c_2 = 3$.

The sample size is then determined by

n= $\dfrac{np_1}{p_1} = \dfrac{1.2104}{0.014} = 86.45 \cong 86$

The attributes system is designated as QSS (86, 3,1)

From equation (1)

$$P_a(p_1) = \frac{p(d \le c_1 / p_1)}{1 - p(d \le c_2 / p_1) + p(d \le c_1 / p_1)}$$

For the above system

$$P(d \le c_1 / p_1) = \sum_{i=0}^{c_1} e^{-np_1}(np_1)^i / i! = 0.659$$

$$P(d \le c_2 / p_1) = \sum_{i=0}^{c_2} e^{-np_1}(np_1)^i / i! = 0.9653 \quad (6)$$

Therefore $P_a(p_1) = 0.95$

Similarly

$$P_a(p_2) = \frac{p(d \le c_1 / p_2)}{1 - p(d \le c_2 / p_2) + p(d \le c_1 / p_2)}$$

$$P(d \le c_1 / p_2) = \sum_{i=0}^{c_1} e^{-np_2}(np_2)^i / i! = 0.0699$$

$$P(d \le c_2 / p_2) = \sum_{i=0}^{c_2} e^{-np_2}(np_2)^i / i! = 0.3709 \quad (7)$$

Therefore $P_a(p_2) = 0.10$

Thus the OC curve of the QSS (86, 3, 1) passes through $(p_{0.95}, 0.95)$ and $(p_{0.10}, 0.10)$. To obtain Quick switching sampling variable system we consider the OC function

$$P_a(p_1) = \frac{\varphi(w_2)}{1 - \varphi(w_1) + \varphi(w_2)} = 0.95$$

where

$w_1 = (v_1 - k_{N\sigma})\sqrt{n_\sigma}$ and $w_2 = (v_1 - k_{T\sigma})\sqrt{n_\sigma}$

If $P_a(p_1) = 0.95$ then

$\varphi(w_2) = 0.6590$ and $\varphi(w_1) = 0.9653$

[From (6) & (7)]

$$w_2 = (v_1 - k_{T\sigma})\sqrt{n_\sigma} = 0.41 \text{ and}$$
$$w_1 = (v_1 - k_{N\sigma})\sqrt{n_\sigma} = 1.82 \quad (8)$$

$$P_a(p_2) = \frac{\varphi(w_2)}{1 - \varphi(w_1) + \varphi(w_2)} = 0.10$$

$w_1 = (v_2 - k_{N\sigma})\sqrt{n_\sigma}$ and $w_2 = (v_2 - k_{T\sigma})\sqrt{n_\sigma}$

If $P_a(p_2) = \beta = 0.10$ then

$\phi(w_2) = 0.0699$ and $\phi(w_1) = 0.3709$

[From (8) & (9)]

$$w_2 = (v_2 - k_{T\sigma})\sqrt{n_\sigma} = -1.48 \text{ and}$$
$$w_1 = (v_2 - k_{N\sigma})\sqrt{n_\sigma} = 0.33 \quad (9)$$

If $p_1 = 0.014 \Rightarrow v_1 = 2.2$

$p_2 = 0.05 \Rightarrow v_2 = 1.64$

Through (10) and (11) one can get

$n_\sigma = 13$, $k_{N\sigma} = 1.713$ and $k_{T\sigma} = 2.068$

From the given Quick switching sampling attribute inspection system (86;3,1) one can obtain the QSVSS as

1. $n_\sigma = 13.011 = 13$
2. $k_{N\sigma} = 1.713$
3. $k_{T\sigma} = 2.068$
4. $v_1 = 2.2$, $v_2 = 1.64$

Thus the quick switching variable sampling system (13: 1.7132, 2.068) is such that the OC curve passes through $(p_{0.95}, 0.95)$ and $(p_{0.10,} 0.10)$. The equivalence of the OC curves is illustrated in Table 3. Though in practice the sample size n_σ would have to be rounded to 13, the OC curve for the Quick switching variable sampling system has been computed with the fractional value n in order to better demonstrate the closeness of the agreement of the OC curves. One can imagine the saving of the variable system with attributes system when the sample size 13 is compared with 86.

5. Selection of variable Systems Matching Quick Switching Attribute System Indexed by AQL and LQL

Table 2 provides matched variables system to that of attributes systems indexed by AQL and LQL. For example, for given $p_1 = 0.012$, $\alpha = 0.05$ $p_2 = 0.05$ and $\beta = 0.10$, the "OR" value is 4.1666, the nearest value of 4.1666 in the Table 3.9 is 4.2069, which has associated with the attribute system n=91, $c_2 = 3$ and $c_1 = 2$ one can obtain the corresponding matched Quick switching sampling variable system from Table 2 as $n_\sigma = 16$, $k_{N\sigma} = 1.7623$, and $k_{T\sigma} = 1.9030$. Table 1 provides $\frac{p_2}{p_1}$ together with the value of np_1, c_2 *and* c_1 $(\alpha = 0.05, \beta = 0.10)$.

Table 1: Values of $\frac{p_2}{p_1}$ for QSS

c_2	c_1	$\frac{p_2}{p_1}$ **for** $(\alpha = 0.05, \beta = 0.10)$	np_1
1	0	8.2128	0.3078
2	1	5.3013	0.7695
2	0	4.3834	0.6437
3	2	4.1703	1.3178
3	1	3.5815	1.2104
3	0	3.1344	1.0046
6	5	2.9126	3.2363
5	3	2.8568	2.4721
7	6	2.7163	3.9318
6	4	2.6543	3.1486
4	0	2.5404	1.3754
6	3	2.4340	3.0089
5	1	2.3707	2.0915
9	7	2.2849	5.2963
6	2	2.2509	2.8056
5	0	2.1989	1.7504
9	6	2.1444	5.1774
6	1	2.1001	2.5239
8	4	2.0837	4.2658
9	5	2.0220	5.0129
8	3	1.9662	4.0358
7	1	1.9160	2.9508
8	2	1.8669	3.7427
7	0	1.8259	2.5051
8	1	1.7833	3.3725
9	2	1.7491	4.1974
9	1	1.6281	3.7899
9	0	1.6281	3.2613
11	2	1.5871	5.0852
11	1	1.5436	4.6135
12	0	1.4611	4.3941

Table 2 .Quick Switching Variable Sampling System Matched with Quick Switching Attribute Sampling System

$\frac{p_2}{p_1}$	Attribute QSS			Variable QSS		
	n	c_2	c_1	n_σ	$k_{N\sigma}$	$k_{T\sigma}$
8.3333	128	1	0	9	2.2330	2.5769
5.2857	110	2	1	11	1.9597	2.1714
4.4615	99	2	0	10	1.9064	2.4358
4.2069	91	3	2	16	1.7623	1.9030
3.5714	86	3	1	13	1.7164	2.0700
3.1333	67	3	0	10	1.5446	2.2538
2.9250	81	6	5	22	1.3985	1.4828
2.8438	77	5	3	18	1.4526	1.6736
2.7200	79	7	6	23	1.3060	1.3855
2.6522	69	6	4	20	1.3028	1.4945
2.5238	66	4	0	11	1.4100	2.2126
2.4390	73	6	3	19	1.3241	1.6401
2.3793	72	5	1	15	1.3794	1.9467
2.2857	76	9	7	24	1.1380	1.2789
2.2500	78	6	2	18	1.3542	1.8165
2.2000	70	5	0	13	1.3293	2.1944
2.1447	68	9	6	23	1.0638	1.2883
2.0952	60	6	1	15	1.1758	1.8582
2.0789	56	8	4	19	1.0100	1.3703
2.0263	66	9	5	22	1.3592	1.0450
1.9692	62	8	3	21	1.0651	1.5342
1.9286	70	7	1	17	1.1847	1.9097
1.8654	72	8	2	19	1.1419	1.7512
1.8095	119	7	0	18	1.4385	2.3461
1.7742	109	8	1	12	1.2344	2.2838
1.7600	168	9	2	28	1.5433	2.1039
1.7037	140	9	1	23	1.1431	2.1716
1.6316	172	9	0	25	1.5130	2.4070
1.6000	170	11	2	27	1.4242	2.1047
1.4762	209	12	0	28	1.4421	2.4526

Table 3. Values of [p, L (p)] for Quick Switching Attribute Sampling System and the equivalent variable system of the proposed method

p	L(p) of attributes QSS	L(p) of Variables QSS
0.01	0.9846	0.9855
0.02	0.8331	0.8107
0.03	0.5044	0.4518
0.04	0.2393	0.2224
0.05	0.1041	0.0956
0.06	0.0444	0.0434
0.07	0.0191	0.0203
0.08	0.0087	0.0099
0.09	0.0042	0.0048
0.10	0.0021	0.0024

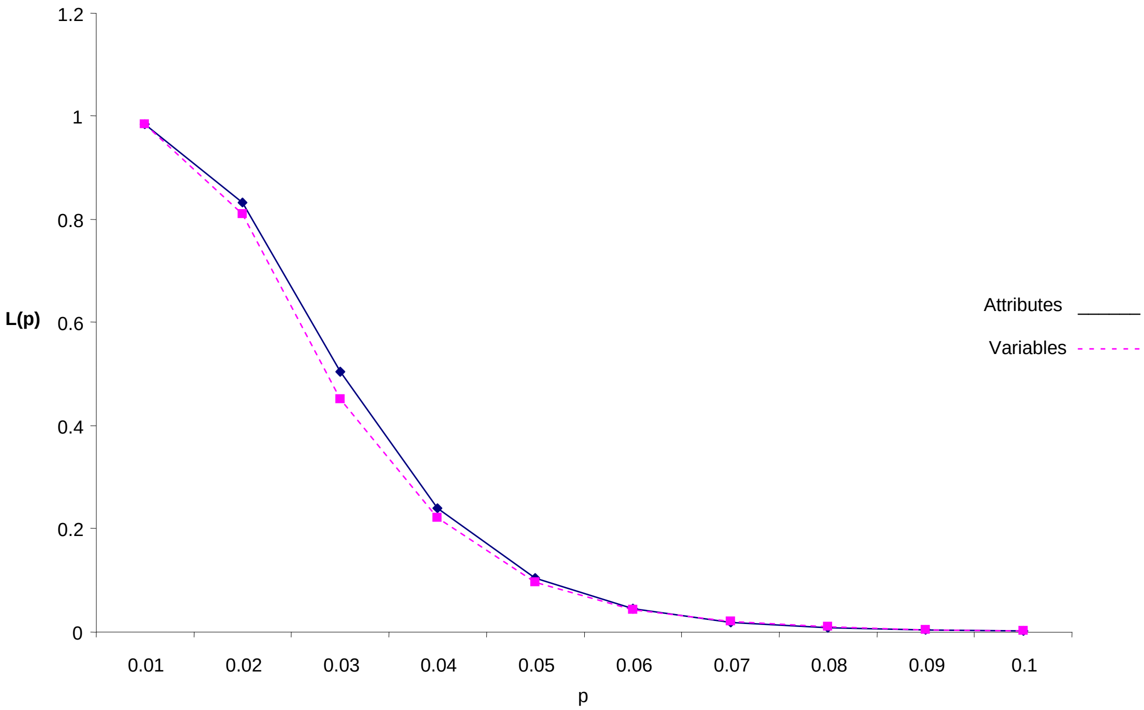

Figure. 1 OC curves - Attributes QSS vs. Variables QSS

6. Conclusions

The methodology used in this paper could be adopted for other developed sampling system to match with other sampling system. In the shop floor, we could apply both the given attribute quick switching systems as well as variable quick switch systems. From the Table 2 and Table 3, one can conclude that the variable quick switching sampling system is better one.

Acknowledgement

The authors are grateful to the Management of Dr.Mahalingam College of Engineering & Technology for the encouragement and support in preparation of this research paper.

Correspondence to:
Sriramachandran G.V
Senior Lecturer in Mathematics
Dr.Mahalingam College of Engineering and Technology
Pollachi-642 003
Tamil Nadu
India
Cellular phone: 091-9245250502;
Email: gvsriramachandran@yahoo.com

Reference

[1] Bender A.J. Single Sampling by Variables to Control the Fraction Defective. Journal of Quality Technology. 1975; 7(5): 71-75.
[2] Dodge H.F. A New Dual System of Acceptance Sampling. Technical report No.16, The Statistics center, Rutgers – The State University. New Brunswick, NJ. 1967
[3] Hamaker H.C. Acceptance Sampling for Percent Defective by Variable and Attributes. Journal of Quality Technology. 1979; 11(3): 139-148.
[4] MIL-STD-105D. Military Standard Sampling Procedures and Tables for Inspection by Attributes. *US Government Printing Press*, Washington D.C. 1963
[5] Romboski L.D. An Investigation of Quick Switching Acceptance Sampling System. Doctoral Dissertation Rutgers- The State University, New Brunswick, New Jersey. 1969
[6] Soundarajan V., Arumainayagam S.D. Construction and Selection of Modified Quick Switching System. Journal of Applied Statistics. 1990; 17(1): 83- 114
[7] Soundarajan V., Palanivel M. Quick Switching Variables Single Sampling (QSVSS) System indexed by AQL and LQL Acceptance Criterion Tightening. Journal of Applied Statistics Science.1997;6(1): 45-57.
[8] Soundarajan V., Palanivel M. Quick Switching Variables Single Sampling (QSVSS) System indexed by AQL and AOQL. Journal of Applied Statistics. 2000; 27(7): 771-778
[9] Taylor W.A. Quick Switching Systems. Journal of Quality Technology.1996; 28(4):460-472.

10/12/2009