A New Approach to Special Relativity and its Consequences

Dr. Wolfgang Gaudig

Address: Schwarzwaldstrasse 102, D-70569 Stuttgart, Germany
E-Mail: wolfgang.gaudig@yahoo.de

Abstract: The theory of special relativity by Albert Einstein is extended by the requirement that not only the coordinate points co-moving with the moving inertial frame shall fulfil the transformation formulae, but also the coordinate points resting with the rest frame. It turns out that the present new theory, although derived by strictly employing Einstein's original light beam procedure, confirms the ad hoc generalized Galilean transformation: The clock paradox is inherently avoided, without having to invoke Einstein's general theory of relativity. However, there are severe consequences: (i) the velocity of the rest frame as observed in the moving frame is not equal to the velocity of the moving frame as observed in the rest frame; (ii) furthermore, the one-way light signal speed is not a universal constant any more, but has to be assumed different in the moving frame. This leads to the definition of the rest frame to be a preferred frame, where the assumption of an isotropic light signal speed still holds. The light signal speed in the moving frame is then anisotropic and dependent on the frame velocity. Several applications are discussed in comparison to Einstein's original theory of special relativity: Light aberration effect, length contraction, time dilation, Maxwell's equations, the electric Lorentz force, the relativistic law of motion, the electromagnetic wave equation, and the relativistic Doppler frequency shift of electromagnetic radiation. It is pointed out that, in the moving frame, it must be distinguished between the light signal speed (ray velocity) and the phase velocity of light. Another issue is the fact that the interpretation of Maxwell's equations in the moving frame is not unequivocal. - However, despite of reasonable and interesting results, the final judgement of the theory will only be possible when reliable evaluations of one-way light signal speed measurements are available.
[Wolfgang Gaudig. **A New Approach to Special Relativity and its Consequences.** Nature and Science 2011;9(4):89-111]. (ISSN: 1545-0740). http://www.sciencepub.net.

Keywords: Extension of Einstein's special relativity, preferred rest frame, unequal relative velocities of moving frame and rest frame, anisotropic one-way light speed in a moving frame, Maxwell's equations.

1. Introduction

It took a couple of decades of years until Einstein's theory of special relativity (Einstein, 1905) was widely excepted by the physical community. However, there has always been criticism of this theory. This has frequently to do with the well known clock paradox (twin paradox) which implies that an observer in either of two inertial frames which are in relative motion to each other sees time in the other frame elapse slower. It is impossible to solve this problem only by the original theory of special relativity because the Lorentz transformation is symmetrical. Einstein resolved the clock paradox (Einstein, 1918) by invoking the general theory of relativity. For this purpose, he had to admit that the rest frame is not at all equivalent to the moving frame, what is of course contradictory to the original idea of relativity. Furthermore, the correction supplied by the general theory of relativity is not a continuous one but is effective only during the turnaround of the moving frame, and thus corrects only for the final state, i. e. the return of the moving frame to the position of the rest frame. To eliminate these problems, several so called test theories of special relativity have been developed: Robertson (1949) replaced Einstein's way of deduction by an experimentally supported approach, Mansouri and Sexl (1977) generalized the Lorentz transformation, and Chang (1979) reconsidered an older approach to special relativity, i. e. the "ether" theory, in the new form of the "Generalized Galilean transformation" which is an ad hoc modification of the Galilean transformation.

The present paper returns to Einstein's original light beam procedure of special relativity, figure 1 a (Einstein, 1905), modified however by extension, figure 1 b: It is required that not only the coordinate points P' of the moving frame should fulfil the transformation formula, but also the coordinate points P of the rest frame, thereby leaving the way of Einstein's logical deduction unchanged. In this way, a new transformation can very easily and straightforwardly be derived, together with all of its consequences. It turns out that this "extended Einsteinian theory of special relativity" approves the "Generalized Galilean Transformation".

2. Extension of Einstein's original theory of special relativity

The basis of Einstein's theory of special relativity (Einstein, 1905) has been a thought experiment, depicted in figure 1 a. An inertial frame $\Sigma'[x',y',z']$ moves uniformly with a velocity of v along the X-axis of the rest frame Σ(x,y,z), where Σ and Σ' are axiparallel Cartesian coordinate systems. A light signal is emitted from the origin O' of Σ' when O' passes through the origin O of Σ. The path of this light signal is given as O'P' in the moving frame, and OP' in the rest frame, where P' is a representative point of the moving frame. The spatial coordinates of P' are expressed in both frames, along with time t and t' elapsed in Σ and Σ', respectively.

Figure 1 b shows the same situation as figure 1 a, except a representative point P of the rest frame is considered. Again, the spatial and temporal coordinates of P are expressed in a double way, firstly when measured in the rest frame, secondly when measured in the moving frame.

Einstein's theory of special relativity is now modified by the extended requirement that the transformation shall be valid for both, any coordinate point P' co-moving with the moving frame and any coordinate point P resting with the rest frame.

Corresponding to the initial condition,

$$t=0,\quad x=0,\quad y=0,\quad z=0,\quad t'=0,\quad x'=0,\quad y'=0,\quad z'=0 \quad , \tag{1}$$

the desired transformation is taken as

$$x' = A_1\,x + A_2\,y + A_3\,z + A_4\,t \quad , \tag{2 a}$$
$$y' = B_1\,x + B_2\,y + B_3\,z + B_4\,t \quad , \tag{2 b}$$
$$z' = C_1\,x + C_2\,y + C_3\,z + C_4\,t \quad , \tag{2 c}$$
$$t' = D_1\,x + D_2\,y + D_3\,z + D_4\,t \quad . \tag{2 d}$$

Because of rotational symmetry about the x(x')-axis, it follows:

$$A_2 = A_3 = B_1 = B_4 = C_1 = C_4 = D_2 = D_3 = 0\,,\quad B_3 = -C_2\,,\quad C_3 = B_2 \quad . \tag{3}$$

Hence,

$$x' = A_1\,x + A_4\,t \quad , \tag{4 a}$$
$$y' = B_2\,y - C_2\,z \quad , \tag{4 b}$$
$$z' = C_2\,y + B_2\,z \quad , \tag{4 c}$$
$$t' = D_1\,x + D_4\,t \quad . \tag{4 d}$$

Table 1 shows the further procedure in finding out the still open parameters A_1, A_4, C_2, D_1, and D_4 (except B_2), step by step: Special point events P' and P, and the origins O' and O, whose spatial and temporal coordinates are straightforward in both frames, are plugged into the transformation formulae, equations. 4 a-d. In this way, every step yields four equations, through which the results in the right column of table 1 are derived. Steps 1 to 5 are necessary and sufficient to derive the well known Lorentz transformation, assuming the light signal speed to be a universal constant c in both frames. However, proceeding with step 6 to 10, in order to get the new modified transformation, requires more open parameters to be adjusted: (i) The light signal speed in the moving frame, c', has to be assumed as a function of the angle α' between the X'-axis and the light beam, figure 1 b, and (ii) the amount of the velocity of the rest frame with respect to the moving frame, u, has not necessarily to be equal to the amount of the velocity of the moving frame with respect to the rest frame, v. Step 10 serves to determine the function c'(α'), and yields two equations, table 1, where the angle α means the angle between the X-axis and the light beam in the rest frame, figure 1 a. These two equations are solved for c'(α'), cos(α'), and sin(α'):

$$\frac{c'(\alpha')}{c} = \frac{-\cos(\alpha')\cdot\frac{c'(90°)}{c}\cdot\frac{v}{c}\cdot\frac{v}{u}\cdot\gamma^2 + \sqrt{\cos^2(\alpha')\cdot\left(\frac{c'(90°)^2}{c^2}\cdot\frac{v^2}{u^2}\cdot\gamma^4 - 1\right)+1}}{\cos^2(\alpha')\cdot\left(\frac{c'(90°)^2}{c^2}\cdot\frac{v^2}{u^2}\cdot\gamma^2 - 1\right)+1} \quad , \tag{5}$$

$$\cos(\alpha') = \frac{c}{c'(\alpha')} \cdot \frac{c}{c'(90°)} \cdot \frac{u}{v} \cdot \left(\cos(\alpha) - \frac{v}{c}\right) \quad , \tag{6}$$

$$\sin(\alpha') = \frac{c}{c'(\alpha')} \cdot \gamma \cdot \sin(\alpha) \quad , \tag{7}$$

where

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad . \tag{8}$$

For α'=90°, equation 5 yields

$$c'(90°) = c \quad . \tag{9}$$

As can be figured from table 1, right column, and equation 9, the parameters A_1, A_4, C_2, D_1, and D_4 are

$$A_1 = B_2 \cdot \frac{u}{v} \cdot \frac{1}{\gamma}, \quad A_4 = -B_2 \cdot \frac{u}{\gamma}, \quad C_2 = 0, \quad D_1 = 0, \quad D_4 = B_2 \cdot \frac{1}{\gamma} \quad , \tag{10}$$

so that the transformation, equations 4 a-d, becomes

$$x' = B_2 \cdot \frac{1}{\gamma} \cdot \frac{u}{v} \cdot (x - v \cdot t) \quad , \tag{11 a}$$

$$y' = B_2\, y \quad , \tag{11 b}$$

$$z' = B_2\, z \quad , \tag{11 c}$$

$$t' = B_2 \cdot \frac{1}{\gamma} \cdot t \quad . \tag{11 d}$$

In order to determine the ratio u/v, another definition is used, the average two-way speed of a light signal in the moving frame Σ' (Reichenbach, 1969):

$$\frac{1}{\bar{c}'} = \frac{1}{2} \cdot \left[\frac{1}{c'(\alpha')} + \frac{1}{c'(\alpha' + 180°)}\right] \quad . \tag{12}$$

This means, the reciprocal two-way speed of a light signal is defined as an average of the reciprocal one-way speed of a light signal travelling in the forward (α') direction and the one-way speed of a light signal travelling in the backward (α'+180°) direction. Taking into account equations 5 and 9, equation 12 yields:

$$\frac{\bar{c}'}{c} = \frac{1}{\sqrt{\left(\frac{v^2}{u^2} \cdot \gamma^4 - 1\right) \cdot \cos^2(\alpha') + 1}} \quad . \tag{13}$$

The Michelson-Morley optical experiment (Michelson and Morley, 1887) says that this two-way light speed in the moving frame Σ' is independent of the direction (angle α') of the light beam, and according to Kennedy and

Thorndike (1932), the two-way light speed is also independent of the velocity v of Σ’ relative to Σ. The result of those measurements was always the same, i. e. the constant two-way light speed c.

$$\bar{c}' = c = \mathrm{const.}(\alpha', v) \quad . \tag{14}$$

From these experimental facts, it can be concluded that the term in parentheses in equation 13 must be zero: Thus it follows that

$$\frac{u}{v} = \gamma^2 \quad . \tag{15}$$

The last unknown parameter B_2 can be determined by the requirement that transverse effects should not occur, i. e. y’ and z’ should not be affected by motion of Σ’ in the X-direction, see equations 11 b and c. This means that

$$B_2 = 1 \quad . \tag{16}$$

Equation 16 was confirmed by Ives and Stilwell (1938) who quantitatively detected the relativistic second-order Doppler shift.

Finally, the transformation, equations 11 a-d, together with the light signal speed in the moving frame Σ’, equation 5, and the light aberration, equations 6 and 7, can be written down as:

$$x' = \gamma\,(x - v \cdot t) \quad , \tag{17 a}$$

$$y' = y \quad , \tag{17 b}$$

$$z' = z \quad , \tag{17 c}$$

$$t' = \frac{1}{\gamma} \cdot t \quad , \tag{17 d}$$

$$u = v \cdot \gamma^2 \quad , \tag{17 e}$$

$$c'(\alpha') = \frac{c}{1 + \frac{v}{c} \cdot \cos(\alpha')} \quad , \tag{17 f}$$

$$\cos(\alpha') = \frac{\cos(\alpha) - \frac{v}{c}}{1 - \frac{v}{c} \cdot \cos(\alpha)} \quad , \tag{17 g}$$

$$\sin(\alpha') = \frac{1}{\gamma} \cdot \frac{\sin(\alpha)}{1 - \frac{v}{c} \cdot \cos(\alpha)} \quad . \tag{17 h}$$

The direction dependent light signal speed in Σ’, equation 17 f, is plotted in polar coordinates in figure 2. This gives an idea of how the velocity v of Σ’ affects the light speed in Σ’, for v/c=0.5 as an example. As can be figured from equation 17 f, the polar plot is an ellipse with one focal point at the origin, eccentricity v/c, semimajor axis $c\gamma^2$, and semiminor axis $c\gamma$. However, it should be noted that this is a theoretical finding which has still to be confirmed experimentally.

The equations 17 a-d form modified Lorentz transformation equations which are in accordance with the “Generalized Galilean Transformation”. Equations 17 g and h express Einstein’s well known light aberration effect (Einstein, 1905).

3. Applications and interpretations

3.1. Clock synchronisation

The initial condition, equation 1 (t=0,t’=0), implies that clocks can be synchronised in both frames. In case of the rest frame, the standard synchronisation procedure by light signals, according to Einstein (Einstein, 1905), can be employed. However, this method is not applicable for internally synchronising clocks in the moving frame. The reason for this complication is the direction-dependent one-way light speed. Instead of the standard synchronisation procedure, a non-standard synchronisation method is then necessary to be applied. A simple external synchronisation is as follows: Both frames shall be thought of being equipped with a rigid arrangement of close-spaced clocks, like a lattice with clocks at the lattice points. Once the rest frame is synchronised (e. g. t=0), the moving frame simultaneously is too (e. g. t'=0), at any spatial position, just by transmission of the time setting from the nearest-by clock in the rest frame. This definition of time synchronisation can be called an absolute one, and makes the theory self-consistent, without internal contradictions.

3.2. Length contraction and time dilation

Length contraction of a moving rigid object as measured in the rest frame was already predicted by Einstein (1905) using the Lorentz transformation. He transformed the equation of the surface of a sphere in the moving frame back to the equation of the surface of a rotational ellipsoid in the rest frame, and found out that the axis of the ellipsoid in the direction of motion was shrunk by a factor of γ. By following his deduction, but using the present transformation, equations 17 a-d, this result is readily confirmed.

Dilation of time in the moving frame, compared to the time elapsed in the rest frame, was predicted using the Lorentz transformation (Einstein, 1905) to amount a factor of 1/γ. The same result is confirmed by using the present transformation, equation 17 d.

In both theories, the rest frame must correctly be chosen to achieve the right result. If the reference frame is chosen to be identical with the rest frame, then the Lorentz transformation yields correct results. However, if the reference frame is chosen to be the moving frame, then the Lorentz transformation yields wrong (inverse) results, i. e. the clocks in the rest frame are slow compared to the clocks in the moving frame. This is the well known clock paradox caused by the symmetrical form of the Lorentz transformation. Contrary to this erroneous result, the present theory, equation 17 d, gives directly the right answer, no matter which of the two frames is chosen to be the reference frame: The clocks in the rest frame are fast compared to those in the moving frame.

3.3. Transformation from one moving frame to another

Three inertial frames are considered: frame no. 1, Σ'(x',y',z';t'), moving with velocity **v**', the rest frame Σ(x,y,z;t), and frame no. 2, Σ''(x'',y'',z'';t''), moving with velocity **v**''. Wanted is the transformation from Σ' to Σ''. The transformation from Σ to Σ'' is given by the equations 17 a-d, what can be written in a general vector formulation as:

$$\mathbf{r}'' = \mathbf{r} + (\gamma''-1)\cdot\frac{\mathbf{r}\cdot\mathbf{v}''}{v''^2}\cdot\mathbf{v}'' - \gamma''\cdot\mathbf{v}''\cdot t \quad , \tag{18}$$

$$t'' = \frac{1}{\gamma''}\cdot t \quad . \tag{19}$$

The transformation from Σ' to Σ is given by the inverse of the transformation from Σ to Σ':

$$\mathbf{r} = \mathbf{r}' - \left(1-\frac{1}{\gamma'}\right)\cdot\frac{\mathbf{r}'\cdot\mathbf{v}'}{v'^2}\cdot\mathbf{v}' + \gamma'\cdot\mathbf{v}'\cdot t' \quad , \tag{20}$$

$$t = \gamma'\cdot t' \quad . \tag{21}$$

The transformation from Σ' to Σ'' is then obtained by plugging the right hand expressions of equations 20 and 21 for **r** and t into equations 18 and 19:

$$\begin{aligned}\mathbf{r}'' = \mathbf{r}' &- \left[\left(1-\frac{1}{\gamma'}\right)\cdot\frac{\mathbf{r}'\cdot\mathbf{v}'}{v'^2} - \gamma'\cdot t'\right]\cdot\left[\mathbf{v}' + (\gamma''-1)\cdot\frac{\mathbf{v}'\cdot\mathbf{v}''}{v''^2}\cdot\mathbf{v}''\right] \\ &+ \left[(\gamma''-1)\cdot\frac{\mathbf{r}'\cdot\mathbf{v}''}{v''^2} - \gamma'\cdot\gamma''\cdot t'\right]\cdot\mathbf{v}''\end{aligned} \quad , \tag{22}$$

$$t'' = \frac{\gamma'}{\gamma''} \cdot t' \quad , \tag{23}$$

where

$$\gamma' = \frac{1}{\sqrt{1-\frac{v'^2}{c^2}}} \quad , \quad \gamma'' = \frac{1}{\sqrt{1-\frac{v''^2}{c^2}}} \quad , \tag{24}$$

and the radius vectors **r**' and **r**'' define the locations in the moving frames Σ' and Σ'', respectively.

3.4. Transformation of velocity and acceleration of a particle

The transformation equations, equations 17 a-d, are given in differential form by:

$$dx' = \gamma \cdot (dx - v \cdot dt) \quad , \tag{25 a}$$

$$dy' = dy \quad , \tag{25 b}$$

$$dz' = dz \quad , \tag{25 c}$$

$$dt' = \frac{1}{\gamma} \cdot dt \quad . \tag{25 d}$$

From equations 25 a-d it is easy to derive the transformation formulae for the first and second time derivatives of the spatial coordinates:

$$\frac{dx'}{dt'} = \gamma^2 \cdot \left(\frac{dx}{dt} - v \right) \quad , \tag{26 a}$$

$$\frac{dy'}{dt'} = \gamma \cdot \frac{dy}{dt} \quad , \tag{26 b}$$

$$\frac{dz'}{dt'} = \gamma \cdot \frac{dz}{dt} \quad , \tag{26 c}$$

$$\frac{d^2x'}{dt'^2} = \gamma^3 \cdot \frac{d^2x}{dt^2} \quad , \tag{27 a}$$

$$\frac{d^2y'}{dt'^2} = \gamma^2 \cdot \frac{d^2y}{dt^2} \quad , \tag{27 b}$$

$$\frac{d^2z'}{dt'^2} = \gamma^2 \cdot \frac{d^2z}{dt^2} \quad . \tag{27 c}$$

Provided that the motion of a particle in the moving frame is given by sufficiently small (non-relativistic) velocities dx'/dt', dy'/dt', and dz'/dt' (all << c), it can be figured from equations 26 a-c that approximately

$$\frac{1}{c} \cdot \frac{dx}{dt} = \frac{v}{c} \quad , \quad \frac{1}{c} \cdot \frac{dy}{dt} = 0 \quad , \quad \frac{1}{c} \cdot \frac{dz}{dt} = 0 \quad . \tag{28}$$

Thus, equations 27 a-c yield the components of the acceleration in the moving frame in form of:

$$\frac{d^2x'}{dt'^2} = \left(1 - \frac{v^2}{c^2} \right)^{-3/2} \cdot \frac{d^2x}{dt^2} = \left[1 - \frac{1}{c^2} \cdot \left(\frac{dx}{dt} \right)^2 \right]^{-3/2} \cdot \frac{d^2x}{dt^2} \quad , \tag{29 a}$$

$$\frac{d^2y'}{dt'^2} = \left(1 - \frac{v^2}{c^2}\right)^{-1} \cdot \frac{d^2y}{dt^2} = \left[1 - \frac{1}{c^2} \cdot \left(\frac{dx}{dt}\right)^2\right]^{-1} \cdot \frac{d^2y}{dt^2} \quad , \tag{29 b}$$

$$\frac{d^2z'}{dt'^2} = \left(1 - \frac{v^2}{c^2}\right)^{-1} \cdot \frac{d^2z}{dt^2} = \left[1 - \frac{1}{c^2} \cdot \left(\frac{dx}{dt}\right)^2\right]^{-1} \cdot \frac{d^2z}{dt^2} \quad , \tag{29 c}$$

where (dx/dt)/c is substituted for v/c (first of equations 28). These results coincide with the results achieved by using the Lorentz transformation (Einstein, 1905).

The components of the driving force **F**' in the moving frame, F'_x, F'_y, F'_z, are equal to the products of the rest mass m of the particle and the second time derivatives of the spatial coordinates, d^2x'/dt'^2, d^2y'/dt'^2, d^2z'/dt'^2, respectively (Newton's law of motion for sufficiently small velocities):

$$F'_x = m \cdot \frac{d^2x'}{dt'^2} \quad , \quad F'_y = m \cdot \frac{d^2y'}{dt'^2} \quad , \quad F'_z = m \cdot \frac{d^2z'}{dt'^2} \quad . \tag{30}$$

The driving force **F**' of an electromagnetic field on a co-moving electro-charged particle as observed in the moving frame is determined in the next section 3.5.

3.5. Maxwell's equations in free space, electric Lorentz force, relativistic law of motion

The Maxwell equations of electrodynamics in empty space are given in the rest frame by:

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad , \tag{31 a}$$

$$\nabla \times \mathbf{B} = \frac{1}{c^2} \cdot \frac{\partial \mathbf{E}}{\partial t} \quad , \tag{31 b}$$

$$\nabla \cdot \mathbf{E} = 0 \quad , \tag{31 c}$$

$$\nabla \cdot \mathbf{B} = 0 \quad , \tag{31 d}$$

where **E** is the electric field vector, **B** is the magnetic field vector, and ∇ denotes the Nabla operator which is defined as the formal vector (∂/∂x,∂/∂y,∂/∂z). Written in components, equations 31 a-d are given by:

$$\begin{pmatrix} \dfrac{\partial E_Z}{\partial y} - \dfrac{\partial E_Y}{\partial z} \\ \dfrac{\partial E_X}{\partial z} - \dfrac{\partial E_Z}{\partial x} \\ \dfrac{\partial E_Y}{\partial x} - \dfrac{\partial E_X}{\partial y} \end{pmatrix} = \begin{pmatrix} -\dfrac{\partial B_X}{\partial t} \\ -\dfrac{\partial B_Y}{\partial t} \\ -\dfrac{\partial B_Z}{\partial t} \end{pmatrix} \quad , \tag{32 a}$$

$$\begin{pmatrix} \dfrac{\partial B_Z}{\partial y} - \dfrac{\partial B_Y}{\partial z} \\ \dfrac{\partial B_X}{\partial z} - \dfrac{\partial B_Z}{\partial x} \\ \dfrac{\partial B_Y}{\partial x} - \dfrac{\partial B_X}{\partial y} \end{pmatrix} = \begin{pmatrix} \dfrac{1}{c^2} \cdot \dfrac{\partial E_X}{\partial t} \\ \dfrac{1}{c^2} \cdot \dfrac{\partial E_Y}{\partial t} \\ \dfrac{1}{c^2} \cdot \dfrac{\partial E_Z}{\partial t} \end{pmatrix} \quad , \tag{32 b}$$

$$\frac{\partial E_X}{\partial x}+\frac{\partial E_Y}{\partial y}+\frac{\partial E_Z}{\partial z}=0 \quad , \tag{32 c}$$

$$\frac{\partial B_X}{\partial x}+\frac{\partial B_Y}{\partial y}+\frac{\partial B_Z}{\partial z}=0 \quad . \tag{32 d}$$

The transformation to the moving frame is achieved through the equations 17 a-d, and by using the chain rule of differentiation:

$$\frac{\partial}{\partial x}=\frac{\partial}{\partial x'}\cdot\frac{\partial x'}{\partial x}+\frac{\partial}{\partial y'}\cdot\frac{\partial y'}{\partial x}+\frac{\partial}{\partial z'}\cdot\frac{\partial z'}{\partial x}+\frac{\partial}{\partial t'}\cdot\frac{\partial t'}{\partial x}=\gamma\cdot\frac{\partial}{\partial x'} \quad , \tag{33 a}$$

$$\frac{\partial}{\partial y}=\frac{\partial}{\partial x'}\cdot\frac{\partial x'}{\partial y}+\frac{\partial}{\partial y'}\cdot\frac{\partial y'}{\partial y}+\frac{\partial}{\partial z'}\cdot\frac{\partial z'}{\partial y}+\frac{\partial}{\partial t'}\cdot\frac{\partial t'}{\partial y}=\frac{\partial}{\partial y'} \quad , \tag{33 b}$$

$$\frac{\partial}{\partial z}=\frac{\partial}{\partial x'}\cdot\frac{\partial x'}{\partial z}+\frac{\partial}{\partial y'}\cdot\frac{\partial y'}{\partial z}+\frac{\partial}{\partial z'}\cdot\frac{\partial z'}{\partial z}+\frac{\partial}{\partial t'}\cdot\frac{\partial t'}{\partial z}=\frac{\partial}{\partial z'} \quad , \tag{33 c}$$

$$\frac{\partial}{\partial t}=\frac{\partial}{\partial x'}\cdot\frac{\partial x'}{\partial t}+\frac{\partial}{\partial y'}\cdot\frac{\partial y'}{\partial t}+\frac{\partial}{\partial z'}\cdot\frac{\partial z'}{\partial t}+\frac{\partial}{\partial t'}\cdot\frac{\partial t'}{\partial t}=-\gamma\cdot v\cdot\frac{\partial}{\partial x'}+\frac{1}{\gamma}\cdot\frac{\partial}{\partial t'} \quad . \tag{33 d}$$

The application of these transformation rules, equations 33 a-d, to equations 32 a-d, yields the Maxwell equations in the moving frame, written in components:

$$\begin{pmatrix} \frac{\partial E_Z}{\partial y'}-\frac{\partial E_Y}{\partial z'}-\gamma\cdot v\cdot\frac{\partial B_X}{\partial x'} \\ \frac{\partial E_X}{\partial z'}-\gamma\cdot\frac{\partial E_Z}{\partial x'}-\gamma\cdot v\cdot\frac{\partial B_Y}{\partial x'} \\ \gamma\cdot\frac{\partial E_Y}{\partial x'}-\frac{\partial E_X}{\partial y'}-\gamma\cdot v\cdot\frac{\partial B_Z}{\partial x'} \end{pmatrix}=\begin{pmatrix} -\frac{1}{\gamma}\cdot\frac{\partial B_X}{\partial t'} \\ -\frac{1}{\gamma}\cdot\frac{\partial B_Y}{\partial t'} \\ -\frac{1}{\gamma}\cdot\frac{\partial B_Z}{\partial t'} \end{pmatrix} \quad , \tag{34 a}$$

$$\begin{pmatrix} \frac{\partial B_Z}{\partial y'}-\frac{\partial B_Y}{\partial z'}+\gamma\cdot v\cdot\frac{1}{c^2}\cdot\frac{\partial E_X}{\partial x'} \\ \frac{\partial B_X}{\partial z'}-\gamma\cdot\frac{\partial B_Z}{\partial x'}+\gamma\cdot v\cdot\frac{1}{c^2}\cdot\frac{\partial E_Y}{\partial x'} \\ \gamma\cdot\frac{\partial B_Y}{\partial x'}-\frac{\partial B_X}{\partial y'}+\gamma\cdot v\cdot\frac{1}{c^2}\cdot\frac{\partial E_Z}{\partial x'} \end{pmatrix}=\begin{pmatrix} \frac{1}{\gamma}\cdot\frac{1}{c^2}\cdot\frac{\partial E_X}{\partial t'} \\ \frac{1}{\gamma}\cdot\frac{1}{c^2}\cdot\frac{\partial E_Y}{\partial t'} \\ \frac{1}{\gamma}\cdot\frac{1}{c^2}\cdot\frac{\partial E_Z}{\partial t'} \end{pmatrix} \quad , \tag{34 b}$$

$$\gamma\cdot\frac{\partial E_X}{\partial x'}+\frac{\partial E_Y}{\partial y'}+\frac{\partial E_Z}{\partial z'}=0 \quad , \tag{34 c}$$

$$\gamma\cdot\frac{\partial B_X}{\partial x'}+\frac{\partial B_Y}{\partial y'}+\frac{\partial B_Z}{\partial z'}=0 \quad . \tag{34 d}$$

Equations 34 a-d can be written as

$$\begin{pmatrix} \frac{\partial E'_Z}{\partial y'} - \frac{\partial E'_Y}{\partial z'} \\ \frac{\partial E'_X}{\partial z'} - \frac{\partial E'_Z}{\partial x'} \\ \frac{\partial E'_Y}{\partial x'} - \frac{\partial E'_X}{\partial y'} \end{pmatrix} = \begin{pmatrix} -\frac{\partial B'_X}{\partial t'} \\ -\frac{\partial B'_Y}{\partial t'} + \frac{v}{c^2} \cdot \frac{\partial E'_Z}{\partial t'} \\ -\frac{\partial B'_Z}{\partial t'} - \frac{v}{c^2} \cdot \frac{\partial E'_Y}{\partial t'} \end{pmatrix} , \qquad (35\ a)$$

$$\begin{pmatrix} \frac{\partial B'_Z}{\partial y'} - \frac{\partial B'_Y}{\partial z'} \\ \frac{\partial B'_X}{\partial z'} - \frac{\partial B'_Z}{\partial x'} \\ \frac{\partial B'_Y}{\partial x'} - \frac{\partial B'_X}{\partial y'} \end{pmatrix} = \begin{pmatrix} \frac{1}{c^2} \cdot \frac{\partial E'_X}{\partial t'} \\ \frac{1}{c^2} \cdot \frac{\partial E'_Y}{\partial t'} + \frac{v}{c^2} \cdot \frac{\partial B'_Z}{\partial t'} \\ \frac{1}{c^2} \cdot \frac{\partial E'_Z}{\partial t'} - \frac{v}{c^2} \cdot \frac{\partial B'_Y}{\partial t'} \end{pmatrix} , \qquad (35\ b)$$

$$\frac{\partial E'_X}{\partial x'} + \frac{\partial E'_Y}{\partial y'} + \frac{\partial E'_Z}{\partial z'} = -v \cdot \left(\frac{\partial B'_Z}{\partial y'} - \frac{\partial B'_Y}{\partial z'} \right) , \qquad (35\ c)$$

$$\frac{\partial B'_X}{\partial x'} + \frac{\partial B'_Y}{\partial y'} + \frac{\partial B'_Z}{\partial z'} = -\frac{v}{c^2} \cdot \left(\frac{\partial E'_Y}{\partial z'} - \frac{\partial E'_Z}{\partial y'} \right) , \qquad (35\ d)$$

where the primed field quantities are defined as:

$$E'_X = E_X \quad , \quad E'_Y = \gamma \cdot (E_Y - v \cdot B_Z) \quad , \quad E'_Z = \gamma \cdot (E_Z + v \cdot B_Y) \quad , \qquad (36\ a)$$

$$B'_X = B_X \quad , \quad B'_Y = \gamma \cdot \left(B_Y + \frac{v}{c^2} \cdot E_Z \right) \quad , \quad B'_Z = \gamma \cdot \left(B_Z - \frac{v}{c^2} \cdot E_Y \right) \quad . \qquad (36\ b)$$

The equations 36 a-b are just the well known Lorentz transformation equations of the electromagnetic field quantities from the rest frame (unprimed) to the moving frame (primed) (Einstein, 1905). The equations 35 a-d and 36 a-b can be written in a general vector form:

$$\nabla' \times \mathbf{E}' = -\frac{\partial \mathbf{B}'}{\partial t'} - \frac{\partial}{\partial t'} \left(\frac{\mathbf{v}}{c^2} \times \mathbf{E}' \right) , \qquad (37\ a)$$

$$\nabla' \times \mathbf{B}' = \frac{\partial}{\partial t'} \left(\frac{\mathbf{E}'}{c^2} \right) - \frac{\partial}{\partial t'} \left(\frac{\mathbf{v}}{c^2} \times \mathbf{B}' \right) , \qquad (37\ b)$$

$$\nabla' \cdot \mathbf{E}' = \nabla' \cdot (\mathbf{v} \times \mathbf{B}') , \qquad (37\ c)$$

$$\nabla' \cdot \mathbf{B}' = -\nabla' \cdot \left(\frac{\mathbf{v}}{c^2} \times \mathbf{E}' \right) , \qquad (37\ d)$$

where **E**’ and **B**’ are given by

$$\mathbf{E}' = \gamma \cdot (\mathbf{E} + \mathbf{v} \times \mathbf{B}) - (\gamma - 1) \cdot \frac{(\mathbf{v} \cdot \mathbf{E}) \cdot \mathbf{v}}{\mathbf{v}^2} , \qquad (37\ e)$$

$$\mathbf{B}' = \gamma \cdot (\mathbf{B} - \frac{1}{c^2} \cdot \mathbf{v} \times \mathbf{E}) - (\gamma - 1) \cdot \frac{(\mathbf{v} \cdot \mathbf{B}) \cdot \mathbf{v}}{\mathbf{v}^2} , \qquad (37\ f)$$

and ∇’ is the Nabla-operator in the moving frame, defined as the formal vector (∂/∂x’,∂/∂y’,∂/∂z’). The equations 37 a-d correspond to the extended notation of Maxwell’s equations, formally including terms of the quasi electric volume charge density, ρ_e', and the quasi magnetic volume charge density, ρ_m', along with their quasi current densities, $\mathbf{j}_e$' and $\mathbf{j}_m$', respectively:

$$\nabla' \times \mathbf{E}' = -\frac{\partial \mathbf{B}'}{\partial t'} - \mu_0 \cdot \mathbf{j}'_m \quad , \tag{38 a}$$

$$\nabla' \times \mathbf{B}' = \frac{\partial}{\partial t'}\left(\frac{\mathbf{E}'}{c^2}\right) + \mu_0 \cdot \mathbf{j}'_e \quad , \tag{38 b}$$

$$\nabla' \cdot \mathbf{E}' = \frac{\rho'_e}{\varepsilon_0} \quad , \tag{38 c}$$

$$\nabla' \cdot \mathbf{B}' = \mu_0 \cdot \rho'_m \quad , \tag{38 d}$$

where

$$\rho'_e = \varepsilon_0 \cdot \nabla' \cdot (\mathbf{v} \times \mathbf{B}') \quad , \quad \rho'_m = -\frac{1}{\mu_0} \cdot \frac{1}{c^2} \cdot \nabla' \cdot (\mathbf{v} \times \mathbf{E}') \quad , \tag{39 a}$$

$$\mathbf{j}'_e = -\frac{1}{\mu_0} \cdot \frac{\partial}{\partial t'}\left(\frac{\mathbf{v}}{c^2} \times \mathbf{B}'\right) \quad , \quad \mathbf{j}'_m = \frac{1}{\mu_0} \cdot \frac{\partial}{\partial t'}\left(\frac{\mathbf{v}}{c^2} \times \mathbf{E}'\right) \quad , \tag{39 b}$$

and

$$\varepsilon_0 \cdot \mu_0 = \frac{1}{c^2} \quad , \tag{40}$$

with ε_0 and μ_0 meaning the permittivity and the permeability of vacuum, respectively. Using the equations 37 a-b, and taking into account that $\mathbf{v}$ is assumed to be constant, the equations 39 a can be rewritten as:

$$\rho'_e = -\varepsilon_0 \cdot \left(\frac{\mathbf{v}}{c^2} \cdot \frac{\partial \mathbf{E}'}{\partial t'}\right) \quad , \tag{41 a}$$

$$\rho'_m = -\frac{1}{\mu_0} \cdot \left(\frac{\mathbf{v}}{c^2} \cdot \frac{\partial \mathbf{B}'}{\partial t'}\right) \quad . \tag{41 b}$$

Thus, in the moving frame, an oscillating electric field generates a quasi electric volume charge density according to equation 41 a, and an oscillating magnetic field generates a quasi magnetic volume charge density according to equation 41 b.

It should be noted that the quasi charge densities ρ_e' and ρ_m' and quasi current densities $\mathbf{j}_e$' and $\mathbf{j}_m$' fulfil the respective continuity equations:

$$\nabla' \cdot \mathbf{j}'_e = -\frac{\partial \rho'_e}{\partial t'} \quad , \quad \nabla' \cdot \mathbf{j}'_m = -\frac{\partial \rho'_m}{\partial t'} \quad . \tag{42}$$

Thus, the requirement of form invariance of physical laws in the theory of special relativity, equations 31 a-d versus equations 38 a-d, is considered to be fulfilled. An exact coincidence cannot be expected under the modified transformation (equations 17 a-d), but rather some extended form, since the rest frame and the moving frame are not equivalent. Therefore, the equations 36 a-b are taken unchanged to be the valid transformation rules for the electromagnetic field quantities, in the new theory as well as in Einstein's special relativity.

The force of an electromagnetic field on a co-moving test particle with electric charge q_e is given in the moving frame by

$$\mathbf{F}' = q_e \cdot \mathbf{E}' \quad . \tag{43}$$

Since the particle is assumed to co-move with the moving frame, i. e. dx'/dt'=0, the velocity v in the equations 36 a-b can be replaced by dx/dt. Then the equations 43, 36 a, 30, and 29 a-c yield:

$$\frac{q_e}{m}\cdot E_X = \left[1-\frac{1}{c^2}\cdot\left(\frac{dx}{dt}\right)^2\right]^{-3/2}\cdot\frac{d^2x}{dt^2} \quad , \tag{44 a}$$

$$\frac{q_e}{m}\cdot\left[1-\frac{1}{c^2}\cdot\left(\frac{dx}{dt}\right)^2\right]^{-1/2}\cdot\left(E_Y-\frac{dx}{dt}\cdot B_Z\right) = \left[1-\frac{1}{c^2}\cdot\left(\frac{dx}{dt}\right)^2\right]^{-1}\cdot\frac{d^2y}{dt^2} \quad , \tag{44 b}$$

$$\frac{q_e}{m}\cdot\left[1-\frac{1}{c^2}\cdot\left(\frac{dx}{dt}\right)^2\right]^{-1/2}\cdot\left(E_Z+\frac{dx}{dt}\cdot B_Y\right) = \left[1-\frac{1}{c^2}\cdot\left(\frac{dx}{dt}\right)^2\right]^{-1}\cdot\frac{d^2z}{dt^2} \quad , \tag{44 c}$$

or

$$q_e\cdot E_X = m\cdot\left[1-\frac{1}{c^2}\cdot\left(\frac{dx}{dt}\right)^2\right]^{-3/2}\cdot\frac{d^2x}{dt^2} \quad , \tag{45 a}$$

$$q_e\cdot\left(E_Y-\frac{dx}{dt}\cdot B_Z\right) = m\cdot\left[1-\frac{1}{c^2}\cdot\left(\frac{dx}{dt}\right)^2\right]^{-1/2}\cdot\frac{d^2y}{dt^2} \quad , \tag{45 b}$$

$$q_e\cdot\left(E_Z+\frac{dx}{dt}\cdot B_Y\right) = m\cdot\left[1-\frac{1}{c^2}\cdot\left(\frac{dx}{dt}\right)^2\right]^{-1/2}\cdot\frac{d^2z}{dt^2} \quad . \tag{45 c}$$

These equations can be written in vector form to obtain the well known relativistic law of motion:

$$q_e\cdot\left(\mathbf{E}+\frac{d\mathbf{r}}{dt}\times\mathbf{B}\right) = \frac{d}{dt}\left[\frac{m\cdot\frac{d\mathbf{r}}{dt}}{\sqrt{1-\frac{1}{c^2}\cdot\left(\frac{d\mathbf{r}}{dt}\right)^2}}\right] \quad , \tag{46}$$

where **r** denotes the radius vector of the particle's location in the rest frame. The expression on the left side of equation 46 is straightforwardly defined to be the force of an electromagnetic field acting on a moving charged particle when measured by an observer resting in the rest frame.

3.6. Electromagnetic wave equations in free space, Doppler frequency effect, normal velocity and signal velocity of light

The classic electromagnetic wave equations in the rest frame can be derived, as is well known, from the corresponding Maxwell equations 31 a-d. Taking the curl of equations 31 a-b, and taking into account equations 31 c-d, yields:

$$\nabla\times(\nabla\times\mathbf{E}) = -\frac{\partial}{\partial t}(\nabla\times\mathbf{B}) = -\frac{1}{c^2}\cdot\frac{\partial^2\mathbf{E}}{\partial t^2} \quad , \tag{47 a}$$

$$\nabla\times(\nabla\times\mathbf{B}) = \frac{1}{c^2}\cdot\frac{\partial}{\partial t}(\nabla\times\mathbf{E}) = -\frac{1}{c^2}\cdot\frac{\partial^2\mathbf{B}}{\partial t^2} \quad . \tag{47 b}$$

On the other hand, by using a general vector identity, and taking into account equations 31 c-d, it follows that

$$\nabla\times(\nabla\times\mathbf{E}) = \nabla\cdot(\nabla\cdot\mathbf{E}) - (\nabla\cdot\nabla)\cdot\mathbf{E} = -(\nabla\cdot\nabla)\cdot\mathbf{E} \quad , \tag{48 a}$$

$$\nabla\times(\nabla\times\mathbf{B}) = \nabla\cdot(\nabla\cdot\mathbf{B}) - (\nabla\cdot\nabla)\cdot\mathbf{B} = -(\nabla\cdot\nabla)\cdot\mathbf{B} \quad . \tag{48 b}$$

The resulting wave equations are received by combining the equations 47 a-b, with equations 48 a-b, respectively:

$$\frac{\partial^2\mathbf{E}}{\partial t^2} = c^2\cdot\left(\frac{\partial^2\mathbf{E}}{\partial x^2} + \frac{\partial^2\mathbf{E}}{\partial y^2} + \frac{\partial^2\mathbf{E}}{\partial z^2}\right) \quad , \tag{49 a}$$

$$\frac{\partial^2\mathbf{B}}{\partial t^2} = c^2\cdot\left(\frac{\partial^2\mathbf{B}}{\partial x^2} + \frac{\partial^2\mathbf{B}}{\partial y^2} + \frac{\partial^2\mathbf{B}}{\partial z^2}\right) \quad . \tag{49 b}$$

The plane wave solutions of these equations, in complex form, are:

$$\mathbf{E} = \mathbf{E}_0\cdot\exp\left[i\cdot 2\cdot\pi\cdot f\cdot\left(t - \frac{1}{c}\cdot(x\cdot n_X + y\cdot n_Y + z\cdot n_Z)\right)\right] \quad , \tag{50 a}$$

$$\mathbf{B} = \mathbf{B}_0\cdot\exp\left[i\cdot 2\cdot\pi\cdot f\cdot\left(t - \frac{1}{c}\cdot(x\cdot n_X + y\cdot n_Y + z\cdot n_Z)\right)\right] \quad , \tag{50 b}$$

where $\mathbf{E}_0$ and $\mathbf{B}_0$ are complex vector amplitudes, f is the frequency, and n_X, n_Y, n_Z are the direction cosines of the plane wave normal **n**. For transformation of the equations 49 a-b to the moving frame, the second spatial and time derivatives must be transformed. This is done through differentiation of equations 33 a-d:

$$\frac{\partial^2}{\partial x^2} = \gamma^2\cdot\frac{\partial^2}{\partial x'^2} \quad , \tag{51 a}$$

$$\frac{\partial^2}{\partial y^2} = \frac{\partial^2}{\partial y'^2} \quad , \tag{51 b}$$

$$\frac{\partial^2}{\partial z^2} = \frac{\partial^2}{\partial z'^2} \quad , \tag{51 c}$$

$$\frac{\partial^2}{\partial t^2} = \gamma^2\cdot v^2\cdot\frac{\partial^2}{\partial x'^2} + \frac{1}{\gamma^2}\cdot\frac{\partial^2}{\partial t'^2} - 2\cdot v\cdot\frac{\partial^2}{\partial x'\,\partial t'} \quad . \tag{51 d}$$

The transformed wave equations are achieved by applying these transformation rules to the equations 49 a-b:

$$\frac{\partial^2\mathbf{E}}{\partial t'^2} = \gamma^2\cdot c^2\cdot\left(\frac{\partial^2\mathbf{E}}{\partial x'^2} + \frac{\partial^2\mathbf{E}}{\partial y'^2} + \frac{\partial^2\mathbf{E}}{\partial z'^2}\right) + 2\cdot\gamma^2\cdot v\cdot\frac{\partial^2\mathbf{E}}{\partial x'\,\partial t'} \quad , \tag{52 a}$$

$$\frac{\partial^2 \mathbf{B}}{\partial t'^2} = \gamma^2 \cdot c^2 \cdot \left(\frac{\partial^2 \mathbf{B}}{\partial x'^2} + \frac{\partial^2 \mathbf{B}}{\partial y'^2} + \frac{\partial^2 \mathbf{B}}{\partial z'^2} \right) + 2 \cdot \gamma^2 \cdot v \cdot \frac{\partial^2 \mathbf{B}}{\partial x' \, \partial t'} \quad . \tag{52 b}$$

This means that any component of the vectors **E** or **B** fulfils the equations 52 a-b. Hence any linear combination of two of these components, e. g. the components of **E**’ and **B**’, equations 36 a-b, does too:

$$\frac{\partial^2 \mathbf{E}'}{\partial t'^2} = \gamma^2 \cdot c^2 \cdot \left(\frac{\partial^2 \mathbf{E}'}{\partial x'^2} + \frac{\partial^2 \mathbf{E}'}{\partial y'^2} + \frac{\partial^2 \mathbf{E}'}{\partial z'^2} \right) + 2 \cdot \gamma^2 \cdot v \cdot \frac{\partial^2 \mathbf{E}'}{\partial x' \, \partial t'} \quad , \tag{53 a}$$

$$\frac{\partial^2 \mathbf{B}'}{\partial t'^2} = \gamma^2 \cdot c^2 \cdot \left(\frac{\partial^2 \mathbf{B}'}{\partial x'^2} + \frac{\partial^2 \mathbf{B}'}{\partial y'^2} + \frac{\partial^2 \mathbf{B}'}{\partial z'^2} \right) + 2 \cdot \gamma^2 \cdot v \cdot \frac{\partial^2 \mathbf{B}'}{\partial x' \, \partial t'} \quad . \tag{53 b}$$

These equations can be rewritten in general form:

$$\frac{\partial^2 \mathbf{E}'}{\partial t'^2} = \gamma^2 \cdot c^2 \cdot \nabla'^2 \mathbf{E}' + 2 \cdot \gamma^2 \cdot (\mathbf{v} \cdot \nabla') \cdot \frac{\partial \mathbf{E}'}{\partial t'} \quad , \tag{54 a}$$

$$\frac{\partial^2 \mathbf{B}'}{\partial t'^2} = \gamma^2 \cdot c^2 \cdot \nabla'^2 \mathbf{B}' + 2 \cdot \gamma^2 \cdot (\mathbf{v} \cdot \nabla') \cdot \frac{\partial \mathbf{B}'}{\partial t'} \quad . \tag{54 b}$$

Equations 54 a-b are the theoretical basis for the evaluation of guided wave experiments to measure the one-way light signal speed in a moving frame (Sfarti, 2007).

The transformed plane wave solutions of these equations are easily obtained through transformation of the corresponding plane wave solutions in the rest frame, equations 50 a-b, by using the inversed equations 17 a-d: After employing some algebra, the final result is:

$$\mathbf{E} = \mathbf{E}_0 \cdot \exp\left[i \cdot 2 \cdot \pi \cdot f \cdot \left(\gamma \cdot (1 - \frac{v}{c} \cdot n_X) \cdot t' - \frac{1}{c} \cdot (x' \cdot \frac{1}{\gamma} \cdot n_X + y' \cdot n_Y + z' \cdot n_Z) \right) \right] \quad , \tag{55 a}$$

$$\mathbf{B} = \mathbf{B}_0 \cdot \exp\left[i \cdot 2 \cdot \pi \cdot f \cdot \left(\gamma \cdot (1 - \frac{v}{c} \cdot n_X) \cdot t' - \frac{1}{c} \cdot (x' \cdot \frac{1}{\gamma} \cdot n_X + y' \cdot n_Y + z' \cdot n_Z) \right) \right] \quad . \tag{55 b}$$

Because of the linearity of the equations 36 a-b, this again means, like above, that the electromagnetic quantities of the moving frame, **E**’ and **B**’, also fulfil the equations 55 a-b:

$$\mathbf{E}' = \mathbf{E}'_0 \cdot \exp\left[i \cdot 2 \cdot \pi \cdot f \cdot \left(\gamma \cdot (1 - \frac{v}{c} \cdot n_X) \cdot t' - \frac{1}{c} \cdot (x' \cdot \frac{1}{\gamma} \cdot n_X + y' \cdot n_Y + z' \cdot n_Z) \right) \right] \quad , \tag{56 a}$$

$$\mathbf{B}' = \mathbf{B}'_0 \cdot \exp\left[i \cdot 2 \cdot \pi \cdot f \cdot \left(\gamma \cdot (1 - \frac{v}{c} \cdot n_X) \cdot t' - \frac{1}{c} \cdot (x' \cdot \frac{1}{\gamma} \cdot n_X + y' \cdot n_Y + z' \cdot n_Z) \right) \right] \quad . \tag{56 b}$$

On the other hand, the mathematical standard form of a plane electromagnetic wave in the moving frame is given by:

$$\mathbf{E}' = \mathbf{E}'_0 \cdot \exp\left[i \cdot 2 \cdot \pi \cdot f' \cdot \left(t' - \frac{1}{c'_n} \cdot (x' \cdot n'_X + y' \cdot n'_Y + z' \cdot n'_Z) \right) \right] \quad , \tag{57 a}$$

$$\mathbf{B}' = \mathbf{B}'_0 \cdot \exp\left[i \cdot 2 \cdot \pi \cdot f' \cdot \left(t' - \frac{1}{c'_n} \cdot (x' \cdot n'_X + y' \cdot n'_Y + z' \cdot n'_Z) \right) \right] \quad , \qquad (57\ b)$$

where f' is the wave frequency in the moving frame, and c'_n denotes the normal (phase) speed of light in the moving frame, in the direction of the plane wave normal **n**' given by the direction cosines, n'_X, n'_Y, n'_Z. Comparison of equations 56 a-b with equations 57 a-b, respectively, yields:

$$f' = \gamma \cdot f \cdot (1 - \frac{v}{c} \cdot n_X) \quad , \qquad (58\ a)$$

$$\frac{f'}{c'_n} \cdot n'_X = \frac{1}{\gamma} \cdot \frac{f}{c} \cdot n_X \quad , \qquad (58\ b)$$

$$\frac{f'}{c'_n} \cdot n'_Y = \frac{f}{c} \cdot n_Y \quad , \qquad (58\ c)$$

$$\frac{f'}{c'_n} \cdot n'_Z = \frac{f}{c} \cdot n_Z \quad . \qquad (58\ d)$$

For the sake of simplicity, a two spatial dimensions formulation is used (see figures 1 a and b), without restriction of generality. This means:

$$n_X = \cos(\alpha)\ ,\ n_Y = \sin(\alpha)\ ,\ n_Z = 0\ ,\ n'_X = \cos(\alpha'_n)\ ,\ n'_Y = \sin(\alpha'_n)\ ,\ n'_Z = 0 \quad , \qquad (59)$$

where α is the angle between **n** and the X-axis, and α'_n is the angle between **n**' and the X'-axis. It follows from equations 58 a-d, taking into account equation 59:

$$f' = \gamma \cdot f \cdot (1 - \frac{v}{c} \cdot \cos(\alpha)) \quad , \qquad (60\ a)$$

$$\frac{f'}{c'_n} \cdot \cos(\alpha'_n) = \frac{1}{\gamma} \cdot \frac{f}{c} \cdot \cos(\alpha) \quad , \qquad (60\ b)$$

$$\frac{f'}{c'_n} \cdot \sin(\alpha'_n) = \frac{f}{c} \cdot \sin(\alpha) \quad . \qquad (60\ c)$$

Equation 60 a expresses the relativistic Doppler effect, in agreement with Einstein (1905). Furthermore, the equations 60 a-c can be solved for c'_n, $\cos(\alpha'_n)$, and $\sin(\alpha'_n)$:

$$c'_n = c \cdot \gamma \cdot \sqrt{\frac{1 - \frac{v}{c} \cdot \cos(\alpha)}{1 + \frac{v}{c} \cdot \cos(\alpha)}} \quad , \qquad (61\ a)$$

$$\cos(\alpha'_n) = \frac{1}{\gamma} \cdot \frac{1}{\sqrt{1 - \frac{v^2}{c^2} \cdot \cos^2(\alpha)}} \cdot \cos(\alpha) \quad , \qquad (61\ b)$$

$$\sin(\alpha'_n) = \frac{1}{\sqrt{1 - \frac{v^2}{c^2} \cdot \cos^2(\alpha)}} \cdot \sin(\alpha) \quad . \qquad (61\ c)$$

It is interesting to note that the equations 61 a-c do not coincide with the equations 17 f-h, respectively. This means, there are two light speeds, the signal speed c' and the normal speed c'_n, and two respective direction angles, the beam direction α' and the direction α'_n normal to planes of constant phase. This can be understood by assigning the space in the moving frame anisotropic quality. Following Max Born's treatment of the optics of anisotropic media (crystals) (Born, 1985), the signal speed of light can be interpreted as the speed at which light energy is transported along the beam direction given by the angle α' (ray velocity). This interpretation is put to a simple plausibility test: The ratio c'/c'_n is determined in two independent ways. Firstly, c'/c'_n is calculated using the equations 17 f-h and 61 a-c. The result is:

$$\frac{c'}{c'_n} = \frac{1}{\cos(\alpha' - \alpha'_n)} \quad . \tag{62}$$

Secondly, c'/c'_n is determined by the ratio of the wave lengths, λ'/λ'_n:

$$\frac{c'}{c'_n} = \frac{\lambda'}{\lambda'_n} \quad , \tag{63}$$

where λ'/λ'_n can be directly seen in figure 3 to be equal to the right side of equation 62.
In the next section 3.7 it is confirmed that the direction of the light signal in the moving frame (angle α') really agrees with the direction of the light energy flux density, i. e. the Poynting vector.

3.7. Electromagnetic energy transport in free space in a moving frame

The propagation of light in free space in a moving frame is governed by the transformed Maxwell equations 37 a-d. The mathematical form of a plane electromagnetic wave in a moving frame is given by equations 57 a-b. Plugging equations 57 a-b into equations 37 a-b yields in terms of complex field vectors:

$$\frac{1}{c'_n} \cdot \mathbf{n}' \times \mathbf{E}' = (\mathbf{B}' + \frac{\mathbf{v}}{c^2} \times \mathbf{E}') \quad , \tag{64 a}$$

$$\frac{1}{c'_n} \cdot \mathbf{n}' \times \mathbf{B}' = -\frac{1}{c^2} (\mathbf{E}' - \mathbf{v} \times \mathbf{B}') \quad . \tag{64 b}$$

Applying again for simplicity a two dimensional spatial formulation by setting $n'_Z=0$ (equation 59), and orienting the positive X(X')-axis in the direction of **v**, it follows in terms of complex field vector components, without restriction of generality:

$$\frac{1}{c'_n} \cdot \begin{pmatrix} E'_Z \cdot n'_Y \\ -E'_Z \cdot n'_X \\ E'_Y \cdot n'_X - E'_X \cdot n'_Y \end{pmatrix} = \begin{pmatrix} B'_X \\ B'_Y \\ B'_Z \end{pmatrix} + \frac{v}{c^2} \cdot \begin{pmatrix} 0 \\ -E'_Z \\ E'_Y \end{pmatrix} \quad , \tag{65 a}$$

$$\frac{1}{c'_n} \cdot \begin{pmatrix} B'_Z \cdot n'_Y \\ -B'_Z \cdot n'_X \\ B'_Y \cdot n'_X - B'_X \cdot n'_Y \end{pmatrix} = -\frac{1}{c^2} \cdot \begin{pmatrix} E'_X \\ E'_Y \\ E'_Z \end{pmatrix} + \frac{v}{c^2} \cdot \begin{pmatrix} 0 \\ -B'_Z \\ B'_Y \end{pmatrix} \quad . \tag{65 b}$$

Solving equation 65 a for B'_X, B'_Y, B'_Z, and plugging the result into equation 65 b leads to a system of linear homogeneous equations with unknowns E'_X, E'_Y, E'_Z:

$$E'_X \cdot \left[\frac{1}{c^2} - \left(\frac{n'_Y}{c'_n} \right)^2 \right] + E'_Y \cdot \left[\frac{n'_Y}{c'_n} \cdot \left(\frac{n'_X}{c'_n} - \frac{v}{c^2} \right) \right] = 0 \quad , \tag{66 a}$$

$$E'_X \cdot \left[\frac{n'_Y}{c'_n} \cdot \left(\frac{n'_X}{c'_n} - \frac{v}{c^2}\right)\right] + E'_Y \cdot \left[\frac{1}{c^2} - \left(\frac{n'_X}{c'_n} - \frac{v}{c^2}\right)^2\right] = 0 \quad , \tag{66 b}$$

$$E'_Z \cdot \left[\frac{1}{c^2} - \left(\frac{n'_Y}{c'_n}\right)^2 - \left(\frac{n'_X}{c'_n} - \frac{v}{c^2}\right)^2\right] = 0 \quad . \tag{66 c}$$

This system of equations has non-trivial solutions $(E'_X, E'_Y, E'_Z) \neq (0,0,0)$ only if the determinant of the coefficient matrix is zero:

$$\begin{vmatrix} \left[\frac{1}{c^2} - \left(\frac{n'_Y}{c'_n}\right)^2\right] & \left[\frac{n'_Y}{c'_n} \cdot \left(\frac{n'_X}{c'_n} - \frac{v}{c^2}\right)\right] & 0 \\ \left[\frac{n'_Y}{c'_n} \cdot \left(\frac{n'_X}{c'_n} - \frac{v}{c^2}\right)\right] & \left[\frac{1}{c^2} - \left(\frac{n'_X}{c'_n} - \frac{v}{c^2}\right)^2\right] & 0 \\ 0 & 0 & \left[\frac{1}{c^2} - \left(\frac{n'_Y}{c'_n}\right)^2 - \left(\frac{n'_X}{c'_n} - \frac{v}{c^2}\right)^2\right] \end{vmatrix} = 0 \quad . \tag{67 a}$$

That is the case if either

$$\begin{vmatrix} \left[\frac{1}{c^2} - \left(\frac{n'_Y}{c'_n}\right)^2\right] & \left[\frac{n'_Y}{c'_n} \cdot \left(\frac{n'_X}{c'_n} - \frac{v}{c^2}\right)\right] \\ \left[\frac{n'_Y}{c'_n} \cdot \left(\frac{n'_X}{c'_n} - \frac{v}{c^2}\right)\right] & \left[\frac{1}{c^2} - \left(\frac{n'_X}{c'_n} - \frac{v}{c^2}\right)^2\right] \end{vmatrix} = 0 \tag{67 b}$$

or

$$\left[\frac{1}{c^2} - \left(\frac{n'_Y}{c'_n}\right)^2 - \left(\frac{n'_X}{c'_n} - \frac{v}{c^2}\right)^2\right] = 0 \tag{67 c}$$

is fulfilled.

These conditions, equation 67 b and equation 67 c, are equivalent: They lead to the same quadratic equation for determining the normal light velocity c'_n:

$$\frac{1}{c^2} \cdot \left(1 - \frac{v^2}{c^2}\right) \cdot c'^2_n + 2 \cdot n'_X \cdot \frac{v}{c^2} \cdot c'_n - 1 = 0 \quad . \tag{68}$$

The solution is:

$$c'_n = \frac{c}{1 - \frac{v^2}{c^2}} \cdot \left(\sqrt{1 - \frac{v^2}{c^2} \cdot n'^2_Y} - \frac{v}{c} \cdot n'_X\right) \quad , \tag{69 a}$$

or in terms of the direction angle α'_n:

$$c'_n = \frac{c}{1-\frac{v^2}{c^2}} \cdot \left(\sqrt{1-\frac{v^2}{c^2} \cdot \sin^2 \alpha'_n} - \frac{v}{c} \cdot \cos \alpha'_n \right) \qquad . \qquad (69\ b)$$

A second solution, with the root in the equations 69 a and 69 b taken negative, however, has no meaning because it gives negative values of c'_n. Equation 69 b coincides with the result obtained by transformation of the plane wave solution, equations 61 a-c of section 3.6, when the angle α is eliminated.

In the following, the ray direction in the moving frame will be determined: The light ray points per definition in the direction of the energy transport. It is assumed that the energy flux density in the moving frame is still given by the Poynting vector defined as

$$\mathbf{S}' = \mathbf{E}' \times \mathbf{H}' = \frac{1}{\mu_0} \cdot \mathbf{E}' \times \mathbf{B}' \qquad , \qquad (70)$$

with $\mathbf{H}'=\mathbf{B}'/\mu_0$ being the complex magnetic field intensity. For determining the direction of the Poynting vector, it is sufficient to employ its time average which is given for time-harmonic fields by

$$\overline{\mathbf{S}}' = \frac{1}{2} \cdot \mathrm{Re}(\mathbf{E}' \times \mathbf{H}'^*) = \frac{1}{2 \cdot \mu_0} \cdot \mathrm{Re}(\mathbf{E}' \times \mathbf{B}'^*) \qquad , \qquad (71)$$

where $\mathbf{H}'^*$ and $\mathbf{B}'^*$ denote the complex conjugate quantities of $\mathbf{H}'$ and $\mathbf{B}'$, respectively, and Re means the real part of a complex quantity. E'_Z can be chosen freely, equations 66 c and 67 c. E'_X and E'_Y however are linked together according to equations 66 a and 66 b, so that only one of these variables can be arbitrarily chosen. Equations 66 a and b yield

$$\frac{E'_X}{c^2} = \left[E'_X \cdot \frac{n'_Y}{c'_n} - E'_Y \cdot \left(\frac{n'_X}{c'_n} - \frac{v}{c^2} \right) \right] \cdot \frac{n'_Y}{c'_n} \qquad , \qquad (72\ a)$$

$$\frac{E'_Y}{c^2} = -\left[E'_X \cdot \frac{n'_Y}{c'_n} - E'_Y \cdot \left(\frac{n'_X}{c'_n} - \frac{v}{c^2} \right) \right] \cdot \left(\frac{n'_X}{c'_n} - \frac{v}{c^2} \right) \qquad , \qquad (72\ b)$$

which leads to a simple relation between E'_X and E'_Y:

$$E'_X = -\frac{n'_Y}{n'_X - \frac{v}{c^2} \cdot c'_n} \cdot E'_Y \qquad . \qquad (73)$$

E'_Y shall be chosen as the independent quantity. Thus, the Poynting vector is evaluated by plugging B'_X, B'_Y, and B'_Z obtained from equation 65 a into equation 71, and then substituting the right hand side of equation 73 for E'_X in the resulting formula. The final result, in terms of components, is:

$$\overline{S}'_X = \frac{1}{2 \cdot \mu_0} \cdot \mathrm{Re} \left\{ E'_Y \cdot E'^*_Y \cdot \left[1 + \left(\frac{n'_Y \cdot c^2}{n'_X \cdot c^2 - v \cdot c'_n} \right)^2 \right] + E'_Z \cdot E'^*_Z \right\} \cdot \left(\frac{n'_X}{c'_n} - \frac{v}{c^2} \right) \qquad , \qquad (74\ a)$$

$$\overline{S}'_Y = \frac{1}{2 \cdot \mu_0} \cdot \mathrm{Re} \left\{ E'_Y \cdot E'^*_Y \cdot \left[1 + \left(\frac{n'_Y \cdot c^2}{n'_X \cdot c^2 - v \cdot c'_n} \right)^2 \right] + E'_Z \cdot E'^*_Z \right\} \cdot \frac{n'_Y}{c'_n} \qquad , \qquad (74\ b)$$

$$\overline{S}'_Z = 0 \quad . \tag{74 c}$$

The angle between the (time-averaged) Poynting vector and the unit vector **n'** normal to planes of constant phase is then, taking credit of the equations 68 and 69 a, given by:

$$\cos(\mathbf{n}', \overline{\mathbf{S}}') = \frac{\mathbf{n}' \cdot \overline{\mathbf{S}}'}{\sqrt{\overline{\mathbf{S}}' \cdot \overline{\mathbf{S}}'}} = \sqrt{1 - \frac{v^2}{c^2} \cdot n_Y'^2} = \sqrt{1 - \frac{v^2}{c^2} \cdot \sin^2(\alpha_n')} \quad . \tag{75}$$

This expression of cos(**n'**,**S'**) coincides with cos(α'-α'$_n$) of equation 62, taking into account the equations 61 a-c and 17 f-g. That means, the direction of the light signal is indeed identical with the direction of the Poynting vector, as expected. This finding indicates the consistency of the theory.

3.8. Another interpretation of Maxwell's equations in anisotropic space

The interpretation of Maxwell's equations in free space in a moving frame, equations 37 a-d, is not unequivocal. Quasi-electric charges and quasi-magnetic charges (equations 38 a-d) as they occur with the Lorentz transformation of electromagnetic fields, equations 37 e-f, can be avoided by an additional second transformation of the field quantities:

$$\mathbf{E}'' = \mathbf{E}' \quad , \tag{76 a}$$

$$\mathbf{B}'' = \mathbf{B}' + \frac{1}{c^2} \cdot (\mathbf{v} \times \mathbf{E}') \quad , \tag{76 b}$$

so that the total transformation is given by

$$\mathbf{E}'' = \gamma \cdot (\mathbf{E} + \mathbf{v} \times \mathbf{B}) - (\gamma - 1) \cdot \frac{(\mathbf{v} \cdot \mathbf{E}) \cdot \mathbf{v}}{\mathbf{v}^2} \quad , \tag{77 a}$$

$$\mathbf{B}'' = \frac{1}{\gamma} \cdot \mathbf{B} + \frac{1}{c^2} \cdot \frac{\gamma}{\gamma + 1} \cdot (\mathbf{v} \cdot \mathbf{B}) \cdot \mathbf{v} \quad . \tag{77 b}$$

The inversion of equations 76 a-b gives:

$$\mathbf{E}' = \mathbf{E}'' \quad , \quad \mathbf{B}' = \mathbf{B}'' - \frac{1}{c^2} \cdot (\mathbf{v} \times \mathbf{E}'') \quad . \tag{78}$$

Substitution of these expressions for **E'** and **B'** in equations 37 a-d yields:

$$\nabla' \times \mathbf{E}'' = -\frac{\partial \mathbf{B}''}{\partial t'} \quad , \tag{79 a}$$

$$\nabla' \times \mathbf{H}'' = \frac{\partial \mathbf{D}''}{\partial t'} \quad , \tag{79 b}$$

$$\nabla' \cdot \mathbf{D}'' = 0 \quad , \tag{79 c}$$

$$\nabla' \cdot \mathbf{B}'' = 0 \quad , \tag{79 d}$$

with

$$\mathbf{D}'' = \varepsilon_0 \cdot \mathbf{E}'' - \varepsilon_0 \cdot \mathbf{v} \times \mathbf{B}'' + \frac{\varepsilon_0}{c^2} \cdot \mathbf{v} \times (\mathbf{v} \times \mathbf{E}'') \quad , \tag{79 e}$$

$$\mathbf{H}'' = \frac{1}{\mu_0} \cdot \mathbf{B}'' - \frac{1}{\mu_0} \cdot \frac{1}{c^2} \cdot \mathbf{v} \times \mathbf{E}'' \quad , \tag{79 f}$$

where **D**'' denotes the electric displacement field, and **H**'' means the magnetic field intensity, in the moving frame. These formulae, equations 79 a-f, correspond to the Maxwell's equations in anisotropic space by T. Chang (Chang, 1979). Quasi-electric and quasi-magnetic charges are avoided, but instead of the usual vacuum relations, $\mathbf{D}''=\varepsilon_0\mathbf{E}''$ and $\mathbf{H}''=\mu_0^{-1}\mathbf{B}''$, there formally appear unusual "quasi-material properties" of the vacuum, equations 79 e-f. The interpretations of the two formulations, equations 37 a-d, and equations 79 a-f, are quite different. In the first case, an observer in the moving frame would measure the electromagnetic field **E**', **B**', in the second case he would observe **E**'', **B**''. This second interpretation does not affect the formula for the electric Lorentz force since **E**'' equals **E**', but its magnetic counterpart differs, equation 76 b: An electric field in the rest frame does not induce a magnetic field in the moving frame, equation 77 b.

The Poynting vector is no distinguishing feature of the two interpretations because it is invariant under the transformation equations 76 a-b:

$$\mathbf{S}'' = \mathbf{E}'' \times \mathbf{H}'' = \mathbf{E}' \times \frac{\mathbf{B}'}{\mu_0} = \mathbf{S}' \quad , \tag{80}$$

what can readily be proved by employing equation 79 f. Thus, experimental evidence seems to be the only way to decide between the two interpretations.

4. Summary and conclusions

Einstein's theory of special relativity states that there is no preferred frame, i. e. any inertial frame can be chosen to be the rest frame. This is not satisfactory since it yields opposite results dependant on the choice of the rest frame, which is due to the symmetry of the Lorentz transformation. This paradox (clock paradox) cannot be resolved by the standard theory of special relativity itself but needs supplement from the theory of general relativity along with the equivalence principle (equivalence of acceleration and gravity).

In order to make the special relativity an independent theory, the old "ether" theory was reconsidered in the new form of the "Generalized Galilean Transformation". This means to back off repudiating Einstein's work.

In contrast, the present paper is meant to step forward by fully using Einstein's light beam procedure, and even extending it. This is accomplished by the requirement that, for completeness, the coordinate points resting in both of the two inertial frames, not only those ones resting in the moving frame, are to fulfil the transformation formulae. Consequently, more open parameters are needed to meet all of these requirements. This is accomplished by (i) allowing the ratio of the velocities of the frames relative to each other to be an open fit parameter, not necessarily equal to 1, and (ii) by allowing the light signal speed c' in one of the two inertial frames, the "moving frame", to deviate from the constant c. The other frame, the "rest frame", is, contrary to Einstein's statement above, but in accordance with the "Generalized Galilean Transformation", assumed to be a preferred frame defined to be motionless, in which the light signal speed is equal to the constant c in all directions. As a consequence, it turns out that (i) the light signal speed in the moving frame must be direction dependent, (ii) that the relative velocities of the frames must be different, and (iii) that the transformation of time must be independent of the spatial coordinates.

Some applications are carried out to check the new transformation: Light aberration effect, length contraction, time dilation, relativistic law of motion, electric Lorentz force, and electromagnetic Doppler frequency are found to be equal to Einstein's results. However, the clock paradox is avoided. Another effect is rather puzzling at a first glance: The transformation of the plane wave solution of the electromagnetic wave equation yields a light speed and its propagation direction which differ from those found for the light signal. This additionally occurring light speed is interpreted as to mean the phase velocity or normal velocity of light. In contrast, the light signal speed is supposed to be the velocity of electromagnetic energy transportation, i. e. the ray velocity. The existence of these two speeds of light in different directions is obviously due to the optical anisotropy of the moving frame. Another result noteworthy is the formal generation of quasi electric and quasi magnetic volume charge densities by an oscillating electromagnetic field in a moving frame. It is shown how to avoid this effect applying a different interpretation of Maxwell's equations in anisotropic space, but then there formally appears an unusual electromagnetic behaviour of the vacuum.

In real cases, it cannot be expected that one of the frames is the preferred rest frame. In these cases, the adequate transformation formulae are found by a first transformation from one of the moving frames to the rest frame, followed by a second transformation from the rest frame to the other moving frame. However, this procedure can only be evaluated if the velocities of the two moving frames relative to the rest frame are known. A measurement of the velocity of a frame could in principle be carried out by an observer resting in that frame through measurement of the light signal speed in different directions.

Finally, it is surprising that the present theory, although based on Einstein's special relativity, at last approves the "Generalized Galilean transformation", and hence supports any application of it. In other words, the "Generalized Galilean transformation" which was up to now just an ad hoc assumption appears to be a result within the framework of the present deductive theory: The "rest frame" is interpreted to be a "preferred frame", and corresponds to the fiction of the "ether".

Table 1. Transformation from rest frame Σ to moving inertial frame Σ' in standard configuration (Einstein, 1905): Special points are chosen for P' co-moving with the moving frame Σ' (figure 1 a) and for P resting in the rest frame Σ (figure 1 b). The coordinates of each point event, P' as well as P, are expressed in both frames, Σ and Σ', and plugged into the transformation formulae, equations 4 a-d, in order to find out their yet unknown coefficients C_2, A_4, D_4, A_1, D_1, step by step (B_2 remains open). Current results of each step are given in the right column of the table. v is the velocity of Σ' as observed in Σ, -u is the velocity of Σ as observed in Σ'. c denotes the isotropic light signal speed in Σ, c'(α') means the anisotropic light signal speed in Σ' (step 10). γ is defined as $\gamma=(1-v^2/c^2)^{-1/2}$.

Deduction of transformation				
step no.	point event			results obtained through plugging the coordinates of the point event into the transformation formulae, equations 4 a-d:
	category	coordinates		
	O, P: resting in rest frame Σ. O', P': co-moving with moving frame Σ′	t ; x, y, z as observed in rest frame Σ	t' ; x', y', z' as observed in moving frame Σ′	$x' = A_1 x + A_4 t$ $y' = B_2 y - C_2 z$ $z' = C_2 y + B_2 z$ $t' = D_1 x + D_4 t$
1	P'	$t;x,y,0$	$t';x',y',0$	$C_2 = 0$
2	O'	$t;vt,0,0$	$t';0,0,0$	$A_4 = -A_1 \cdot v$
3	P'	$t;\ vt, \sqrt{c^2-v^2}\ t,\ 0$	$t';\ 0,\ c'(90°)\ t',\ 0$	$D_4 = \frac{B_2}{c'(90°)} \cdot \frac{c}{\gamma} - D_1 \cdot v$
4	P'	$t;ct,0,0$	$t';c'(0°)\ t',0,0$	$A_1 = 2 \cdot \frac{c'(0°) \cdot c'(180°)}{c'(0°)+c'(180°)} \cdot \frac{B_2 \cdot \gamma}{c'(90°)}$
5	P'	$t;-ct,0,0$	$t';-c'(180°)\ t',0,0$	$D_1 = -\left(\frac{c'(0°)-c'(180°)}{c'(0°)+c'(180°)} + \frac{v}{c}\right) \cdot \frac{B_2 \cdot \gamma}{c'(90°)}$
6	O	$t;0,0,0$	$t';-ut',0,0$	$A_1 = -D_1 \cdot u + \frac{B_2 \cdot c}{c'(90°)} \cdot \frac{u}{v} \cdot \frac{1}{\gamma}$
7	P	$t;0,\sqrt{c^2-v^2}\ t,0$	$t';-ut',c'(90°)\ t',0$	$D_1 = 0$
8	P	$t;ct-vt,0,0$	$t';c'(0°)\ t'-ut',0,0$	$c'(0°) = \frac{u}{v} \cdot (c-v)$
9	P	$t;-ct-vt,0,0$	$t';-c'(180°)\ t'-ut',0,0$	$c'(180°) = \frac{u}{v} \cdot (c+v)$
10	P' or P	$t;\ c\ t \cos(\alpha),$ $c\ t \sin(\alpha),0$	$t';c'(\alpha')\ t' \cos(\alpha'),$ $c'(\alpha')\ t' \sin(\alpha'),0$	$c'(\alpha') \cos(\alpha') = \frac{c^2}{c'(90°)} \cdot \frac{u}{v} \cdot \left(\cos(\alpha) - \frac{v}{c}\right)$ $c'(\alpha') \sin(\alpha') = c \cdot \gamma \sin(\alpha)$

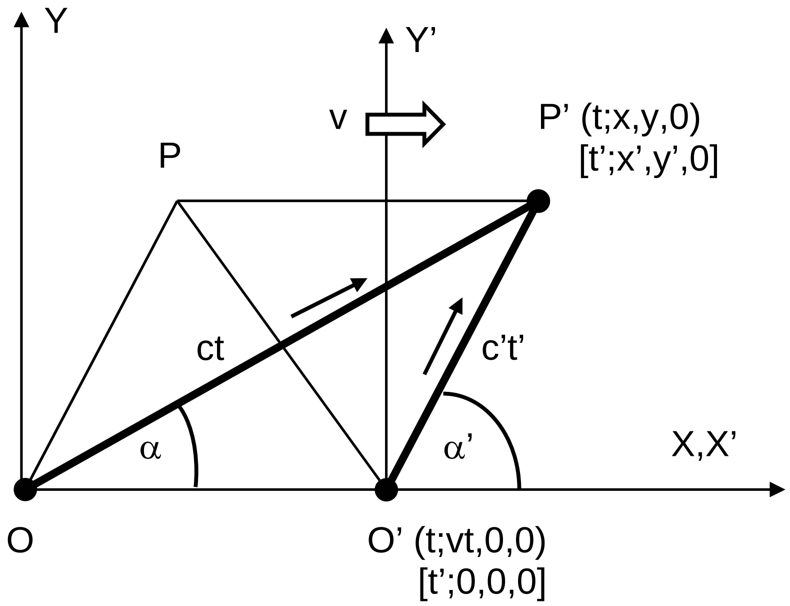

Figure 1 a

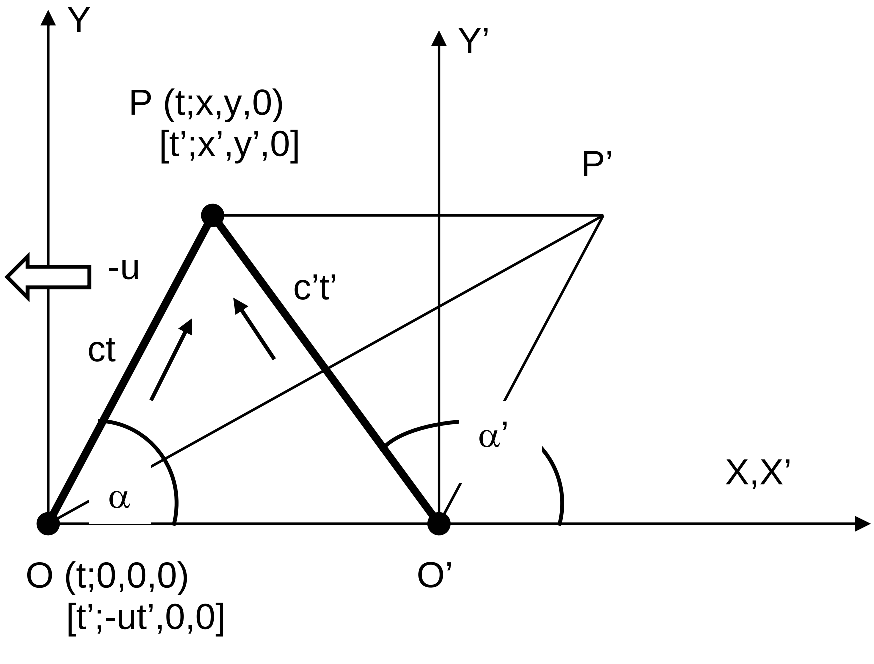

Figure 1 b

Figure 1. Thought experiment (Einstein, 1905): Light is emitted from the origin O' of the inertial frame Σ' moving with uniform velocity v as observed in the rest frame Σ. Σ and Σ' are Cartesian coordinate systems, with the axes X, Y, Z of Σ parallel to the axes X', Y', Z' of Σ', respectively. The plane of drawing shows the X/Y(X'/Y')-plane. The initial condition is: t = 0, x = 0, y = 0, z = 0, t' = 0, x' = 0, y' = 0, z' = 0.

a) A representative point P' is co-moving with Σ', with its coordinates expressed in both frames, Σ and Σ'.

b) A representative point P is resting with Σ, with its coordinates expressed in both frames, Σ and Σ'. -u is the velocity of Σ observed in Σ'.

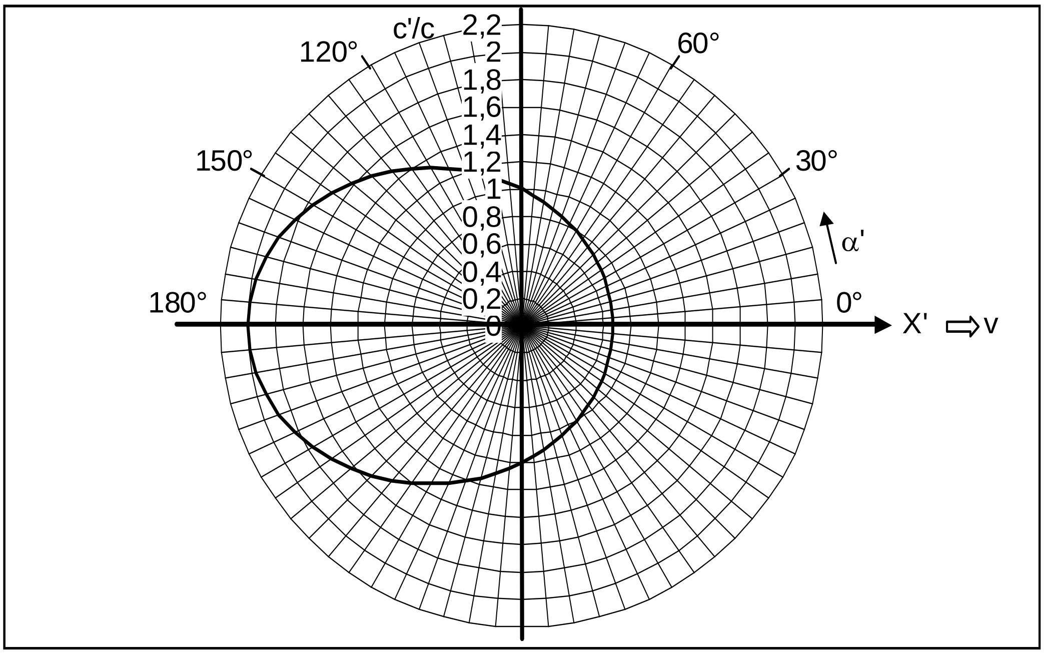

Figure 2. Anisotropy of one-way light signal speed c’ as observed in a moving frame Σ’: c’/c is depicted as function of the angle α’ between light beam in Σ’ and velocity **v** of Σ’, for v/c=0.5 as an example (c = isotropic light signal speed in the rest frame Σ).

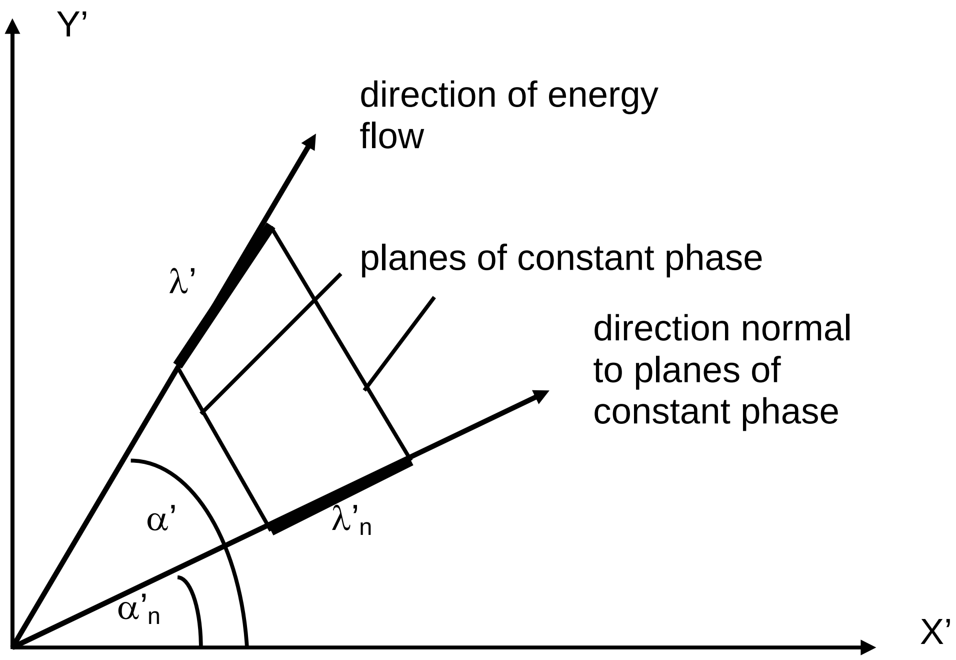

Figure 3. Plausibility test of the definition of two different light speeds, the velocity of energy flow (ray velocity, signal speed), c’, and the normal (phase) velocity, c'_n: This ambiguity is caused by different wave-lengths in two propagation directions: The ratio c'/c'_n is equal to $\lambda'/\lambda'_n=1/\cos(\alpha'-\alpha'_n)$. The plane of drawing displays the (x’,y’)-plane of the moving frame.

References

1. Einstein A. Zur Elektrodynamik bewegter Körper. Annalen der Physik 1905;17:891-921.
2. Einstein A. Dialog über Einwände gegen die Relativitätstheorie. Die Naturwissenschaften 1918;48:697-702.
3. Robertson H P. Postulate versus Observation in the Special Theory of Relativity. Reviews of Modern Physics 1949;21:378-382.
4. Mansouri R, Sexl R U. A test theory of special relativity. I: Simultaneity and clock synchronization. General Relativity and Gravitation 1977;8 (7):497-513.
5. Chang T. Maxwell's equations in anisotropic space. Physics Letters 1979;70 A(1):1-2.
6. Sfarti A. Wave Guide Astronomical Experiments for One Way Light Speed Isotropy Measurements. Nature and Science 2007;5:66-71.
7. Reichenbach H. Axiomatization of the Theory of Relativity. University of California Press 1969.
8. Michelson A A, Morley E W. On the Relative Motion of the Earth and the Luminiferous Ether. American Journal of Science 1887;34:333-345.
9. Kennedy R J, Thorndike E M. Experimental Establishment of the Relativity of Time. Physical Review. 1932;42:400-418.
10. Ives H E, Stilwell G R. An experimental study of the rate of a moving clock. Journal of the Optical Society of America 1938;28:215. 1941;31:369.
11. Born M. Optik. Berlin, Heidelberg, New York, Tokyo: Springer. 1985; reprint of 3rd edition:218-223.

Date: January 17, 2011