Integration of Interval TOPSIS and Fuzzy AHP for Technology Selection

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Abstract: Selecting the right technology is always a difficult task for decision-makers. Technologies have varied strengths and weaknesses which require careful assessment by the purchasers. The purpose of this paper is applying a new integrated method to technology selection. Proposed approach is based on fuzzy AHP and Interval TOPSIS methods. FAHP method is used in determining the weights of the criteria by decision makers and then rankings of technologies are determined by Interval TOPSIS method. A numerical example demonstrates the application of the proposed method.

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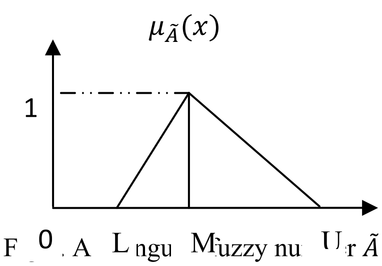
1. Introduction

Selection of technologies is one of the most challenging decision making areas the management of a company encounters. It is difficult to clarify the right technology alternatives because the number of technologies is increasing and the technologies are becoming more and more complex. However, right technologies could create significant competitive advantages for a company in a complex business environment. The aim of technology selection is to obtain new know-how, components, and systems which will help the company to make more competitive products and services and more effective processes, or create completely new solutions (Farzipoor Saen, 2006). The rest of the paper is organized as follows: The following section presents a concise treatment of the basic concepts of fuzzy set theory. Section 3 presents the methodology of Fuzzy AHP and Interval TOPSIS. The application of the proposed framework to technology selection is addressed in Section 4. Finally, conclusions are provided in Section 5.

2. Fuzzy sets and Fuzzy Numbers

Fuzzy set theory, which was introduced by Zadeh (1965) to deal with problems in which a source of vagueness is involved, has been utilized for incorporating imprecise data into the decision framework. A fuzzy set $\tilde{A}$ can be defined mathematically by a membership function $\mu_{\tilde{A}}(X)$, which assigns each element x in the universe of discourse X a real number in the interval [0,1]. A triangular fuzzy number $\tilde{A}$ can be defined by a triplet (a, b, c) as illustrated in Fig 1.



F . A ngu uzzy nu r $\tilde{A}$

The membership function $\mu_{\tilde{A}}(X)$ is defined as

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a}{b-a} & a \le x \le b \\ \frac{x-c}{b-c} & b \le x \le c \\ 0 & oterwise \end{cases} \quad (1)$$

Basic arithmetic operations on triangular fuzzy numbers $A_1 = (a_1,b_1,c_1)$, where $a_1 \le b_1 \le c_1$, and $A_2 = (a_2,b_2,c_2)$, where $a_2 \le b_2 \le c_2$, can be shown as follows:

Addition: $A_1 \oplus A_2 = (a_1 + a_2, b_1 + b_2, c_1 + c_2)$ (2)

Subtraction: $A_1 \ominus A_2 = (a_1 - c_2, b_1 - b_2, c_1 - a_2)$ (3)

Multiplication: if k is a scalar

$$k \otimes A_1 = \begin{cases} (ka_1, kb_1, kc_1), & k > 0 \\ (kc_1, kb_1, ka_1), & k < 0 \end{cases}$$

$$A_1 \otimes A_2 \approx (a_1a_2, b_1b_2, c_1c_2), \text{ if } a_1 \ge 0, a_2 \ge 0 \quad (4)$$

Division: $A_1 \oslash A_2 \approx (\frac{a_1}{c_2}, \frac{b_1}{b_2}, \frac{c_1}{a_2})$, if $a_1 \geq 0$, $a_2 \geq 0$ (5)

Although multiplication and division operations on triangular fuzzy numbers do not necessarily yield a triangular fuzzy number, triangular fuzzy number approximations can be used for many practical applications (Kaufmann & Gupta, 1988). Triangular fuzzy numbers are appropriate for quantifying the vague information about most decision problems including personnel selection (e.g. rating for creativity, personality, leadership, etc.). The primary reason for using triangular fuzzy numbers can be stated as their intuitive and computational-efficient representation (Karsak, 2002). A linguistic variable is defined as a variable whose values are not numbers, but words or sentences in natural or artificial language. The concept of a linguistic variable appears as a useful means for providing approximate characterization of phenomena that are too complex or ill defined to be described in conventional quantitative terms (Zadeh, 1975).

3. Research Methodology

In this paper, the weights of each criterion are calculated using of Fuzzy AHP. After that, Interval TOPSIS is utilized to rank the alternatives. Finally, we select the best technology based on these results.

3.1. Fuzzy AHP

Despite of its wide range of applications, the conventional AHP approach may not fully reflect a style of human thinking. One reason is that decision makers usually feel more confident to give interval judgments rather than expressing their judgments in the form of single numeric values. As a result, fuzzy AHP and its extensions are developed to solve alternative selection and justification problems. Although FAHP requires tedious computations, it is capable of capturing a human's appraisal of ambiguity when complex multi-attribute decision making problems are considered. In the literature, many FAHP methods have been proposed ever since the seminal paper by Van Laarhoven and Pedrycz (1983). In his earlier work, Saaty (1980) proposed a method to give meaning to both fuzziness in perception and fuzziness in meaning. This method measures the relativity of fuzziness by structuring the functions of a system hierarchically in a multiple attribute framework. Later on, Buckley (1985) extends Saaty's AHP method in which decision makers can express their preference using fuzzy ratios instead of crisp values. Chang (1996) developed a fuzzy extent analysis for AHP, which has similar steps as that of Saaty's crisp AHP. However, his approach is relatively easier in computation than the other fuzzy AHP approaches. In this paper, we make use of Chang's fuzzy extent analysis for AHP. Kahraman et al. (2003) applied Chang's (1996) fuzzy extent analysis in the selection of the best catering firm, facility layout and the best transportation company, respectively. Let $O = \{o_1, o_2, ..., o_n\}$ be an object set, and $U = \{g_1, g_2, ..., g_m\}$ be a goal set. According to the Chang's extent analysis, each object is considered one by one, and for each object, the analysis is carried out for each of the possible goals, g_i. Therefore, m extent analysis values for each object are obtained and shown as follows:

$$\tilde{M}^1_{g_i}, \tilde{M}^2_{g_i}, \ldots, \tilde{M}^m_{g_i}, \ i=1,2,\ldots,n$$

Where $\tilde{M}^j_{g_i}$ (j=1,2,3,…, m) are all triangular fuzzy numbers. The membership function of the triangular fuzzy number is denoted by $M_{(x)}$. The steps of the Chang's extent analysis can be summarized as follows:

Step 1: The value of fuzzy synthetic extent with respect to the ith object is defined as:

$$S_i = \sum_{j=1}^{m} \tilde{M}^j_{g_i} \otimes \left[\sum_{i=1}^{n}\sum_{j=1}^{m} \tilde{M}^j_{g_i}\right]^{-1} \quad (6)$$

Where $\otimes$ denotes the extended multiplication of two fuzzy numbers. In order to obtain $\sum_{j=1}^{m} \tilde{M}^j_{g_i}$ We perform the addition of m extent analysis values for a particular matrix such that,

$$\sum_{j=1}^{m} \tilde{M}^j_{g_i} = \left(\sum_{j=1}^{m} l_j, \sum_{j=1}^{m} m_j, \sum_{j=1}^{m} u_j\right) \quad (7)$$

And to obtain $\left[\sum_{i=1}^{n}\sum_{j=1}^{m} \tilde{M}^j_{g_i}\right]^{-1}$ we perform the fuzzy addition operation of $\tilde{M}^j_{g_i}$ (j =1,2,…,m) values such that,

$$\sum_{i=1}^{n}\sum_{j=1}^{m} \tilde{M}^j_{g_i} = \left(\sum_{i=1}^{n} l_i, \sum_{i=1}^{n} m_i, \sum_{i=1}^{n} u_i\right) \quad (8)$$

Then, the inverse of the vector is computed as,

$$\left[\sum_{i=1}^{n}\sum_{j=1}^{m} \tilde{M}^j_{g_i}\right]^{-1} = \left(\frac{1}{\sum_{i=1}^{n} u_i}, \frac{1}{\sum_{i=1}^{n} m_i}, \frac{1}{\sum_{i=1}^{n} l_i}\right) \quad (9)$$

Where u_i , m_i , $l_i > 0$

Finally, to obtain the S_j , we perform the following multiplication:

$$S_i = \sum_{j=1}^{m} \tilde{M}^j_{g_i} \otimes \left[\sum_{i=1}^{n}\sum_{j=1}^{m} \tilde{M}^j_{g_i}\right]^{-1}$$
$$= \left(\sum_{j=1}^{m} l_j \otimes \sum_{i=1}^{n} l_i, \sum_{j=1}^{m} m_j \otimes \sum_{i=1}^{n} m_i,\right.$$

$\sum_{j=1}^{m} u_j \quad \otimes \quad \sum_{i=1}^{n} u_i)$ (10)

Step 2: The degree of possibility of $\tilde{M}_2 = (l_2, m_2, u_2) \geq \tilde{M}_1 = (l_1, m_1, u_1)$ is defined as

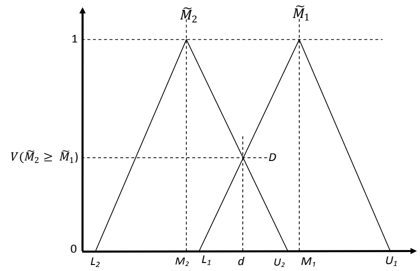


Fig 2: The degree of possibility of $\tilde{M}_1 \geq \tilde{M}_2$

$$V(\tilde{M}_2 \geq \tilde{M}_1) = sup[\min(\tilde{M}_1(x), \tilde{M}_2(y))] \quad (11)$$

This can be equivalently expressed as,

$$V(\tilde{M}_2 \geq \tilde{M}_1) = \text{hgt}(\tilde{M}_1 \cap \tilde{M}_2) = \tilde{M}_2(d)$$
$$= \begin{cases} 1 & if\ m_2 \geq m_1 \\ 0 & if\ l_1 \geq u_2 \\ \frac{l_1 - u_2}{(m_2 - u_2) - (m_1 - l_1)}, & otherwise \end{cases} \quad (12)$$

Fig. 2 illustrates $V(\tilde{M}_2 \geq \tilde{M}_1)$ for the case d for the case $m_l < l_l < u_2 < m_1$, where d is the abscissa value corresponding to the highest crossover point D between $\tilde{M}_1$ and $\tilde{M}_2$, To compare $\tilde{M}_1$ and $\tilde{M}_2$, we need both of the values $V(\tilde{M}_1 \geq \tilde{M}_2)$ and $V(\tilde{M}_2 \geq \tilde{M}_1)$.

Step 3: The degree of possibility for a convex fuzzy number to be greater than k convex fuzzy numbers M_i (I=1, 2… K) is defined as

$V(\tilde{M} \geq \tilde{M}_1, \tilde{M}_2, \ldots, \tilde{M}_k) = \min V(\tilde{M} \geq \tilde{M}_i)$, i =1,2,…,k

Step 4: Finally, W=(min V($s_1 \geq s_k$) min V($s_2 \geq s_k$),….,min V($s_n \geq s_k$))^T, is the weight vector for k = 1,…,n.

In order to perform a pairwise comparison among the parameters, a linguistic scale has been developed. Our scale is depicted in Fig.3 and the corresponding explanations are provided in Table 1. Similar to the importance scale defined in Saaty's classical AHP (Saaty, 1980), we have used five main linguistic terms to compare the criteria: ''equal importance'', ''moderate importance'', ''strong importance'', ''very strong importance'' and ''demonstrated importance''. We have also considered their reciprocals: ''equal unimportance'', ''moderate unimportance'', ''strong unimportance'', ''very strong unimportance'' and ''demonstrated unimportance''. For instance, if criterion A is evaluated ''strongly important'' than criterion B, then this answer means that criterion B is ''strongly unimportant'' than criterion A.

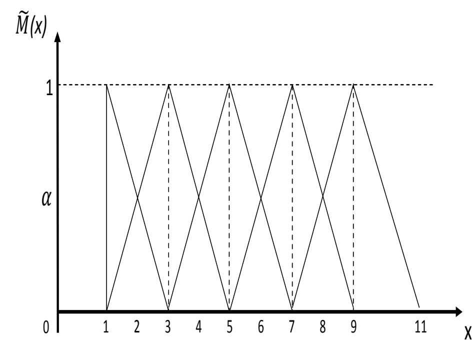


Fig 3: Membership functions of triangular fuzzy numbers corresponding to the linguistic scale

Table 1: The linguistic scale and corresponding triangular fuzzy numbers

Linguistic scale	Explanation	triangular fuzzy numbers	The inverse of triangular fuzzy numbers
Equal Importance	Two activities contribute equally to the objective	(1, 1, 1)	(1, 1, 1)
Moderate Importance	Experience and judgment slightly favor one activity over another	(1, 3, 5)	(1/5, 1/3, 1)
Strong importance	Experience and judgment strongly favor one activity over another	(3, 5, 7)	(1/7, 1/5, 1/3)
Very strong importance	An activity is favored very strongly over another; its dominance demonstrated in practice	(5, 7, 9)	(1/9, 1/7, 1/5)
Demonstrated importance	The evidence favoring one activity over another is highest possible order of affirmation	(7, 9, 11)	(1/11, 1/9, 1/7)

3.2. The TOPSIS Method with Interval Data

Considering the fact that, in some cases, determining precisely the exact value of the attributes is difficult and that, as a result of this, their values are considered as intervals, therefore, Jahanshahloo et al. (2006) extend TOPSIS for these interval data. Suppose $A_1,A_2, \dots ,Am$ are m possible alternatives among which decision makers have to choose, $C_1,C_2, \dots ,C_n$ are criteria with which alternative performance are measured, x_{ij} is the rating of alternative A_i with respect to criterion C_j and is not known exactly and only we know $x_{ij} \epsilon [x_{ij}^L x_{ij}^U]$. A MCDM problem with interval data can be concisely expressed in matrix format as follow:

Table 2: The interval decision matrix

A_i \ C_j	C_1	…	C_j	…	C_n
A_1	$[X_{11}{}^L, X_{11}{}^U]$	…	$[X_{1j}{}^L, X_{1j}{}^U]$	…	$[X_{1n}{}^L, X_{1n}{}^U]$
⋮	⋮		⋮		⋮
A_i	$[X_{i1}{}^L, X_{i1}{}^U]$	…	$[X_{ij}{}^L, X_{ij}{}^U]$	…	$[X_{in}{}^L, X_{in}{}^U]$
⋮	⋮		⋮		⋮
A_m	$[X_{m1}{}^L, X_{m1}{}^U]$	…	$[X_{mj}{}^L, X_{mj}{}^U]$	…	$[X_{mn}{}^L, X_{mn}{}^U]$
W	W_1	…	…	…	W_n

where w_j is the weight of criterion C_j.

A systematic approach to extend the TOPSIS to the interval data is proposed in this section by Jahanshahloo et al. (2006). First they calculate the normalized decision matrix as follows:

The normalized values $\bar{n}_{ij}^L$ and $\bar{n}_{ij}^U$ are calculated as

$$\bar{n}_{ij}^L = x_{ij}^L / \sqrt{\sum_{j=1}^{m} \left(\left(x_{ij}^L\right)^2 + \left(x_{ij}^U\right)^2 \right)}, \quad j = 1, \dots, m, \quad i = 1, \dots, n, \tag{13}$$

$$\bar{n}_{ij}^U = x_{ij}^U / \sqrt{\sum_{j=1}^{m} \left(\left(x_{ij}^L\right)^2 + \left(x_{ij}^U\right)^2 \right)}, \quad j = 1, \dots, m, \quad i = 1, \dots, n. \tag{14}$$

Now interval $[\bar{n}_{ij}^L \bar{n}_{ij}^U]$ is normalized of interval$[x_{ij}^L x_{ij}^U]$. The normalization method mentioned above is to preserve the property that the ranges of normalized interval numbers belong to [0, 1].

Considering the different importance of each criterion, the weighted normalized interval decision matrix construct as follow:

$$\bar{u}_{ij}^L = w_i \bar{n}_{ij}^L \quad j = 1, \dots, m, \quad i = 1, \dots, n, \tag{15}$$

$$\bar{u}_{ij}^U = w_i \bar{n}_{ij}^U \quad j = 1, \dots, m, \quad i = 1, \dots, n, \tag{16}$$

where w_i is the weight of the ith attribute or criterion, and $\sum_{i=1}^{n} w_i = 1$ Then, Jahanshahloo et al. (2006) identified positive ideal solution and negative ideal solution as

$$\bar{A}^+ = \{\bar{u}_1^+, \dots, \bar{u}_n^+\} = \left\{\left(\max_j \bar{u}_{ij}^U \middle| i\epsilon I\right), \left(\min_j \bar{u}_{ij}^L \middle| i\epsilon J\right)\right\}, \tag{17}$$

$$\bar{A}^- = \{\bar{u}_1^-, \dots, \bar{u}_n^-\} = \left\{\left(\min_j \bar{u}_{ij}^L \, i\epsilon I\right), \left(\max_j \bar{u}_{ij}^U \middle| i\epsilon J\right)\right\}, \tag{18}$$

Where I is associated with benefit criteria, and J is associated with cost criteria. The separation of each alternative from the positive ideal solution, using the n-dimensional Euclidean distance, can be currently calculated as:

$$\bar{d}_j^+ = \left\{\sum_{i\epsilon I}\left(\bar{u}_{ij}^L - \bar{u}_i^+\right)^2 + \sum_{i\epsilon J}\left(\bar{u}_{ij}^U - \bar{u}_i^+\right)^2\right\}^{\frac{1}{2}}, \quad j = 1, \dots, m. \tag{19}$$

Similarly, the separation from the negative ideal solution can be calculated as

$$\bar{d}_j^- = \left\{\sum_{i\epsilon I}\left(\bar{u}_{ij}^U - \bar{u}_i^-\right)^2 + \sum_{i\epsilon J}\left(\bar{u}_{ij}^L - \bar{u}_i^-\right)^2\right\}^{\frac{1}{2}},$$

$$\mathrm{j} = 1, \ldots, \mathrm{m}. \quad (20)$$

A closeness coefficient is defined to determine the ranking order of all alternatives once the $\bar{d}_j^+$ and $\bar{d}_j^-$ of each alternative A_j has been calculated. The relative closeness of the alternative A_j with respect to $\bar{A}^+$ is defined as

$$\bar{R}_j = \bar{d}_j^- / (\bar{d}_j^- + \bar{d}_j^+), \quad j = 1, \ldots, m. \quad (21)$$

Obviously, an alternative A_j is closer to the $\bar{A}^+$ and farther from $\bar{A}^-$ as $\bar{R}_j$ approaches to 1. Therefore, according to the closeness coefficient, we can determine the ranking order of all alternatives and select the best one from among a set of feasible alternatives. In sum, an algorithm to determine the most preferable choice among all possible choices, when data is interval, with extended TOPSIS approach is given in the following:

Step 1: Establishing system evaluation criteria that relate system capabilities to goals (identification the evaluation criteria).

Step 2: Developing alternative systems for attaining the goals (generating alternatives).

Step 3: Evaluating alternatives in terms of criteria (the values of the criterion functions which are intervals).

Step 4: Identifying the weight of criteria.

Step 5: Construct the interval decision matrix and the interval normalized decision matrix (using the formulas (13) and (14)).

Step 6: Construct the interval weighted normalized decision matrix (using the formulas (15) and (16)).

Step 7: Determine positive ideal solution and negative ideal solution (identification of $\bar{A}^+$ and $\bar{A}^-$, using the formulas (17) and (18)).

Step 8: Calculate the separation of each alternative from positive ideal solution and negative ideal solution, respectively (identification of $\bar{d}_j^+$ and $\bar{d}_j^-$, using the formulas (19) and (20)).

Step 9: Calculate the relative closeness of each alternative to positive ideal solution (identification of $\bar{R}_j$, using the formula (21)).

Step 10: Rank the preference order of all alternatives according to the closeness coefficient.

3.3. Incorporation of ordinal preference information into the interval TOPSIS model

Wang et al (2005) proposed a method to deal with both interval and ordinal data in DEA models. Wang used an innovative method to transform the ordinal inputs or outputs into interval data ,and then solved the DEA model with only interval data .one of the contributions of our paper is to use Wang's strategy to translate ordinal criteria into interval criteria. Suppose data of some criteria (C_j) for alternatives (A_i) are given in the form of ordinal preference information. Usually, there may exist three types of ordinal preference information: (1) strong ordinal preference information such as $X_{ij} > X_{ik}$ which can be further expressed as $X_{ij} \geq \chi X_{ik}$, where $\chi > 1$ is the parameter on the degree of preference intensity provided by decision maker (DM); (2) weak ordinal preference information such as $X_{ip} \geq X_{iq}$; (3) indifference relationship such as $X_{il} = X_{it}$. We can conduct a scale transformation to ordinal criteria so that its best ordinal datum is less than or equal to unity and then give an interval estimate for each ordinal datum. For transforming ordinal scale to interval scale, we use the following formula:

$$X_{ij} \in \left[\sigma_j {\chi_j}^{m-j}, {\chi_j}^{1-j}\right], \; j = 1, \ldots, m \qquad whit \quad \sigma_j \leq {\chi_j}^{1-n} \quad (22)$$

Where χ is a preference intensity parameter satisfying $\chi_j > 1$ provided by the DM and σ_j is the ratio parameter also provided by the DM. According to the simplest order relation between two interval numbers, i.e. $A \leq B$ if and only if $a^L \leq b^L$ and $a^U \leq b^U$, where $A = [a^L, a^U]$ and $B = [b^L, b^U]$ are two interval numbers, the transformed interval data still reserve the original ordinal preference relationships (Wang et al, 2005).

Through the scale transformation above and the estimation of permissible intervals, all the ordinal preference information is converted into interval data and can thus be incorporated into interval TOPSIS models.

4. A Numerical Application of Proposed Approach

This paper, the proposed methodology that may be applied to a wide range of technology selection problems is used for robot selection. We considered cost as a non-beneficial attribute and Vendor reputation, Load capacity and Velocity and as beneficial attributes for Technology selection. These attributes are taken from Farzipoor saen (2006). These attributes are shown in Table 3.

Table 3: Attributes for robot selection

criteria	Attributes
C_1	Cost (10000$)
C_2	Vendor reputation
C_3	Load capacity(kg)
C_4	Velocity(m/s)

In this paper, the weights of criteria are calculated using of FAHP, and these calculated weight values are used as Interval TOPSIS inputs. Then, after TOPSIS calculations, evaluation of the alternatives and selection of technology is realized.

Fuzzy AHP: In fuzzy-AHP, firstly, the criteria and alternatives' importance weights must be compared. For this reason, there must be linguistic terms and their equivalent fuzzy numbers denoting comparison measures. The linguistic comparison terms and their equivalent fuzzy numbers considered in this paper are shown in Table 1. Afterwards, for the first step, the comparisons about the criteria and alternatives, and the weight calculation need to be made. Thus, the evaluation of the criteria according to the main goal and the evaluation of the alternatives for these criteria must be realized. Then, after all these evaluation procedure, the weights of the alternatives can be calculated. In the second step, these weights are used to Interval TOPSIS calculation for the final evaluation. The comparison matrix for the criteria can be seen from Table 4.

Table 4: Fuzzy comparison matrix

Criteria	C_1	C_2	C_3	C_4
C_1	(1.00, 1.00, 1.00)	(3.00, 4.33, 6.00)	(5.00, 6.33, 8.00)	(0.20, 2.13, 6.00)
C_2	(0.17, 0.25, 0.33)	(1.00, 1.00, 1.00)	(1.00, 3.25, 5.00)	(0.14, 0.83, 2.00)
C_3	(0.13, 0.16, 0.20)	(0.20, 0.51, 1.00)	(1.00, 1.00, 1.00)	(0.20, 1.18, 3.00)
C_4	(0.17, 3.39, 6.00)	(0.50, 2.83, 5.00)	(0.33, 0.85, 7.00)	(1.00, 1.00, 1.00)

After forming fuzzy pair-wise comparison matrix, we calculate the weight of all criteria. The weight calculation details are given below. Because of the other calculations are similar for each comparison matrix, these are not given here and can be done simply according the computations below. The value of fuzzy synthetic extent with respect to the ith object (i = 1,2, . . . ,4) is calculated as

S_1= (9.20, 13.80, 21) $\otimes$ (0.0176, 0.0332, 0.0665) = (0.01627, 0.4592, 1.3967)
S_2= (2.31, 5.33, 8.33) $\otimes$ (0.0176, 0.0332, 0.0665) = (0.0408, 0.1772, 0.5542)
S_3= (1.53, 2.85, 8.20) $\otimes$ (0.0176, 0.0332, 0.0665) = (0.0269, 0.0949, 0.5454)
S_4= (2.00, 8.07, 19) $\otimes$ (0.0176, 0.0332, 0.0665) = (0.0353, 0.2686, 0.1.2637)

Then the V values calculated using these vectors are shown in Table 3.

Table 5: V values result

V	S_1	S_2	S_3	S_4
S_1	-	1	1	1
S_2	0.3645	-	1	0.4585
S_3	0.3805	1.1337	-	0.4797
S_4	0.9404	1	1	-

Thus, the weight vector from Table 5 is calculated and normalized as
W^t = (0.372355, 0.135759, 0.141711, 0.350174)

Interval TOPSIS: The weights of the criteria are calculated by fuzzy AHP up to now, and then these values can be used in Interval TOPSIS. The decision matrix with ordinal and cardinal data for each robot is shown in Table 6. These data are taken from Farzipoor saen (2006).
Attributes which are ranked Using of Eq. (22) transformed into interval scale. These intervals identify the range of attributes. For example, interval scale of A_1 with ($\chi = 1.12$، $\sigma = .1$) is calculated as below:

$$A_{22} \in [(0.1)1.12^{10-1}, 1.12^{1-1}] = [0.277,1]$$

Similar to A_{22}, the interval scale of other alternatives are calculated and shown in the Table 7.

The interval decision matrix and interval normalized decision matrix are shown in Tables 8 and 9, respectively.

In the next step, the positive ideal solution and the negative ideal solution are then determined as:
$\bar{A}^{+}$= [0.002847, 0.170673, 0.266347, 0.160721]
$\bar{A}^{-}$= [0.14233, 0.017067, 0.001211, 0.012054]

A comparison between the normalized performance ratings of each alternative A_i and $\bar{A}^{+}$ by Eq. (19) (that is shown in Table 11), and between that of A_i and $\bar{A}^{-}$ by Eq. (20) (that is shown in Table 12) would indicate how the Robot is performing as compared with the best performance and the worst performance of all the robots with respect to each criterion.

After that we obtain the interval Weighted normalized decision matrix that is shown in Table 10.

Table 6: Related attributes for 10 robots (The decision matrix with ordinal and cardinal data)

Robot (No)	Cost(10000$)	Vendor reputation	Load capacity(kg)	Velocity(m/s)
1	0.16	5	[1,4]	0.8
2	8	1	[10,18]	2
3	4.8	7	[60,70]	1.1
4	6.9	10	[10,15]	0.15
5	2.4	2	[5,8]	1
6	1.76	8	[4,5]	1
7	1.07	3	[1,2]	0.3
8	6.72	9	[9,12]	1.1
9	4	6	[190,220]	0.75
10	3.63	4	[8,12]	1

Table 7: Ordinal scale and Interval Scale for A_i in C_2

Robot (No)	Vendor reputation ordinal	Vendor reputation cardinal
1	5	[0.176, 0.636]
2	1	[0.277,1.000]
3	7	[0.140, 0.507]
4	10	[0.100, 0.361]
5	2	[0.248, 0.893]
6	8	[0.125, 0.452]
7	3	[0.221, 0.797]
8	9	[0.112, 0.404]
9	6	[0.157, 0.567]
10	4	[0.197, 0.712]

Table 8: The Interval decision matrix of 10 alternatives

Robot(No)	Cost(10000$)		Vendor reputation		Load capacity(kg)		Velocity(m/s)	
	X_{1j}^L	X_{1j}^U	X_{2j}^L	X_{2j}^U	X_{3j}^L	X_{3j}^U	X_{4j}^L	X_{4j}^U
A_1	0.16	0.16	0.176	0.636	1	4	0.8	0.8
A_2	8	8	0.277	1	10	18	2	2
A_3	4.8	4.8	0.14	0.507	60	70	1.1	1.1
A_4	6.9	6.9	0.1	0.361	10	15	0.15	0.15
A_5	2.4	2.4	0.248	0.893	5	8	1	1
A_6	1.76	1.76	0.125	0.452	4	5	1	1
A_7	1.07	1.07	0.221	0.797	1	2	0.3	0.3
A_8	6.72	6.72	0.112	0.404	9	12	1.1	1.1
A_9	4	4	0.157	0.567	190	220	0.75	0.75
A_{10}	3.63	3.63	0.197	0.712	18	12	1	1

Table 9: The Interval normalized decision matrix

Robot(No)	Cost(10000$)		Vendor reputation		Load capacity(kg)		Velocity(m/s)	
	$\bar{n}_{1j}^{L}$	$\bar{n}_{1j}^{U}$	$\bar{n}_{2j}^{L}$	$\bar{n}_{2j}^{U}$	$\bar{n}_{3j}^{L}$	$\bar{n}_{3j}^{U}$	$\bar{n}_{4j}^{L}$	$\bar{n}_{4j}^{U}$
A_1	0.008	0.008	0.081	0.292	0.003	0.013	0.173	0.173
A_2	0.382	0.382	0.127	0.458	0.033	0.059	0.432	0.432
A_3	0.229	0.229	0.064	0.232	0.195	0.228	0.237	0.237
A_4	0.330	0.330	0.046	0.165	0.033	0.049	0.032	0.032
A_5	0.115	0.115	0.114	0.409	0.016	0.026	0.216	0.216
A_6	0.084	0.084	0.057	0.207	0.013	0.016	0.216	0.216
A_7	0.051	0.051	0.101	0.365	0.003	0.007	0.065	0.065
A_8	0.321	0.321	0.051	0.185	0.029	0.039	0.237	0.237
A_9	0.191	0.191	0.072	0.260	0.618	0.715	0.162	0.162
A_{10}	0.173	0.173	0.090	0.326	0.059	0.039	0.216	0.216

Table 10: The Interval weighted normalized decision matrix

Robot(No)	Cost(10000$)		Vendor reputation		Load capacity(kg)		Velocity(m/s)	
	$\bar{u}_{1j}^{L}$	$\bar{u}_{1j}^{U}$	$\bar{u}_{2j}^{L}$	$\bar{u}_{2j}^{U}$	$\bar{u}_{3j}^{L}$	$\bar{u}_{3j}^{U}$	$\bar{u}_{4j}^{L}$	$\bar{u}_{4j}^{U}$
A_1	0.003	0.003	0.030	0.109	0.001	0.005	0.064	0.064
A_2	0.142	0.142	0.047	0.171	0.012	0.022	0.161	0.161
A_3	0.085	0.085	0.024	0.087	0.073	0.085	0.088	0.088
A_4	0.123	0.123	0.017	0.062	0.012	0.018	0.012	0.012
A_5	0.043	0.043	0.042	0.152	0.006	0.010	0.080	0.080
A_6	0.031	0.031	0.021	0.077	0.005	0.006	0.080	0.080
A_7	0.019	0.019	0.038	0.136	0.001	0.002	0.024	0.024
A_8	0.120	0.120	0.019	0.069	0.011	0.015	0.088	0.088
A_9	0.071	0.071	0.027	0.097	0.230	0.266	0.060	0.060
A_{10}	0.065	0.065	0.034	0.122	0.022	0.015	0.080	0.080

Table 11: Distance of each alternative from the positive ideal solution

$\bar{d}_1^+$	$\bar{d}_2^+$	$\bar{d}_3^+$	$\bar{d}_4^+$	$\bar{d}_5^+$	$\bar{d}_6^+$	$\bar{d}_7^+$	$\bar{d}_8^+$	$\bar{d}_9^+$	$\bar{d}_{10}^+$
0.315	0.315	0.267	0.353	0.304	0.313	0.327	0.327	0.192	0.298

Table 12: Distance of each alternative from the negative ideal solution

$\bar{d}_1^-$	$\bar{d}_2^-$	$\bar{d}_3^-$	$\bar{d}_4^-$	$\bar{d}_5^-$	$\bar{d}_6^-$	$\bar{d}_7^-$	$\bar{d}_8^-$	$\bar{d}_9^-$	$\bar{d}_{10}^-$
0.175	0.215	0.144	0.052	0.182	0.144	0.172	0.096	0.290	0.148

Finally we calculate the relative closeness of each alternative to positive ideal solution and we rank the alternatives that show in Table 13.

According to Table 13, A_9 is the best technology among other technologies and other alternatives ranked as follow: $A_9 > A_2 > A_5 > A_1 > A_3 > A_7 > A_{10} > A_6 > A_8 > A_4$.

Table 13: Closeness coefficient and ranking

Alternatives	$\bar{R}_j$	Rank
A_1	0.357	4
A_2	0.405	2
A_3	0.351	5
A_4	0.127	10
A_5	0.374	3
A_6	0.315	8
A_7	0.344	6
A_8	0.227	9
A_9	0.602	1
A_{10}	0.331	7

5. Conclusions

Selection of technologies is one of the most challenging decision making areas the management of a company encounters. It is difficult to clarify the right technology alternatives because the number of technologies is increasing and the technologies are becoming more and more complex

This paper illustrates an application of fuzzy AHP along with Interval TOPSIS in selecting best technology. Fuzzy set theory is incorporated to overcome the vagueness in the preferences. A two step fuzzy-AHP and Interval TOPSIS methodology is structured here that Interval TOPSIS uses fuzzy-AHP result weights as input weights. Then a numerical example is presented to show applicability and performance of the methodology. It can be said that using linguistic variables makes the evaluation process more realistic. Because evaluation is not an exact process and has fuzziness in its body. Here, the usage of fuzzy-AHP weights in Interval TOPSIS makes the application more realistic and reliable.

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