A Simple Laboratory Experiment for Measuring the Expansivity of Tungsten at Elevated Temperatures

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Abstract: A laser pointer is used to backlight the tungsten filament of an ordinary P21W automotive light bulb, gradually driven to incandescence by direct current. Dimensional changes in the filament shadowgraphs are measured, then integrated to voltage - current data, in order to calculate the coefficient of thermal expansion CTE of tungsten, in the temperature range from 500 to 3500K. This low cost experiment is a simple activity that enables science students to understand the thermometric behavior of tungsten as a refractory metal to the extent of its melting temperature.

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1. Introduction

One of the most important thermo-physical properties of solids is the thermal expansion. It can be defined as the tendency of matter to change in volume in response to a change in temperature [1, 2]. The thermal expansion of uniform linear objects of length L is proportional to temperature change ΔT according to the relation:

$$\Delta L/L = \alpha\, \Delta T \qquad (1)$$

Where α is the thermal coefficient of linear expansion CTE. The thermal expansion can be expressed as a polynomial function of temperature on the form:

$$\Delta L/L = c_o + c_1 T + c_2 T^2 + \ldots + c_n T^n \qquad (2)$$

The CTE can be obtained by differentiating the previous equation with respect to T yielding to the expression:

$$\alpha = c_1 + 2c_2 T + 3c_3 T^2 + \ldots + n\, c_n T^{n-1} \qquad (3)$$

At most cases, the CTE of a metal is described by a polynomial of degree $n \geq 3$, in order to comprise the non linearity of the expansion behavior over most of the solid range to melting point. Different substances expand by different amounts over small temperature ranges. This relative expansion can be demonstrated and measured by means of dilatometers. (Devices capable of performing extensometry and thermometry to a solid sample simultaneously)

Unfortunately, classrooms' dial gauge dilatometers [3], are having a very narrow temperature window ranging from 20 to 90°C. At this extent the CTE remains constant, and equals c_1, or $n = 1$, as an unaccepted approximation. Moreover, the students cannot notice the expansion process unless following carefully the dial gauge, the physical sense of nonlinearity is then missing, and the educational profit from this laboratory activity is poor. This work presents a low cost method to overcome the mentioned problems.

2. Theory

As a refractory metal, tungsten has the highest melting point (3683K) and the lowest thermal expansion among all metals (only 2.2% of its initial length near melting) [4], making it very suitable, to demonstrate elevated temperatures non linear expansivity. We selected an ordinary P21W incandescent automotive light bulb as an inexpensive test sample, previously used in different works [5]. Fig. (1) shows the coiled filament inside the bulb that is made of commercial grade tungsten, surrounded by an inert gas atmosphere to prevent the metal from oxidization.

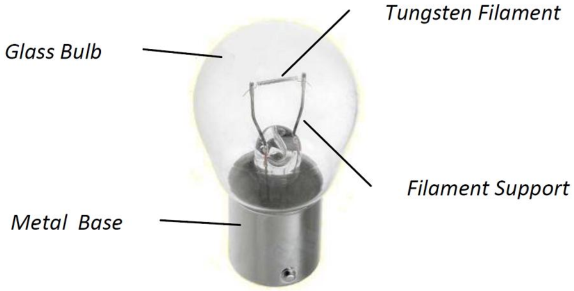

Figure 1. Structure of a standard P21W automotive incandescent light bulb

The filament is made of a wire of length l and diameter d, having a resistance R and resistivity ρ, winded in form of a uniform coil having N turns, mean diameter D and, attached between two rigid and thick supports with neglected resistance.

Figure 2. Geometry of single coiled linear tungsten filament.

From Fig. (2) the filament is considered to be made of a chained perimeters of N circles, hence the total wire length equals:

$$l = \pi N D \tag{4}$$

Assuming room temperature initial conditions, where $T=T_o=300\text{K}$, $R=R_o$, $l=l_o$ and $D=D_o$; Eq. (4) yields:

$$l_o = \pi N D_o \tag{5}$$

Substituting Eq. (4) and Eq. (5) in Eq. (1) to obtain ;

$$\Delta l/l_o = \Delta D/D_o \tag{6}$$

The temperature of the filament is then given by the relation [6]:

$$T = 293 \times (R/R_o)^{0.83} \tag{7}$$

When the coiled wire sample is heated under the action of DC current flow injected across the rigid thick supports, a temperature gradient will arise along the coil because of the heat conduction loss through supports ends (simply called end loss) [7]. At the same time, a temperature dependant stiffness gradient distribution is also observed along the coiled sample that causes in return, the bending of the filament starting from its less stiff hot part at the middle, as result of its confined expansion as shown in Fig. (3).

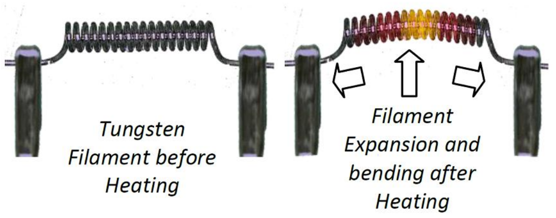

Figure 3. Bending of a confined filament between the rigid supports under the effect of thermal expansion

As the diameter of the coil increases, the shear on the wire cross section decreases, by consequences the thermal strain of the coil decreases. Therefore, we will consider the filament as a fixed-fixed compression spring and we will define the spring index [8] of a coil as its expansion index μ, which is inversely proportional to d. Hence,

$$\mu = D_o / d \tag{8}$$

It is better to choose the test sample that have maximum expansion index to minimize the error in measuring ΔD caused by filament bending under thermal strain.

3. Experimental Work

3.1. Sample selection

Table (1) shows a variety of P21W automotive bulb selected as test samples, produced by different manufacturers, having almost the same wire diameter, and made of the same tungsten grade but differ only in their coils diameters.

Table 1. Selected test samples

Sample	d *(mm)*	D *(mm)*	$\mu = D_o / d$
[A] *TAIWAN* P21W6V	0.08	0.31	3.8
[B] *ZEEMOH* 12V21W	0.08	0.345	4.3
[C] *MASA* 21W6V	0.08	0.35	4.3
[D] *AU-LITE* P21W12V E13	0.08	0.58	6.5
[E] *TARIFA* 21W12V	0.08	0.57	7.1
[F] *TUNGSRAM* 12V 21W	0.08	0.59	7.3
[G] *EAGLEYE* 12V21W	0.09	0.71	7.8

3.2. Setup

It is easy to generate shadowgrams [9, 10] for the tungsten filament, by backlighting the selected light bulb using a commercial 1000 mw laser pointer. The color of the laser is not critical, but green laser (wavelength = 532 nm) seems to be comfortable for the human eye rather than the blue laser that may causes visual illusions and after effects. Red laser is also excluded due to its weak contrast relative to incandescent objects. The filament image is projected by a throw lens to a white wall screen as shown in Fig. (4).

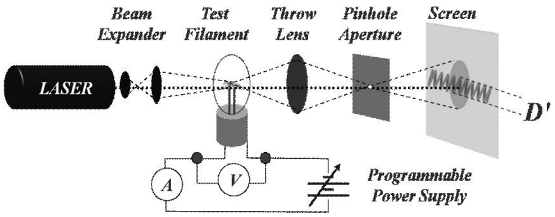

Figure 4. Incandescent filament laser shadowgraphy setup

In the previous setup, the beam expander is built in the laser pointer head and the pinhole aperture is used to discard filament color during incandescence (optional). The test filament is connected to a programmable power supply capable of delivering 5 amps at 25 volts to overdrive the light bulb beyond it nominal rating, forcing it to achieve a working temperature of 3500 K instead of 2400 K. The magnification M of the projected image can be calculated easily from Eq. (9) as follows:

$$M = D_o' / D_o \qquad (9)$$

Where D_o' is the coil diameter at the filament image. Practically, M should be between 500 and 2000 at max to obtain a significant shadowgraph.

3.3. Precautions

After the installation and adjustment of the optical setup in a dark room and before starting the measurement process it is important to make sure that:

- All crucial data specifically R_o , d_o and D_o have been measured for each sample.
- The laser beam is always targeting the center of the tungsten filament through carefully cleaned light bulb glass envelop.
- The shadow of the expanded coil must be always placed inside the screen area, no adjustments are allowed after starting the measurements.
- The generated shadow is projected to a wall screen to obtain more magnification according to the throw distance, also to enable manual mark tracing to ease the measurements.

The filament current should not excide 135% of its nominal operating value to prevent sample burnout.

3.4. Procedure

A simple method to measure $\Delta D'$ directly from the shadowgraph of the bending coil is shown in Fig. (5).

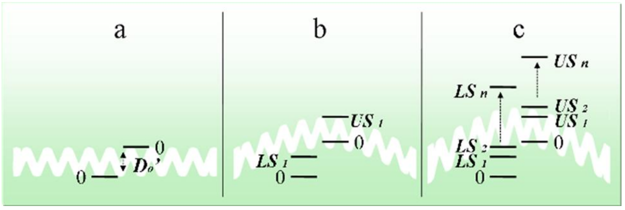

Figure 5. Mark tracing method to determine $\Delta D'$

This method is based on the following measures:

- Tracing two limiting marks indicating the upper and the lower edge of the coil diameter D_o' , as shown in Fig. (5a).
- This pair of marks represents the zero current coil position or the datum points, while their span represents D' from which M is calculated using Eq. (9).
- To start measurements, the DC current I, is gradually increased on stepping increments of 100 mA per step.
- Every increment, marking, the new coil locus on the screen, as shown at Fig. (5b), while recording the value of the current I and the voltage V to obtain R and T using Ohm's law and Eq. (7).
- By the end of the experiment, two set of marks are obtained, the first is the upper set (*US*) and the second is the lower set (*LS*) as shown in Fig. (5c).
- At each current increment, the net values of $\Delta D'$ are obtained by taking the difference value of every pair of analogue readings in both sets, as shown at Table.2 where all essential data are reduced in a report sheet.
- Hence $\Delta D'$ is used (in light of the known optical magnification) to deduce ΔD.
- Calculating $\Delta D/D$ that represents the linear expansion of the sample $\Delta l/l_o$ according to Eq. (6), plotting it versus T, and then a best-fit curve is traced.
- Almost the same procedure can be repeated to cool down the sample gradually, in order to study the effect of re-crystallization of tungsten, on its expansion behavior during cooling, to reveal any probable hysteresis phenomena. (optional).

Table 2. Sample report sheet where basic data are reduced

I (*A*)	*V* (*V*)	*R* (*Ω*)	*T* (*K*)	*US* (*mm*)	*LS* (*mm*)	*ΔD'* (*mm*)
0	*0*	R_o	T_o	*0*	*0*	*0*
I_1	V_1	R_1	$T_o(R_1/R_o)^{0.83}$	US_1	LS_1	$US_1\text{-}LS_1$
I_2	V_2	R_2	$T_o(R_2/R_o)^{0.83}$	US_2	LS_2	$US_2\text{-}LS_2$
¦	¦	¦	¦	¦	¦	¦
I_n	V_n	R_n	$T_o(R_n/R_o)^{0.83}$	US_n	LS_n	$US_n\text{-}LS_n$

3.5. Data refining

- The resulting best-fit curve should be refined and smoothed by regression to obtain the linear expansion as a function of T on the form of polynomial equation of degree n (where $3 \leq n \leq 5$) as described by Eq. (2).

- To start the regression, a set of n data pairs that represents the fitted curve $\{T_i, (\Delta l/l_o)_i\}$ is selected. ($i = 0,1,2,3,.., n$)
- Then, using Eq. (2) and the set of selected data to generate a set of n equation in order to be solved simultaneously to obtain ($c_o, c_1, c_2, ..., c_n$) or:

$$
\begin{aligned}
c_o + c_1 T_o + c_2 T_o^2 + \ldots + c_n T_o^n &= (\Delta l/l_o)_o \\
c_o + c_1 T_1 + c_2 T_1^2 + \ldots + c_n T_1^n &= (\Delta l/l_o)_1 \\
c_o + c_1 T_2 + c_2 T_2^2 + \ldots + c_n T_2^n &= (\Delta l/l_o)_2 \\
\vdots \quad \vdots \quad\quad \vdots \quad\quad \vdots \quad\quad \vdots \quad &\quad\quad \vdots \\
c_o + c_1 T_n + c_2 T_n^2 + \ldots + c_n T_n^n &= (\Delta l/l_o)_n
\end{aligned}
$$

- Finally, by introducing the constants ($c_1, c_2, ..., c_n$) into Eq. (3), the CTE function can be obtained, plotted versus T, and compared with the standard reference data.

4. Results and Discussion

4.1. Expansion visualization

Due to the supreme sharpness of shadowgraph image, it was easy to notice the expansion of filament at the moment of incandescence with bared eye, this could be probably an exclusive classroom demonstration. Fig. (6) shows the shadowgraph of 2 sample filament with different expansion index before and after incandescence. Fig. (6a & 6b) represents the expansion of sample F with $\mu = 7.3$, While Fig. (6c & 6d) represents the expansion of sample B with $\mu = 4.3$. Obviously the expansion of later sample is very remarkable as predicted before.

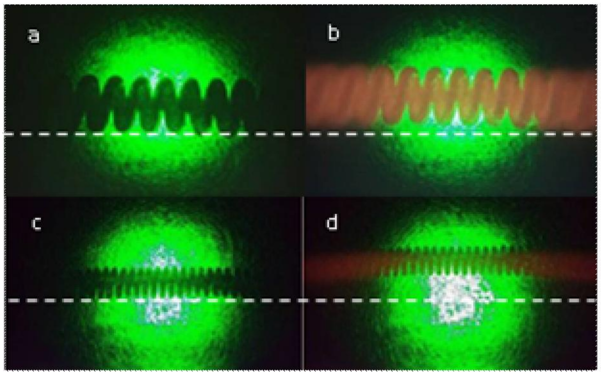

Figure 6. Shadowgraph of 2 sample filament with different expansion index before and after incandescence.

4.2. Expansion measurement

In order to evaluate the experimental results obtained by laser shadowgraphy, Fig. (7) Shows the measured value of linear expansion percent of different test samples in comparison with the reference data obtained by White and Minges [11,12]

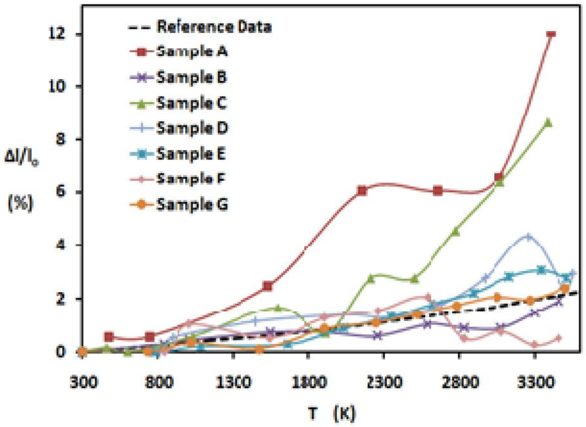

Figure 7. Linear expansion percent vs. temperature compared to tungsten reference data

As shown in Fig.(6), experimental results reveal that the thermometric curves of the tungsten filaments are not quite regular. To understand this behavior, samples must be classified at first according to their expansion profiles into three groups:

- Group I: *Conservative samples* (sample D, E and G), where the expansion behavior follows the reference model.
- Group II: *Progressive samples* (sample A and C), where the expansion process leaps the reference model.
- Group III: *Regressive samples* (sample B and F), where the expansion process reveals retardation from the reference model.

The factors behind the deviation of expansion of the samples from the reference model are numerous but it could be summarized as follows:

- The non linear thermal distribution along the filament length [13] due to heat loss in the thick supports.
- All measurements take place at the filament center, which is the hotter part of the sample, hence it becomes the softer part during heating; it loses its strength quickly leaving the filament extremities more stiff.
- As the sample continue to expand, internal stresses are being generated inside the confined filament, forcing the middle diameter to decrease until the thermal distribution along the filament become homogenous and the diameter continue to expand uniformly.
- The presence of clamping pre-stresses exerted by the supports on the filaments tends to reduce diameters of the filaments.

- The contribution of filament supports in the expansion process causing changes in the span of the filaments.
- Mechanical softening of the filament causes gravity to oppose the deflection caused by thermal strain.
- In addition to those factors, the behavior of potassium doped tungsten used as filament material differs slightly [14] from that of pure tungsten of the reference data.

Table.3 shows the coefficients (c_1 , c_2 ,c_3 ,c_4) of the 3rd degree polynomial regression equations for different expansion groups.

Table 3. Coefficients of polynomial regression equations for different linear expansion groups

Group	*C_0*	*C_1*	*C_2*	*C_3*
Reference	-1.41E-03	4.77E-06	-1.89E-10	2.03E-13
I	-1.70E-03	5.36E-06	-7.41E-10	3.93E-13
II	-2.69E-06	-1.76E-06	5.75E-09	-1.09E-12
III.	-1.16E-02	3.84E-05	-2.21E-08	6.01E-12
I & III	-1.70E-03	5.36E-06	-7.41E-10	3.93E-13
I, II & III	3.26E-05	-1.20E-06	5.46E-09	-6.44E-13

The precedent coefficients are used to model smoothly the linear expansion and the CTE of different sample groups as shown below in Fig. (8) and Fig. (9) successively.

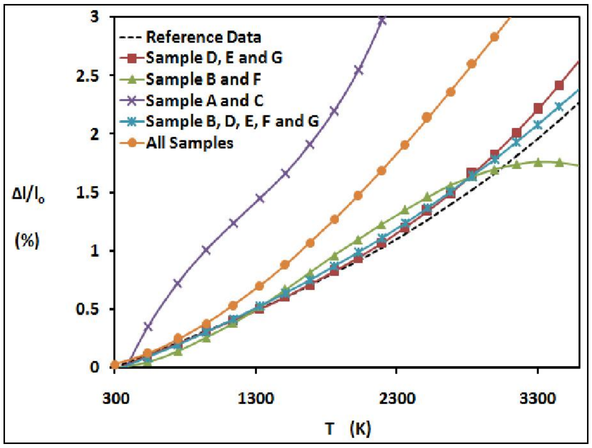

Figure 8. Linear Expansion Percent vs. Temperature Compared to Tungsten Reference Data for Different Groups of Samples

From the previous results, progressive samples must be excluded from the work (6 volts P21W bulbs was the worst samples), only 12 volts P21W incandescent light bulbs has prove to be the best samples for this application.

During test runs, this method enabled the observation of the predicted deflection of the tungsten filament at incandescence temperatures up to 3500 k, allowing the detection of 0.1 mm shadow displacement, at total optical magnification of about 2000X, which was equivalent to a resolution of 0.5 nm.

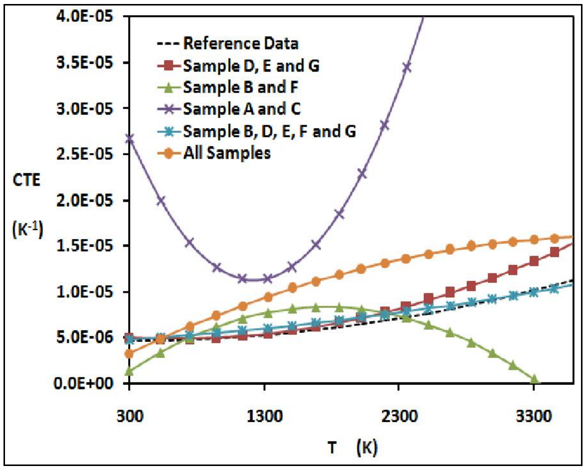

Figure 9. CTE vs. Temperature Compared to Tungsten Reference Data for Different Groups of Samples

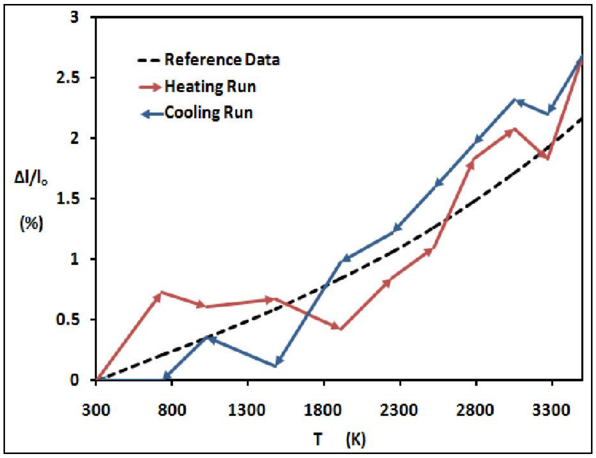

Figure 10. Linear expansion of sample G on heating and cooling

4.3. Expansion hysteresis

Tungsten is fabricated and alloyed by powder metallurgy process [15, 16], hence the fabricated filaments are very brittle due to their poor polycrystalline characteristics and lattice dislocations. When filament samples are heated for the first time it start to re-crystallize and therefore their thermal expansion behavior differs on cooling and showing thermal expansion hysteresis [17, 18] as seen in Fig. (10). In order to minimize this phenomena all samples must be preheated to 1000K for at least one hour before testing to allow metallic grain size to stabilize. For this reason and due to phase changes, a negative CTE is noticed for a while during some test runs.

5. Conclusions

- Laser shadowgraphy of simple 12 volts P21W automotive incandescent light bulb can be used to study the thermal expansivity of tungsten at elevated temperatures.
- Measurements with this method may take place at undergraduate laboratory for the first time, enabling the demonstration of the near melting thermometric properties of tungsten.
- The measured tungsten CTE came in good agreement with the previously published data.
- A one watt green (532 nm) commercial semiconductor laser has prove to be a suitable backlighting source enabling the detection of expansion and bending of confined filament under the effect of thermal strain at incandescence temperatures. This could explain the buzzing sound of some filament bulbs [19, 20].
- The expansion behavior of the samples shows some hysteresis on cooling, a phenomena that was reported before by many literatures.
- This low cost method successfully attained a resolution of 5 nm and can be adapted for professional use, if the light bulb is replaced by a sealed test cavity, with transparent windows, conductive supports to clamp the sample, vacuum pump, optical pyrometer for thermometry, and an automated imaging interface.

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