Sinusoidal Frequency Estimation in Strong Chaotic Noise

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Abstract: The problem of sinusoidal frequency estimation in chaotic noise is considered in this paper. Since sinusoidal signal and a class of chaotic signals have different forth-order- cumulant character. Construct a high-order- cumulant data series of mixed signal, the cumulant data series has a new sinusoidal signal whose frequency is in proportion to the sinusoidal signal to be detected. Using empirical mode decomposition (EMD) method ,the cumulant data series can be decomposition to a series of intrinsic mode functions (IMFs),among which one IMF is the recovered sinusoidal signal .One merit of the algorithm is that it suit for the hybrid noises which is the mixture of chaotic signal and strong Gaussian colored noise, another merit is that it need not to know the mathematical model of the chaotic noise. The Simulation results show that the proposed algorithm is easy to implement , robust, and less complicated in calculation. [New York Science Journal. 2008;1(3):12-19]. (ISSN: 1554-0200).

Keywords: empirical mode decomposition; chaos; sinusoidal signal, forth-order-cumulant

1. Introduction

Signal generated by chaotic systems represent a potentially rich class of signals processing problems because of its highly complexity and randomicity .Recent years , a number of scholars have investigated the chaotic signal processing because a wide range of signal processes including sea clutter[1], electrocardiograph signals [2] , indoor multipath [3] and speech [4] have been demonstrate to be chaotic rather than purely random .A variety of problems involving frequency estimation in chaotic noise therefore arises in potential application context .In some cases, this problem occurs when a chaotic signal is used purposely such as narrowband interference cancellation in a chaotic direct sequence code division multiple access communication systems and communication channel identification .in other scenarios such as radar surveillance in a ocean environment[1] and angle of arrival estimation in multi- path .Obviously, the sinusoidal frequency estimation from a chaotic noise is very important in theory and application. In this research field, the phase space volume method is introduced to estimate the coefficients of an autoregressive spectrum [5], the detection of a small target in sea clutter is investigated by means of neural network method [6]. The use of nonlinear dynamic (NLD) forecasting is considered to extract messages from chaotic communication systems [7].Base on the geometry of chaotic interference, a method for signal extraction from received data contaminated with strong chaotic interference is proposed [8]. A new nonlinear technique, referred to as empirical mode decomposition (EMD), has recently been pioneered by Huang et al. [9], it was proved to be remarkably more effective than other signal processing methods for nonlinear signals [10][11].

It's easy to estimation sinusoidal frequency when it is contaminated by most common noises. However ,when sinusoidal signal is submerged in the mixture of chaotic signal and strong Gaussian colored noise, most of conventional signal process methods failed, especially when sinusoidal signal frequency is in the middle of the frequency band of chaotic noise.

In this paper we focus our attention on the high-order-cumulant of the mixed signals .Since chaotic noise and sinusoidal signal have different character on forth-order-cumulant. By choosing proper time lags a new forth-order- cumulant data series is available, then using empirical mode decomposition approach, the sinusoidal frequency can be estimated. The following part give detail of the approach , then examples are given on the sinusoidal signals extraction from hybrid noise (chaotic noise and Gaussian colored noise). The simulation results show that the method is effective and satisfied.

2. Basic theories

Chaotic signal is a kind of special signal which is irregular but deterministic motion , and most of the signal processing methods of random signals failed to chaotic noise. For a kind of chaotic signals (Duffing , Lorenz ,Rossler,chen etc) ,which have no significant power beyond certain frequency on power spectrum ,as shown in fig1 ,if sinusoidal frequency is in the center of frequency band of the strong chaotic signal ,it will be difficult to estimation sinusoidal frequency. However, forth-order-cumulant provide a bridge to the problem . For a sinusoidal signal

$$x(t) = A * \sin(\omega_0 t + \varphi) \qquad (1)$$

, whose one dimension diagonal slice of forth- order-cumulant is

$$c_{4x}(\tau) = c_{4x}(\tau,\tau,\tau) = -3 \times A^4 \cos(\omega_0 \tau)/8 \qquad (2)$$

For a hybrid signal $y(t) = x(t) + n(t) + w(t)$,

where n(t) is chaotic signal ,w(t) is Gaussian colored noise, and supposing the chaotic signal is non-correlation with sinusoidal signal ,if time lags are chosen $m \times \tau_0 \times k$ ($k = 1,2,\Lambda\ ,N$) where,m is a positive constant, τ_0 is the sampling interval , a new forth-order-cumulant data sequences of the mixed signal are:

$$[c_{4y}(m\times\tau_0), c_{4y}(m\times\tau_0\times 2), \Lambda\ , c_{4y}(m\times\tau_0\times N)] = [\frac{3\times A^4}{8}\cos(m\omega_0\times\tau_0) + c_{4n}(m\omega_0\times\tau_0), \frac{3\times A^4}{8}\cos(m\omega_0\times 2\tau_0) + c_{4n}(m\omega_0\times 2\tau_0), M \frac{3\times A^4}{8}\cos(m\omega_0\times N\tau_0) + c_{4n}(m\omega_0\times N\tau_0)]$$

(3)

Because the forth-order-cumulant of Gaussian colored noise is zero, The upper data sequences are equal to the sum of the harmonic signal whose frequency is $m\omega_0$ and the forth-order-cumulant of the chaotic signal ,the sampling interval of new data sequence is τ_0. Since the chaotic sequences are non-periodic , and its forth-order-cumulant sequences are non-periodic too. More importantly, the power spectrum of the forth-order-cumulant data sequences of the chaotic signal has low frequency character ,as shown in fig2. Though the harmonic frequency to be estimated is in the center of the bandwidth of chaotic signal , the increase of the harmonic frequencies in the new sequence make it beyond the central bandwidth of the other components. Thus it reduces the difficulty of the detection of the harmonic frequency. Since the harmonic frequency of the new sequence is proportional to the original harmonic frequency, it is possible for the detection of the harmonic frequency.

Empirical mode decomposition is a powerful tool for nonlinear signal processing. It is based on the local characteristic time scale of the data, it is able to decompose complex signals to a collection of intrinsic mode functions (IMFs). Most important of all, it is adaptive.

At any given time, the data may involve more than one oscillatory mode, each mode, for linear or nonlinear, has same numbers of extrema and zero crossings, and only one extremum between the successive zero crossings. These modes should all be orthogonal to each other for a linear decomposition. Thus, an arbitrary signal can be decomposed to a collection of IMFs.

An intrinsic mode function (IMF) is a function that satisfies two conditions:

(1) In the whole data set, the number of extrema and the number of zero crossings must either equal or differ at most by one; and

(2) at any point, the mean value of the envelope defined by the local maxima and the envelope defined by the local minima is zero.

Comparing with simple monotone function, an IMF is a simple vibrating mode. Given a signal x(t), the algorithm of EMD can be summarized as follows:

(1) Identify all extrema of x(t), connect all the local maxima a cubic spline line as the upper envelope;

(2) connect all the local minima a cubic spline line as the upper envelope, the upper and lower envelopes should cover all the data between them;

(3) the mean of upper and lower envelopes is designated as m1, and the difference between the data and m1 is the first component, h_1, i.e.

$$h_1(t)=x(t)-m(t) \qquad (4)$$

If h_1 is an IMF, h_1 is the first component of x(t).

(4) if h1 is not an IMF, h1 is treated as the original data, continue the step (1), (2) and (3), get the mean of upper and lower envelopes, which is designated as m11, if $h_{11}=h_1-m_{11}$ is still not an IMF, continue the steps (1) – (3), until the first component h_{1k} is an IMF, and designated as $c_1 = h_{1k}$. c1 is the first IMF component of x(t);

(5) separate c_1 from the rest of the data by

$$r_1(t)=x(t)-m(t) \qquad (5)$$

Since the residue, r_1, still contains information of longer period components, it is treated as the new data and subjected to the same sifting process as described above, get the second IMF component of x(t) designated as c_2, the above procedure can be repeated to get nth IMF component until the residue, r_n becomes a monotonic function from which no more IMF can be extracted. Thus, we achieved a decomposition of the data x(t) into n-empirical modes, and a residue r_n, where ci, (i = 1, ⋯,n), contain different component of the signal from high to low frequency bands respectively. Frequency components in each band are different to other bands. The residue r_n is the mean trend of signal x(t).

3. Simulations

The chaotic signal which is created by Lorenz equation are adopted as the background signals. The equation can be written as follows:

$$\dot{x}_1 = p_1(x_2 - x_1)$$
$$\dot{x}_2 = p_2 x_1 - x_1 x_3 - x_2$$
$$\dot{x}_2 = x_1 x_2 - p_3 x_3 \qquad (6)$$

where: $p_1 = 10, . p_2 = 8/3, p_3 = 28$.In the paper, the classical fourth order Runge-Kutta algorithm is used to gain the time series of the equation (6). The initiatory 10000 data points are abandoned in the experiments to ensure the chaotic signal x(k) isn't affected by the initial conditions. To validate the effectivity of the method, the harmonic signal is submerged in the background signal which is the mixed of chaotic signal and Gaussian colored noise . When Gaussian white noise pass though a low pass filter, the Gaussian colored noise is available , here the bandwidth of low pass filter is 20hz. In the following , harmonic signals are estimated by using the proposed method in different conditions.

3.1.Simulation 1

Here the interference signals are chaotic signal and colored noise. The initial values of the Lorenz system are（1，5，9），the harmonic signal is ω_0=2.5HZ , A=0.7, the Gaussian colored noise is the noise which Gaussian white noise pass though a low pass filter , the variance of Gaussian white noise is 15 and the signal sampling frequency is 100HZ. The figure 3 shows the sinusoidal signal is submerged by hybrid signals completely.

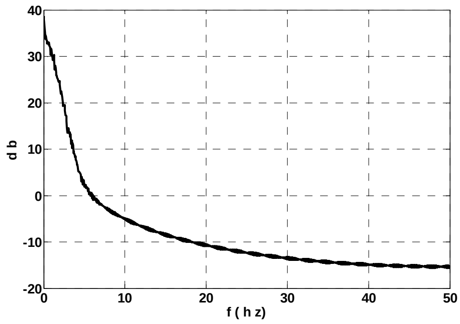

Fig 1. The power spectrum of x1 belonging to Lorenz system

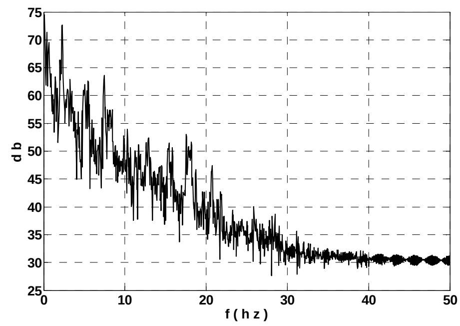

Fig 2. The power spectrum of forth-order -cumulant of x1 belonging to Lorenz system

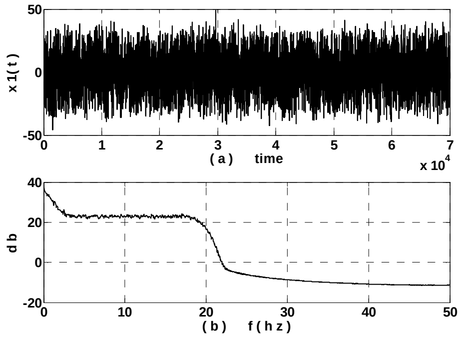

Fig 3. (a) the time series of the hybrid signals (b) the power spectrum of the hybrid signals

In order to make the sinusoidal frequency of the new data sequences does not exist in the central frequency bandwidth of the other components , the value of m should choose a bigger value. Figure 4a shows the part of first mode component c1 which is created by EMD decomposition of the new data sequence, since the modes of the EMD decomposition still contain a few adjacent mode components, the cross power spectrum is used to detect the sinusoidal frequency of the mode components. Figure 4b shows that the sinusoidal frequency to be detected is 25/10=2.5HZ. To validate the reliability of the method, figure 5 shows the cross power spectrum of c1 ,where m=12, the sinusoidal frequency to be detected is also 30/12=2.5HZ.

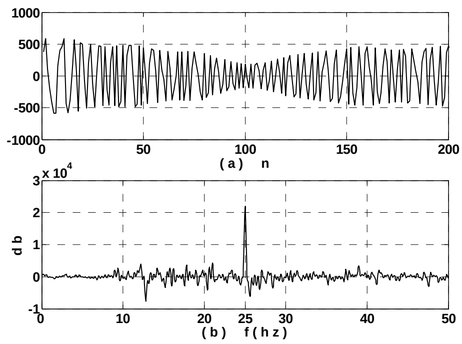

Figure4. (a) part of the first mode component (c1) of new data sequence (b) m=10, the cross power spectrum of c1

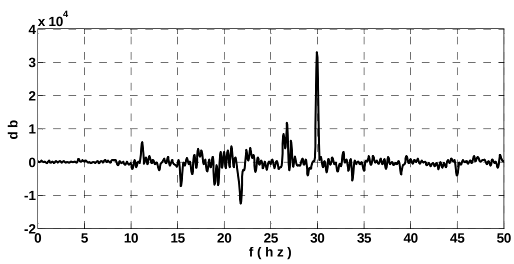

Fig 5. m=12, the cross power spectrum of c1

3.2. Simulation 2

The initial values of the Lorenz system are（10，5，9），the sinusoidal frequency is 3HZ and the amplitude is 0.6 , the variance of the Gaussian white noise is 15 , the signal sampling frequency is 100HZ. Figure 6 shows the mixed signal time series and its power spectrum. Figure 7 shows the cross power spectrum of c1 which is the first mode of new data sequence, where m=10. The sinusoidal frequency to be estimation is f=36/12=3HZ.

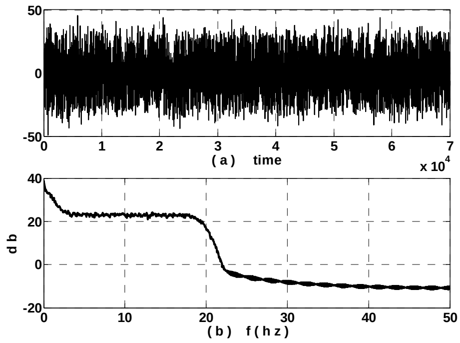

Fig 6. (a) the time series of the mixed signals (b) the power spectrum of the mixed signals

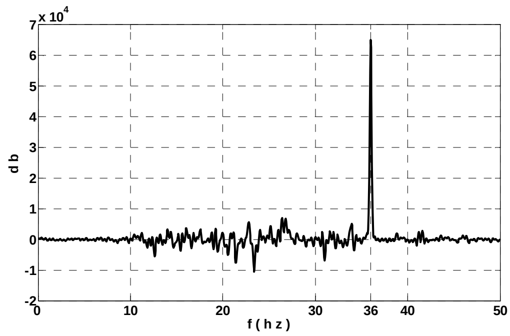

Fig 7. m=12, the cross power spectrum of c1

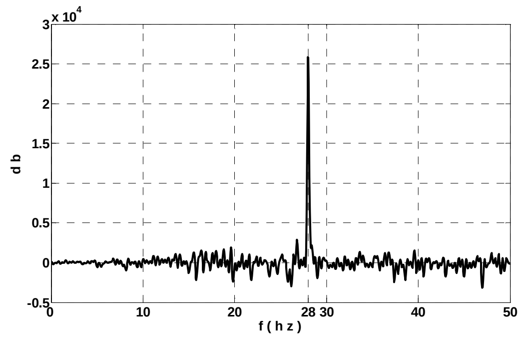

Fig 8. m=8, **the cross power spectrum of c1**

3.3. Simulation 3

Here the interference signals are chaotic signal and Gaussian colored noise. The initial values of the Lorenz system are（1，5，9），the sinusoidal frequency is 3.5HZ, the amplitude is 0.7, the variance of the Gaussian white noise is 15. Figure 8 shows the cross power spectrum of c1 ,where m=8. It's clear that the sinusoidal frequency is f=28/8=3.5HZ.

4. Conclusions

Chaotic signal process is a hot topic in recent years. Because of the high complexity of the chaotic signal, the general methods to detect the signals in the chaotic background are limited. In the paper, a new method is proposed. It uses the forth-order-cumulant character of the chaotic signal and sinusoidal signal . By adopting some specific principles ,a new forth-order- cumulant data sequences is available. In the new data sequences, the increase of the sinusoidal frequency makes it not exist in the central bandwidth of other components .this can reduce the difficulty of detecting sinusoidal signal. Based on empirical mode decomposition theory, the sinusoidal signal frequency is detected from the new reconstructed data sequence . Since the Gaussian colored noise has inherent forth-order- cumulant character , the method can be used to detect harmonic signal frequency which exists in the complex interference background such as the strong chaotic signal and Gaussian colored noise. In the simulating experiments, sinusodial signals are submerged in the strong hybrid noise, which can't be detected either in the time domain or in the frequency domain , however ,the proposed approach can estimation the sinusoidal frequency accurately in different conditions. The method is simple and feasible, it needn't to know the general parameter and initials of chaotic system and the calculations of the algorithms are small, it has important meaning in the practical application of chaotic signals .

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