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Scale Analysis of Fluid Flow and Forced Convective Heat Transfer in the Entrance Region of Elliptic Conduits

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Abstract: The paper presents heuristic scale technique to study fluid flow and heat transfer in the entrance region of elliptic ducts starting with conservation laws of mass, momentum and energy transport. The method provides a powerful scheme to approximate heat transfer parameters required for the design of heat exchangers as in the case of solar collector, automotive radiator, transformer and other heat transfer equipment. A generalized boundary condition in which no mode of heating is specified is imposed on the fluid flow. Generalized equations of thermal entrance length $L_t^{/}$, hydrodynamic entrance length $L_h^{/}$, friction coefficient $f^{//}$, axial pressure drop $dp^{/}$, and Nusselt number $Nu^{/}$ are developed. The results obtained from the analysis show that the generalized Nusselt number, thermal entrance length and hydrodynamic entrance length are independent of fluid properties and therefore depends solely on the eccentricity of the ellipse. The results also indicated that the Nusselt number at the end of minor axis is greater than that of the major axis. Also obtained is hydrodynamic characteristic fRe=16 for e=o whch compares well with the value compiled by Necati Ozsiki for a circular geometry (e=0). [New York Science Journal. 2009;2(3):59-71]. (ISSN: 1554-0200).

Keywords: Analytical, Fluid, Scale, Entrance, Ellipse

INTRODUCTION

The present investigation, using a generalized boundary condition, is to establish the validity of scheme of scale analysis for the solution of engineering problems and to predict parameters required for design of heat exchanger as in the case of solar collector, nuclear reactor, power plant radiator and some other heat transfer equipment. However, the scheme has proved its worth as a hypothesis in engineering and technology by giving an appropriate theoretical analysis to generate a generalized condition to predict both hydrodynamic and thermal entrance lengths in any elliptic configuration. Among previous researchers who had worked in closely related areas are Horneck[1], Shah[2] and Wiginton[3]. They studied laminar flow in entrance region of pipe, rectangular duct and parallel plates respectively. Abdel-Wahed et al.[4] also carried out extensive practical investigation in study of laminar developing and fully developed flows and heat transfer in horizontal elliptic duct. Numerical study of entrance region heat transfer was considered by Webb[5]. Kutcher et al.[6] analyzed some aspect of unglazed transpired solar collector using scale approach. Kutcher[7] also introduced scale analysis in solving some parts of his problem. Analysis of laminar flow and heat transfer in the entrance region of an internally finned circular duct has been investigated by Parakash[8]. Bello-Ochende and Adegun[9] studied heat transfer in polygonal ducts using scale technique. Adegun[10] also investigated heat transfer in the entrance of concentric elliptic annulus using scaling method. Bello-Ochende and Adegun[11] also studied heat transfer in elliptic duct using perturbation technique. Osizik[12] compiled results for generalized thermal and hydrodynamic entrance lengths for various geometries.

Focus of the Work

The present study is an investigation of forced convection and fluid flow in elliptic ducts. The authors adopted an approximate method called scale analysis. A physical model of the problem is shown in Fig.1. The duct is divided into two distinct regions: entrance and fully developed region. Only the entrance region is analyzed, a generalized condition in which no mode of heating is specified is assumed at the entrance region. For each of the problem to be solved, the scale most representative of the appropriate equation was analyzed heuristically. The scale analysis was carried out manually while computer algorithm was developed to generate the result.

2. FORMULATION OF THE GOVERNING EQUATIONS

The physical model and cylindrical-polar coordinate (r,z,φ) system are shown in Fig. 1.

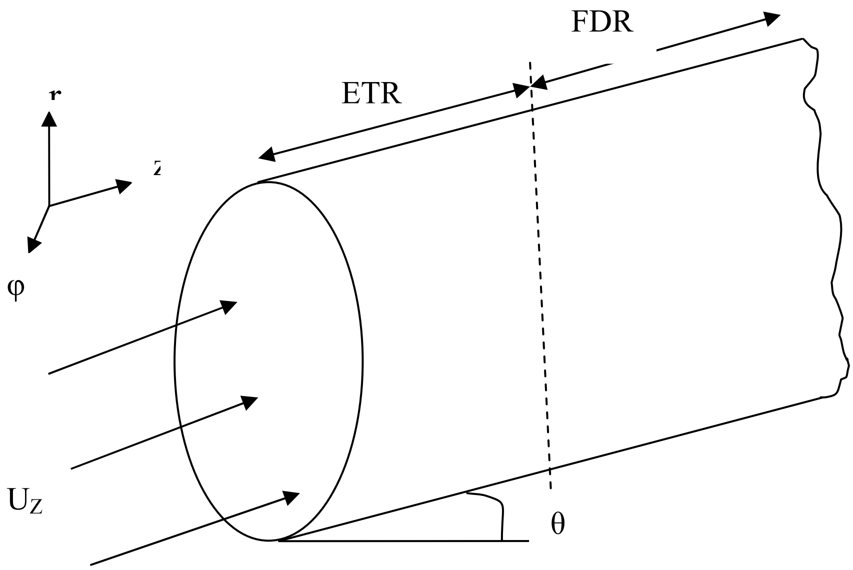

Fig. 1: The Physical Model and the Coordinate Axis

Ur, Uz and Uφ are the velocity components in r, z and φ- coordinates respectively. The working fluid is assumed Newtonian, incompressible with constant property. It is assumed that the velocity profile at the entrance is uniform and the working fluid is air. Effect of inclination is assumed to be negligible because of the potentiality of the flow which is likely to be very high at the entrance of the duct. However, the following specifications are made for the validity of the entrance formulae.

(i) The boundary layer thickness, δ is far less than the characteristic length, L of the duct

$$\delta << L$$

(ii) The following scale of changes and order of proportionality are identified in axial and radial directions:

$$z \sim L, \quad r \sim \delta \text{ and } U_z \sim U_\infty$$

(iii) For a conduit at the transition point,

$$\delta_h \sim \delta_t \text{ for } Pr \sim 1,$$

(iv) At the point where the flow is hydrodynamically fully developed,
$L \sim L_h$,
(v) At the point of thermally developed fluid flow,
$L \sim L_t$.
The following equations govern the fluid flow and heat transfer in the elliptic pipe.

2.1 Continuity Equation:
For a steady state incompressible fluid flow,

$$\frac{\partial U_r}{\partial r}+\frac{U_r}{r}+\frac{\partial U_\phi}{r\partial \phi}+\frac{\partial U_z}{\partial z}=0 \quad (1)$$

2.2. Momentum Transport Equations.
For constant velocity and density, the momentum equations for a steady flow in polar coordinate system are

Radial direction:

$$M_r=-\frac{\partial P^{/}}{\rho\partial r}+\nu\left(\nabla^2 U_r-\frac{U_r}{r^2}-\frac{2\partial U_\phi}{r^2\partial\phi}\right) \quad (2)$$

Azimuthal direction:

$$M_\phi=-\frac{\partial P^{/}}{\rho\partial\phi}+\nu(\nabla^2 U_\phi-\frac{U_\phi}{r^2}+\frac{2\partial U_r}{r^2\partial\phi}) \quad (3)$$

Axial direction:

$$M_Z=-\frac{\partial P^{//}}{\rho\partial Z}+\nu\nabla^2 U_Z \quad (4)$$

Where,

$$M_r=U_r\frac{\partial U_r}{\partial r}+\frac{U_\phi\partial U_r}{r\partial\phi}-\frac{U_\phi^2}{r}$$

$$M_\phi=U_r\frac{\partial U_\phi}{\partial r}+\frac{U_\phi\partial U_\phi}{r\partial\phi}-\frac{U_rU_\phi}{r}$$

$$M_z=U_r\frac{\partial U_z}{\partial r}+\frac{U_\phi\partial U_z}{r\partial\phi}+U_z\frac{\partial U_z}{\partial z}$$

$$\nabla^2=\frac{\partial}{\partial r^2}+\frac{\partial}{r\partial r}+\frac{\partial^2}{r^2\partial\phi^2}+\frac{r^2}{\partial z^2}$$

2.3 Energy Transport Equation

In the absence of energy source and viscous dissipation; if radiation is neglected, the energy transport equation for a steady flow is

$$U_r \frac{\partial T}{\partial r} + U_\phi \frac{\partial T}{r\partial \phi} - U_z \frac{\partial T}{\partial z} = \alpha \nabla^2 T \qquad (5)$$

3 CONCEPT OF HYDRODYNAMIC AND THERMAL BOUNDARY LAYER

This analysis is of practical interest, particularly to the engineers. It establishes the hydrodynamic and thermal characteristics of fluid flow at the entry of conduits. Attention is focused on hydrodynamic and heat transfer results. The governing equations are intuitively scale to yield a generalized form of results under a generalized boundary constraint.

3.1. Hydrodynamic Analysis

The analysis will help to determine the hydrodynamic entrance length, L_h for flow inside conduits. This length is defined as the length required from the duct inlet to achieve a maximum velocity of 0.99 of the corresponding fully developed value. The scale analysis of the continuity equation (1) is:

Thus,

$$\frac{U_r}{\delta_h}, \frac{U_\phi}{\delta_h}, \frac{U_\infty}{L} = 0 \qquad (6)$$

$$\frac{U_r}{\delta_h} \Box \frac{U_\phi}{\delta_h} \Box \frac{U_\infty}{L} \qquad (7)$$

$\delta_h \sim r_h$ the hydraulic radius defined as

$$r_h = \frac{A^*}{P^*} \qquad (8)$$

To complement the above scaling, the momentum equation (4) in the axial direction is also scaled to become,

$$\frac{U_r U_\infty}{\delta_h} \sim \frac{U_\phi U_\infty}{\delta_h} \sim \frac{U_\infty^2}{L} = \frac{P}{\rho L}, \nu \frac{U_\infty}{\delta_h{}^2}, \nu \frac{U_\infty}{L^2} \qquad (9)$$

Where from equation (9),

$$\frac{U_\infty^2}{L} \sim \nu \frac{\mathrm{U}_\infty}{\delta_\mathrm{h}^2}, \ and \ \frac{U_r U_\infty}{\delta_h} \sim \nu \frac{\mathrm{U}_\infty}{\mathrm{L}^2} \ and \ P \sim \mathrm{P}_\infty$$

Hence, the following relations can be deduced

$$\delta_h = L \,\mathrm{Re}_L{}^{1/2} \ and \ U_r \sim U_\infty \mathrm{Re}_L{}^{-3/2} \qquad (10)$$

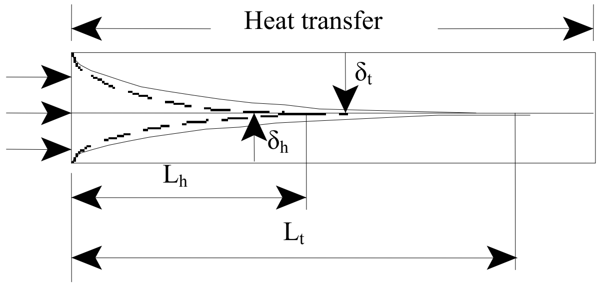

Fig.2: A typical sketch showing hydrodynamic and thermal entrance lengths

3.1.1Estimation of Generalized Hydrodynamic Length

It is obtained from equation (10) that

$$\delta_h \sim L\,\mathrm{Re}_L^{1/2} \qquad (11)$$

Introducing the specified parameters and scale of changes into equation (11) yields

$$L_h^{/} = \frac{1-e^2}{4(1+\sqrt{1-e^2})} \sim \frac{L_h}{\mathrm{Re}_D\, D_h} \qquad (12)$$

Where,

$$a^2 - b^2 = a^2 e^2$$

$$\delta_h = \frac{a\sqrt{1-e^2}}{1+\sqrt{1-e^2}},\quad V_r = \frac{U_r}{U_\infty}$$

The generalized hydrodynamic boundary layer is

$$\delta_h^{/} = \frac{\delta_h}{a} = \frac{\sqrt{1-e^2}}{1+\sqrt{1-e^2}} \qquad (13\ )$$

$L_h^{/}$, $\delta_h^{/}$, and V_r are the generalized non-dimensional hydrodynamic entrance length, hydrodynamic boundary layer thickness and radial velocity respectively.

3.1.2 Generalized Friction Coefficient

The scale most representative of shear stress equation,

$$\tau = \mu\frac{\partial u}{\partial r},\ \text{is}$$

$$\tau \sim \frac{\rho U_\infty V}{\delta_h} \qquad (14)$$

Introducing the appropriate scale of changes gives a generalized form of hydrodynamic characteristic of the duct flow as

$$f^{/}\,\mathrm{Re} = \frac{8\left(1+\sqrt{1-e^{2}}\right)}{\sqrt{1-e^{2}}} \qquad (15)$$

Where,

$$f^{/} \sim \frac{\tau}{\frac{1}{2}\rho U_{\infty}}$$

3.1.3 Generalized Static Pressure Distribution

The axial pressure drop in the entrance region of a horizontal elliptic pipe is obtained from momentum equation in the axial direction. The scaling of the equation yields

$$\frac{U_{\infty}^{2}}{L} \sim -\frac{1}{\rho}\frac{dP}{L} \qquad (16)$$

The generalized pressure drop is obtained as

$$\frac{dP}{L} \sim \frac{\frac{1}{2}\rho U_{\infty}^{2}}{\mathrm{Re}_{D} D_{h}} \qquad (17)$$

where, $dP = P_{\infty} - P_{Lh}$

Equation (17) indicates that the pressure gradient is maximum at the entrance of the pipe and decreases monotonically with downstream distance until it reaches a constant value. The equation also shows that the pressure drop dP increases with axial distance and inversely proportional to Reynolds number, Re_D.

3.2 Concept of Thermal Boundary Layer

The analysis presents a well defined equation for generalized thermal entrance length, $L_t^{/}$ and Nusselt number, $Nu^{/}$ at the end of major and minor axes of the ellipse. The scheme employs the energy transport equation for the derivation of the equations.

3.2.1 Evaluation of Thermal Entrance Length

For the energy transport equation (5), the scale analysis is,

$$\frac{U_r \Delta T}{\delta_t}, \frac{U_{\phi}\Delta T}{\delta_t}, \frac{U_{\infty}\Delta T}{L} \sim K_{\alpha} \qquad (18)$$

Where,

$$K_{\alpha} = \alpha\left[\frac{\Delta T}{\delta_t^{2}}, \frac{\Delta T}{\delta_t^{2}}, \frac{\Delta T}{L^{2}}\right]$$

Thus, the scale most representative of equation (18) is

$$\frac{U_{\infty}\Delta T}{L} \sim \alpha\frac{\Delta T}{\delta_t^{2}} \qquad (19)$$

From equation (19), it is derived that the generalized entrance length is

$$L_t^{/} \sim \frac{L_t}{D\,\mathrm{Re}\,\mathrm{Pr}} \sim \frac{1-e^{2}}{\left(1+\sqrt{1-e^{2}}\right)^{2}} \qquad (20)$$

L_t is local thermal entrance length.

3.2.2 Evaluation of generalized Mean Nusselt number

Nusselt number is a non dimensional parameter indicative of the ratio of energy convection to conduction.

Nusselt number at the end of Major Axis

The fundamental equation for Nusselt number is

$$NuD = \frac{hD}{K} \tag{21}$$

where h is convective heat transfer coefficient. It is given as

$$h = \frac{K}{\delta_t} \tag{22}$$

The generalized Nusselt number at the end of major axis is obtained as,

$$NuD \sim \frac{2\left(1+\sqrt{1-e^2}\right)}{\sqrt{1-e^2}} \tag{23}$$

Nusselt number at the end of Minor Axis

Recall equation (21), and express as

$$NuB \sim \frac{D}{\delta_{tm}} \tag{24}$$

In this case, δ_{tm} is the thermal boundary layer thickness along the minor axis. Fig. 3 is considered for continuity of the boundary layer thickness.

$$\delta_{tmj} = \delta_{tmn} + \delta_{tz} \tag{25}$$

For continuity of the boundary layer thickness, the following expression is employed:

$$\frac{\delta_{tmn}}{\delta_{tmj}} \delta_{tmj} \sim \delta_{tB} \tag{26}$$

Where δ_{tB} is the minor thermal boundary layer thickness at the transition point.

The Nusselt number at the end of minor axis is

$$NuB = \frac{2\left(1+\sqrt{1-e^2}\right)}{1-e^2} \tag{27}$$

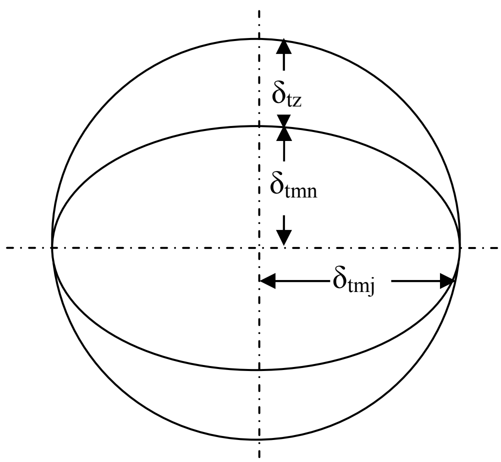

Fig.3: Enlarged Section of the Duct

4. RESULTS AND DISCUSSION

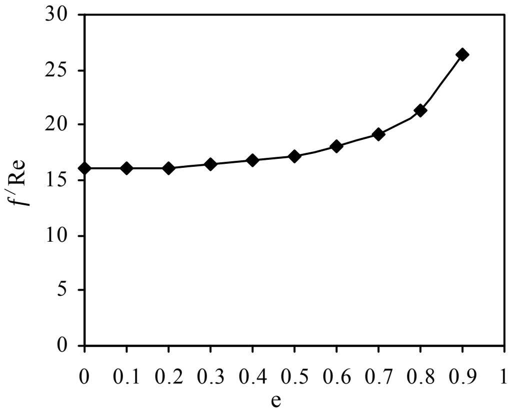

Fig. 4 Effects of eccentricity of the ellipse on f/Re

Fig. 4 shows the variation of hydrodynamic characteristic of the duct flow, f/Re with eccentricity. It is observed that f/Re increases with eccentricity, and f/Re =16 for e = 0, which corresponds to a circular geometry, compares well with those published in literatures.

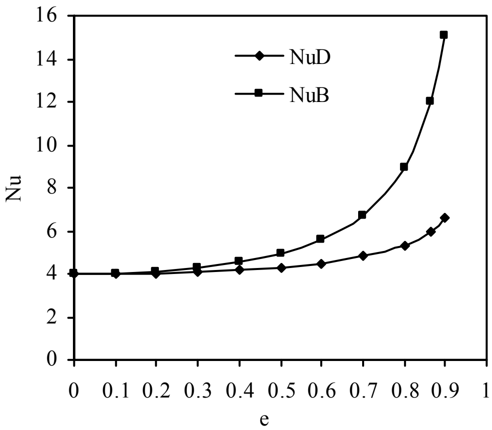

Fig. 5 Nusselt number at the end of major and minor axes

Fig.5 shows the relationship between Nusselt number and eccentricity of the ellipse at the end the minor and major axes. It is noticed in the plots that for $0 \leq e \leq 0.2$, the difference between the heat transfer rate at the end of major and minor axes is insignificant. Above e = 0.2, more heat is transferred from the end of minor axis. It is also noticed that the higher the value of eccentricity, e, the higher the difference between NuB and NuD for a given eccentricity. A critical eccentricity, e=0.866, is obtained for optimum heat transfer by the use of mathematical analysis in combination with the results obtained from the plots.

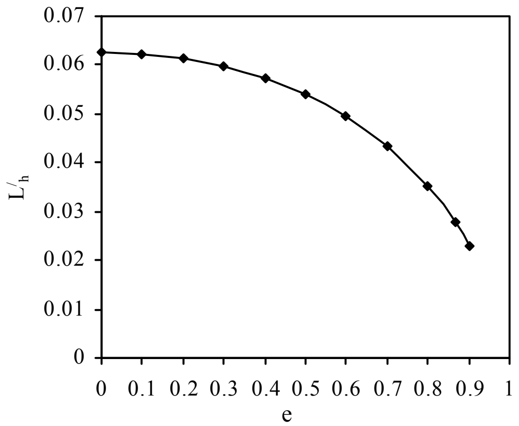

Fig. 6 Hydrodynamic entrance lengths for various eccentricities

Effect of eccentricity on hydrodynamic entrance length is given in Fig.6. The figure shows that the entrance length reduces with increasing value of eccentricity. The present generalized hydrodynamic entrance length 0.0277 lies within the values obtained by McComas (0.02447) and Abdel-Waheed (0.0345) for the same elliptic geometry of e=0.866.

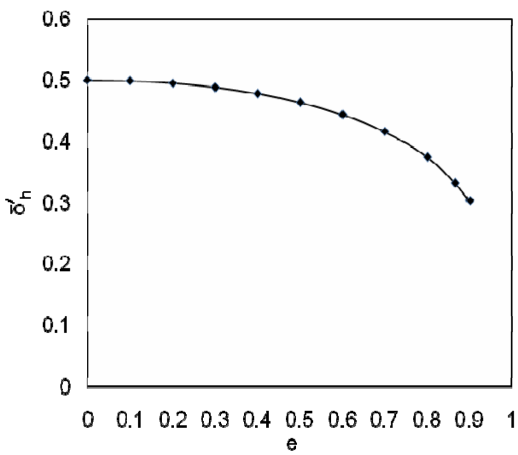

Fig. 7 Effect of eccentricity on hydrodynamic boundary layer thickness

Variation of hydrodynamic boundary layer thickness, δ'_h with eccentricity, e is shown in Fig.7. It shows that at the points of transition to fully developed flow, δ'_h reduces with increase in e.

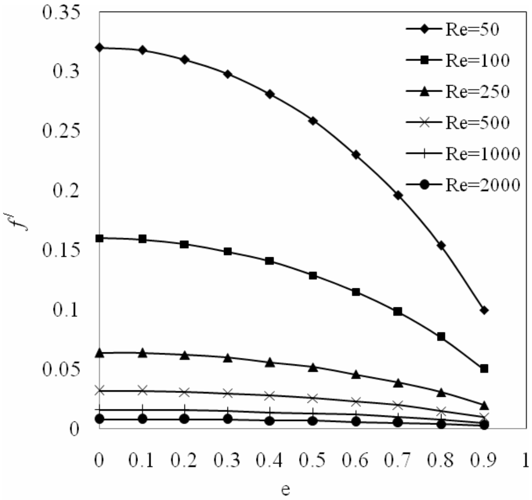

Fig. 8 Influence of eccentricity on friction for various Reynold's numbers

Fig.8 shows that the generalized friction values f' reduces with increase in e and Re. Also, towards Re = 2000 the plots becomes flattened which is an indication that as Re tends towards transition from laminar to turbulent flow, f' becomes almost a constant.

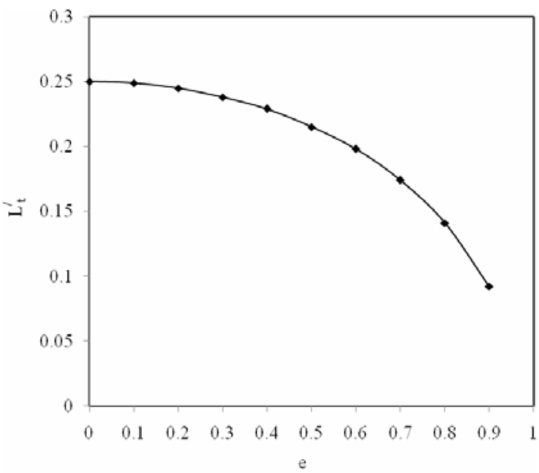

Fig. 9 Variation of thermal entrance length with eccentricity

Fig.9 depict that thermal entrance length and eccentricity are inversely related, with L'_t reduces with e.

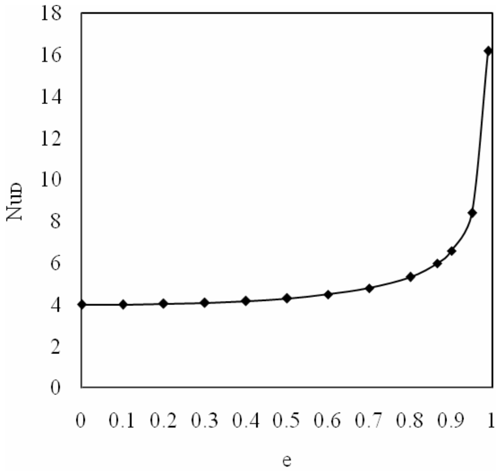

Fig. 10 Nusselt number at the end of major axes

Fig.10 shows a gradual increase in Nusselt number with increasing eccentricity. Above 0.866, that is, towards the collapse of the duct, heat transfer is not by convection alone, conduction plays its own part, and that is why there is sudden rise in the Nusselt number. Therefore 0.866 is the critical value for optimum heat transfer by convection.

CONCLUDING REMARKS

- Scale analysis has been found suitable method for solution of entrance problems.
- Generalized Nusselt numbers are independent of fluid properties. They depend solely on eccentricity of the duct.
- A critical eccentricity, e = 0.866, is attained for optimum heat transfer by convention.
- Mathematical expression to relate hydrodynamic characteristic of the duct flow and Nusselt number had been developed.

$$NuD = 0.25 f'\text{Re}$$

$$NuB = \frac{1}{\sqrt{1-e^2}} f'\text{Re}$$

- The relationship between NuD and NuB has also been established:

$$NuD = 0.25\sqrt{1-e^2}\ NuB$$

Nomenclatures

a Semi major axis of the ellipse
A* Cross sectional area
B Semi minor axis of the ellipse
B Minor diameter
C_p Specific Heat Capacity at constant pressure (J/kgK)

D_h Hydraulic Diameter
D Major Diameter
e Eccentricity of ellipse
ETR Entrance region
$F(r)$ Body force component in radial direction
$F(\phi)$ Body force in θ direction
$F(z)$ Body force component in the axial direction
FDR Fully developed region
g Acceleration due to gravity (m/s^2)
h Convective heat transfer coefficient
K Thermal conductivity of the working fluid
L Characteristic Length
L_h Local hydrodynamic entrance length
LHS Left hand side
L_l Local e thermal entrance length
NuB Nusselt number at the end of minor axis
NuD Nusselt number at the end of major axis
p'' Average pressure over the duct cross section
p' The small pressure variation governing the flow distribution in the cross stream plane
$\Pr$ Prandtl number, ν / a
$P*$ Wetted perimeter
Re Reynolds number
RHS Right hand side
T Dimensional fluid temperature
U_r Dimensional velocity in radial direction
U_ϕ Dimensional velocity in azimuthal direction
U_z Dimensional velocity in z direction
V_r Normalized cross flow velocity
Z Coordinate in the direction of flow (axial direction)

Greek Symbol

α Thermal diffusivity (m^2/s)
β Coefficient of thermal expansion
δ_h Local hydrodynamic boundary layer thickness
δ_l Local thermal boundary layer thickness
λ Angle of inclination

ν Kinematic viscosity of the fluid m^2/s

ρ Density, kg/m^3

Subscripts:

h Hydrodynamic

t Thermal

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