A Mathematical Model For Emission Control Of Industrial Pollution

Nwagwo, A.,[1,*,+] Adeosun, T. A.,[2] and Akora, J. A.[3]

[1]Department of Mathematics, Univerisity of Ibadan, Ibadan, Nigeria
E-mail:- naxader2000@yahoo.com
[2]Department of Mathematics, Yaba College of Technology, P.M.B. 2011, Yaba, Lagos, Nigeria.
E-mail: adebaba2001@yahoo.com
[3]Department of Civil Engineering, Yaba College of Technology, P.M.B. 2011, Yaba, Lagos, Nigeria
akojay@yahoo.com

ABSTRACT: In this paper, a mathematical model for controlling the generation of industrial pollution in a given economic system is presented. For each sector of the economy, the model determines the appropriate technologies that can be used to produce the amount of pollution allowed for the sector's external demand. The conditions for that model are relaxed is this paper. The relaxation makes the new model more realistic and more applicable than the previous one. [New York Science Journal. 2009;2(6):99-104]. (ISSN: 1554-0200).

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1. INTRODUCTION

The need to have a safe and pollutants-free environment is a widely discussed issue in the society today. Consequently, manufacturing companies are looking for ways to reduce the amount of pollutants they emit into the atmosphere. For companies to come to grips with the pollutant emission problem, they must work to eliminate from their production processes, those factors that cause high pollutant emission. In most industrial set-ups, a major factor that influences the amount of pollutants emitted into the atmosphere is technology. It is easy to see that two different sets of machines for producing an item emit different levels of pollutants into the atmosphere. The amount of pollutants produced by a given sector of the economy using a given technology is estimated. From this information, the amount of pollutants per unit output a sector produces using a given technology is calculated. Agencies charged with the responsibility of protecting the environment place restrictions on the amount of pollutants a sector of the economy can produce per unit output. In Nigeria, the Federal Environmental Protection Agency (FEPA) is charged with this responsibility. FEPA shall be the frame of reference in this paper.

* Corresponding: Alexander Nwagwo: P.O. Box 2268, Apapa, Lagos
e-mail: nexander2000@yahoo.com

among different goods [9]. If technology is fixed, then increase in final demand leads to increase in the level of pollutants emitted into the atmosphere. Also pollutants emitted by various industries increase or decrease if the technology matrix and final demand are held constant. Assuming minimal changes in a composition of final demand, choosing the right technology is the most important factor in reducing industrial pollution.

The following problem would be addressed in this paper given different technologies for the production of item j by industry $j, j = 1..., n$, which technology should be chosen by industry j so as to (1) satisfy, FEPA permissible pollutant levels for the industry, and (2) satisfy demand for output j as much as possible. By permissible pollutant level (PPL), we mean pollutant quantities, which place limits on the amount of pollutants an industry can produce; the actual quantities produced by the industries may be different from the PPL values. The PPL can be considered as the pollutant levels that are allowed for demand external to the economic system. Once the PPL values are determined, the system can generate enough pollutants to produce corresponding goods and services. Changing the PPL values lead to new emission standards. The Leontief production model shall play an important role in our formulation of the model.

The structure of the rest of the paper is as follows. In Section 2, the Leontief Input-output model is explained. Terminology associated with this model shall be used extensively in the paper. We formulate the model in section 3 and provide solutions technique in section 4. An example illustrating an application of the model is given in Section 5. In section 6, we summarise our results.

2. LEONTIEF PRODUCTION MODEL

The Lentitive input-output production model describes the inter-relationship among prices, production levels, and demands in a given economic system (8). For a fixed period of activities, the Leontief input-output production model is described by the equation.

$$x_j = \sum_{k=1}^{n} {}^{a} jk^{x} k^{+b} j \tag{1}$$

Where

n = Number of industries / Sectors in the system;

x_j = Total outputs from industry *j*;

b_j = Units available / needed at industry *j* to satisfy demand;

a_{jk} = Technical coefficient representing units of production of industry *j* required by industry k.

The vectors $x = [x_1, ..., x_n]^t$ x and $b = [b_1, ..., b_n]^t$ are called production vector and demand vector, respectively. The associated matrix $[a_{ik}]_{i,k}^{n}$ is called coefficient matrix or technology matrix. In the next section, we extend and modify this model to address the pollution control problem.

3. MODEL FORMULATION

Consider an economy with n industries, each of which must produce an item. Let *mj* ≥ 1 be the number of different technologies available for the production of output j by industry j. Let

x_j = monetary value of the amount of pollutants produce by industry *j*

b_j = Monetary value of the amount of pollutants require to satisfy external demand for industry *j* products

$a^{j}{}_{ik}$ = Monetary value of the output of pollutants by technology *i* resulting from the production of one unit of item *j* by industry *j* for industry *k*.

The requirement that the amount of pollutants produced by sector j is at most equal to the amount allowed for both internal and external demands is equivalent to:

$$xj \le b_i^j + \sum_{k=1}^{n} a_{ik}^j x_{kj} : i = 1, \ldots, m_j xj[\{\ \}] = 0, j = 1, \ldots, n. \tag{2}$$

Observe that the quantity $\sum a_{ik}^{j} x_k$ is the amount of pollutants needed to satisfy internal demands, and that this is not restricted in this model. One argument for not restricting pollutants generated in order to satisfy internal demand is that doing so could weaken the national economy.

Apart from meeting the FEPA requirement on pollution emission, it is important for an industry to meet the demand for its products as much as possible. To ensure that production levels of chosen technologies meet demands for goods and services, we add the condition:

$$xj\left[\max\left\{xj - b_i^j - \sum_{k=1}^{n} a_{ik}^j x_k \; : i = 1,\dots, m_j\right\}\right] = 0,\; j = 1,\dots, n. \qquad (3)$$

The technology chosen by sector j is the one that satisfies Equations (2) and (3), simultaneously. The mathematical model for Emission Control Industrial Pollution is now defined as follows (MMECIP): Given the amount of pollutants b_i^j, in monetary values, needed to satisfy emission restrictions for sector *j* outputs, and the pollution input-output coefficients a_{ik}^j, find the amount of pollutants x_j, in monetary value, that sector *j* should produce so that

MMECIP: $$xj \le b_i^j + \sum_{k=1}^{n} a_{ik}^j x_k,\; j = 1,\dots, n, 1 \le i \le m_j.$$

$$xj\left[\max\left\{xj - b_i^j - \sum_{k=1}^{n} a_{ik}^j x_k \; : i = 1,\dots, m_j\right\}\right] = 0,\; j = 1,\dots, n.$$

$$xj \ge 0,\; j = 1.,,,.n.$$

4. SOLVABILITY OF THE PROBLEM

In this section, we provide procedures for solving equation (2) and (3) simultaneously. We observe that the system in (3) is non-linear. Let

$$E = \begin{bmatrix} e_1 & 0 & \dots & 0 \\ 0 & e_2 & 0\dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & \dots & \dots o & e_n \end{bmatrix} \qquad d^j = \begin{bmatrix} b_i^j \\ . \\ . \\ . \\ b_{mj}^j \end{bmatrix} A^j = \left(a_{ik}^j\right)_{i=1, j=1}^{mj, n}$$

Where e_j is and m_j x 1 column vector of all ones. Moreover, define matrices *A* and *d* by

$$A = \begin{bmatrix} A^1 \\ : \\ A^n \end{bmatrix}, \quad d = \begin{bmatrix} d^1 \\ : \\ d^n \end{bmatrix}$$

Proposition: The vector x^* is a solution of the MP with optimal objective function value $z = 0$ if and only if it is solution of the MMECIP

Proof: We first observe that $G_j \ge 0$ for each *j, j = 1, .., n,* by Equation (2) and (4) suppose x^* is a solution of the *MP* with $z = 0$. Then Equation (2) is satisfied by the constraints of the *MP*. Moreover, we must have.

$$z = \sum_{j=1}^{n} x^*_j G_j = 0$$

since the optimal objective value is zero.

This implies that $x^*_j\ G_j = 0$ for each j, since x^*_j and G^*_j are both nonnegative. By equation (4) and $x^*_j\ G_j = 0,$ equation (3) is satisfied. Thus x^* solves the MMECIP.

Conversely, suppose x^* solves the MMECIP. It is easy to see that the constraints of the *MP* hold by equation (2), By equations (3)

$$x^*_j \left\{ \min_{1 \le i \le m_j} \left(-x^*_j + b_i^j + \sum_{k=1}^{n} a_{ik}^j x_k^* \right) \right\} = 0.$$

This Implies that $x^*_j\ G_j = 0.$ for each j. Consequently,

$$z = \sum_{j=1}^{n} x^*_j\ G_j = 0.$$

This completes the proof.

Remark: G_j is a piecewise linear function. If the minimum is attained at i = p, then

$$G_j = -x_j + b_p^j + \sum_{k=1}^{n} a_{pk}^j x_k$$

for each *j*, G_j can easily be selected by solving at most m_j linear programs. Consequently, the objective function in the *MP* can be transformed into a quadratic function that is easily solved by mathematical programming software such as GINO.

5. EXAMPLE

Consider an economy with 2 industries, say, auto and steel industries. Suppose that the auto industry has 3 technologies for producing cars, and the steel industry has 2 technologies for producing steel. Assume that in this economic system, the input-output pollutant coefficients are as given below

Auto Industry

Technology	Air pollution output coefficients in thousand of tons emitted per \$1m output.	Allowable pollution level for external demand in thousand of tons
Technology 1	.05 .10	405,000
Technology 2	.06 .11	405,000
Technology 3	.05 .09	405,000

Steel Industry

Technology	Air pollution output coefficients in thousand of tons emitted per \$1m output.	Allowable pollution level for external demand in thousand of tons
Technology 1	.08 .0145	600,000
Technology 2	.06 .18	600,000

We want to determine the appropriate technology for each of the two industries that provide pollutant levels not exceeding these PPL requirements. We shall use the *MP*. The constraints corresponding to the auto industry are:

$$\varsigma x_1 \le 405{,}000 + 0.5x_1 + 0.10x_2$$

$$x_1 \le 405{,}000 + 0.6x_1 + 0.11x_2$$
$$x_1 \le 405{,}000 + 0.5x_1 + 0.09x_2$$

The constraints of steel industry are:

$$x_2 \le 600{,}000 + 0.8x_1 + 0.0145x_2$$
$$x_2 \le 600{,}000 + 0.6x_1 + 0.018x_2$$

The *Gs* are computed as follows:

$$G_1 = \min\begin{Bmatrix} -x_1 + 405{,}000 + 0.5x_1 + 0.10x_2, \\ -x_1 + 405{,}000 + 0.6x_1 + 0.11x_2, \\ -x_1 + 405{,}000 + 0.5x_1 + 0.09x_2, \end{Bmatrix}$$

$$= -x_1 + 405{,}000 + 0.5x_1 + 0.10x_2$$

OR

$$G_1 = -x_1 + 405{,}000 + 0.5x_1 + 0.09x_2$$

$$G_2 = \min\begin{Bmatrix} -x_2 + 600{,}000 + 0.8x_1 + 0.0145x_2, \\ -x_2 + 600{,}000 + 0.6x_1 + 0.018x_2 \end{Bmatrix}$$

$$-x_2 + 600{,}000 + 0.6x_1 + 0.018x_2$$

Substituting these into the MP gives the quadratic programming problem:

$$\max z = x_1G_1 + x_2G_2$$

S.T. $\quad 0.95x_1 - 0.10x_2 \le 405{,}00$

$$-0.06x_1 + 0.982x_2 \le 405{,}00$$
$$x_1 \ge 0, \ge 0.$$

Solving the MP, we obtain $x_1 = 493{,}807.31$ and $x_2 = 641{,}169.50$, where G_1 corresponds to choosing technology 1 for the auto industry, and $G_2 = 0$ corresponds to choosing technology 2 for the steel industry. This result is interpreted as follows: The auto industry should choose technology 1. This technology produces 405,000 thousand tons of pollutants for external demand and 88,803.31 thousand tons of pollutants for internal demands. similar interpretation follows for the steel industry.

Note that using $G_1 = x_1 + 405{,}000 + 0.5x_1 + 0.09x_2$ and replacing the first constraint by $0.94x_1 - 0.11x_2 \le 405{,}000$ leads to the same results.

6.0 CONCLUSION

A mathematical model for controlling the generation of industrial pollution is presented. Information needed for the model's construction is available for the US economy. Nevertheless, many industrial nations have undertaken input-output studies of their economies and the concepts of this paper apply to their economies as well.

It is also shown that the model can be solved as a mathematical programming problem. This is advantageous considering the popularity of MP methodology in the manufacturing sectors of most industrial economies.

Finally, it is pertinent to point out that real applicability of any model based on empirical information, such as the one presented here, depends on accurate estimation of the needed numerical data. Fortunately, advances in new technology indicate that this is a task within reach.

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