Mathematical Analysis of Evaporation Rate Laws and Determination of the Duration and Conditions for Latent Drying of Wet Clays

Chukwuka Ikechukwu Nwoye*, Reginald Ejimofor and Okechukwu Onyebuchi Onyemaobi

Department of Materials and Metallurgical, Engineering Federal University of Technology, Owerri, Nigeria.
chikeyn@yahoo.com

Abstract: Mathematical analysis and determination of the duration and conditions of latent drying of wet clays has been carried out based on evaporation rate laws. Results of the analysis indicate that the conditions permitting latent drying of wet clays prevail in the absence of evaporation and shrinkage of the drying clay. These conditions were evaluated as; drying of the clay at room temperature, equality in vapour pressure of water at the drying temperature and partial pressure of water in the surrounding atmosphere ($P_w = P_o$), equality in the initial and final concentrations of water in the clay (within the latent drying period) (C_o –C), evaporation rate being equal to zero (j = 0) and change in the thickness of the drying slab being zero ($\Delta w = 0$). It was also evaluated that the time elapse within which latent drying of the clay occurred just before evaporation resumes is 0.37second. [New York Science Journal. 2010;3(1):38-41]. (ISSN: 1554-0200).

Keywords: Mathematical Analysis, Conditions, Duration, Latent Drying, Wet Clays.

1. Introduction

Reed (1988) described firing as having three stages through which it proceeds; preliminary reactions which include binder burnout, elimination of gaseous product of decomposition and oxidation, sintering as well as cooling which may include thermal and chemical annealing. Several works (Barsoum,1997;Viewey and Larry,1978;Keey, 1978) have been carried out on shrinkage of clay during drying. In all these works, porosity has been shown to influence the swelling and shrinkage behaviour of clay products of different geometry. It has been reported (Reed,1988) that drying occurs in three stages; increasing rate, constant and decreasing rate. He pointed out that during the increasing rate; evaporation rate is higher than evaporating surface hence more water is lost. At constant rate, the evaporation rate and evaporation surface are constant. He posited that shrinkage occurs at this stage. Keey (1978) also in a similar study suggested that at this stage, free water is removed between the particles and the inter-particle separation decreases, resulting in shrinkage. During the decreasing rate, particles make contacts as water is removed, which causes shrinkage to cease.

Model for calculating the volume shrinkage resulting from the initial air-drying of wet clay has been derived (Nwoye, 2008). The model;

$$\theta = \gamma^3 - 3\gamma^2 + 3\gamma \qquad (1)$$

calculates the volume shrinkage when the value of dried shrinkage γ, experienced during air-drying of wet clays is known. The model was found to be third-order polynomial in nature. Olokoro clay was found to have the highest shrinkage during the air drying condition, followed by Ukpor clay while Otamiri clay has the lowest shrinkage. Volume shrinkage was discovered to increase with increase in dried shrinkage until maximum volume shrinkage was reached, hence a direct relationship.

Nwoye et al. (2008) derived a model for the evaluation of overall volume shrinkage in molded clay products (from initial air-drying stage to completion of firing at a temperature of 1200^0C). It was observed that the overall volume shrinkage values predicted by the model were in agreement with those calculated using conventional equations. The model;

$$S_T = \alpha^3+\gamma^3-3(\alpha^2+\gamma^2)+3(\alpha+\gamma) \qquad (2)$$

depends on direct values of the dried γ and fired shrinkage α for its precision. Overall volume shrinkage was found to increase with increase in dried and fired shrinkages until overall volume shrinkage reaches maximum.

Nwoye (2009a) derived a model for calculating the quantity of water lost by evaporation during oven drying of clay at 90^0C. The model;

$$\gamma = \exp[(\ln t)^{1.0638} - 2.9206] \qquad (3)$$

indicated that the quantity of evaporated water, γ during the drying process is dependent on the drying time t, the evaporating surface being constant. The validity of the model was found to be rooted in the expression $(\text{Log}\beta + \ln\gamma)^N = \ln t$.

Model for predictive analysis of the quantity of water evaporated during the primary-stage processing of a bioceramic material sourced from kaolin has been derived by Nwoye (2009b). The model;

$$\alpha = e^{(\ln t/2.1992)} \quad (4)$$

shows that the quantity of water α, evaporated at 110^0C, during the drying process is also dependent on the drying time t, where the evaporating surface is constant. It was found that the validity of the model is rooted on the expression $(\ln t/\ln\alpha)^N = \text{Log}\beta$ where both sides of the expression are correspondingly approximately equal to 3. The respective deviation of the model-predicted quantity of evaporated water from the corresponding experimental value was found to be less than 22% which is quite within the acceptable deviation range of experimental results.

Model for quantifying the extent and magnitude of water evaporated during time dependent drying of clay has been derived (Nwoye et al.,2009). The model;

$$\gamma = \exp((\ln t/2.9206)^{1.4}) \quad (5)$$

indicates that the quantity of evaporated water γ during the drying process (at 90^0C) is dependent on the drying time, t the evaporating surface being constant. It was found that the validity of the model is rooted in the expression $\ln\gamma = (\ln t/\text{Log}\beta)^N$ where both sides of the expression are correspondingly almost equal.

The present work is to carry out mathematical analysis and determination of the duration and conditions for latent drying of wet clays.

2. Evaporation Rate Laws

Philip and Knight [10] following their research findings, stated that after a short period of time, the form of the concentration profile during drying of a wet clay becomes fixed; - j/w equals dc/dt, a constant every where.

2.1 Mathematical analysis of evaporation rate laws

The analysis is based on the two major parameters affecting evaporation of water during drying of wet clays;

(a) Pressure of water in the surrounding atmosphere and at the temperature of drying

(b) Initial and final concentration of water in the clay

Based on the evaporation rate laws,

$$\left(\frac{-j}{w}\right) = \left(\frac{dc}{dt}\right) \quad (6)$$

$$j = k(P_w - P_o) \quad (7)$$

Substituting equation (7) into (6)

$$\left(\frac{-k(P_w - P_o)}{w}\right) = \left(\frac{dc}{dt}\right) \quad (8)$$

Where

j = Volume of water evaporated per unit surface area per second

P_w = Vapour pressure of water at the temperature of drying

P_o = Partial pressure of water in the surrounding atmosphere

w = Thickness of drying slab

k = Evaporating Constant

dc = Change in the concentration of water in the drying clay

dt = Change in time during change in water concentration

$$P_w - P_o = \Delta P \quad (9)$$

Substituting equation (9) into equation (8)

$$\left(\frac{dc}{dt}\right) = -k\left(\frac{\Delta P}{w}\right) \quad (10)$$

$$dc = -k\left(\frac{\Delta P}{w}\right) dt \quad (11)$$

Integrating both sides of equation (11) between limits of C_o to C and 0 to t respectively

$$C - C_o = -k\left(\frac{\Delta P}{w}\right) t \quad (12)$$

Where

C_o = Initial uniform concentration of water in the drying clay at time; t = 0

C = Concentration of water in the drying clay at time; t = t

Multiplying both sides of equation (12) by negative sign, reduces it to;

$$C_o - C = -k\left(\frac{\Delta P}{w}\right) t \quad (13)$$

$$C = -k\left(\frac{\Delta P}{w}\right) t + C_o \quad (14)$$

Substituting equation (9) into equation (14) reduces it to;

$$C = -k\left(\frac{P_w - P_o}{w}\right) t + C_o \quad (15)$$

Equation (15) gives the concentration of water in the drying clay at any time t, during the constant-rate-period. Rearranging equation (15) gives

$$\Delta P = w\left(\frac{C_o - C}{kt}\right) \quad (16)$$

Substituting equation (9) into equation (16) reduces it to;

$$P_w - P_o = w\left(\frac{C_o - C}{kt}\right) \quad (17)$$

$$P_w = w\left(\frac{C_o - C}{kt}\right) + P_o \quad (18)$$

Equation (18) gives the vapour pressure of water in a drying clay at any time t during the constant-rate-period, given a particular drying temperature. The equation indicates that the vapour pressure of water in the clay at the drying temperature depends on;

(1) Thickness of the drying mass of clay

(2) Initial and final concentration of water in the drying clay

(3) Partial pressure of water in the surrounding atmosphere

(4) Time elapse during drying

(5) Evaporation constant

In equation (18), P_w is calculated partly in terms of difference in the concentration of water in the drying mass of clay.

2.2 Detemination of the duration for latent drying of wet clays

Based on the foregoing, assuming $C_o - C = \Delta C = dC$. Equation (18) reduces to;

$$P_w = w\left(\frac{dC}{kt}\right) + P_o \quad (19)$$

From equation (6)

$$dc = \left(\frac{-jdt}{w}\right) \quad (20)$$

Substituting equation (20) into equation (19) and evaluating further,

$$Pw = \left(\frac{-jdt}{kt}\right) + P_o \quad (21)$$

$$Pw - P_o = \left(\frac{-jdt}{kt}\right) \quad (22)$$

Since $Pw - P_o = \Delta P = dp$;

$$dP = \left(\frac{-jdt}{kt}\right) \quad (23)$$

Integrating both sides of equation (23) between limits of P_o to P_w and 0 to t respectively

$$Pw - P_o = \left(\frac{-j}{k}\right) \ln t \quad (24)$$

$$Pw = \left(\frac{-j}{k}\right) \ln t + P_o \quad (25)$$

Substituting equation (7) into equation (25) and evaluating further,

$$\ln t = -1 \quad (26)$$

Evaluating equation (26) for the value of t;

$$t = 0.37\text{second}$$

Therefore equation (25) is correct and satisfied at t = 0.37s or lnt = -1, where lnt is a constant equal to the coefficient of pressure due to evaporation of water (ε).

Equation (25) can be rewritten as;

$$P_w = \left(\frac{-j}{k}\right) \varepsilon + P_o \quad (27)$$

Equation (27) indicates that P_w is calculated in terms of the evaporation rate j. This equation is correct and satisfied at a constant ε = -1 (for water evaporation).

2.3 Conditions for latent drying of clay

Based on the foregoing,

Equations (18) and (25) are equal;

$$w\left(\frac{C_o - C}{kt}\right) + P_o = \left(\frac{-j}{k}\right) \ln t + P_o \quad (28)$$

Substituting equation (7) into equation (28) and evaluating further,

$$w\left(\frac{C_o - C}{kt}\right) = \left(P_w - P_o\right) \ln t \quad (29)$$

Substituting equation (26) into equation (29) and evaluating further,

$$\left(\frac{C_o - C}{P_w - P_o}\right) = t\left(\frac{k}{w}\right) \quad (30)$$

$$t = \frac{w\left(C_o - C\right)}{k\left(P_w - P_o\right)} \quad (31)$$

Equation (31) gives the time for the evaporation process during the constant-rate-period drying of wet clay.

Substituting equation (7) into equation (31) and evaluating further,

$$t = \frac{w\left(C_o - C\right)}{j} \quad (32)$$

Rearranging equation (32);

$$j = \frac{w\left(C_o - C \right)}{t} \qquad (33)$$

From equation (33),
At $j = 0$ and $C_o = C$, $\Delta w = 0$.
Based on these mathematical analyses, it is pertinent to note that equation (33) is similar and satisfies the evaporation equation reported by Philip and Knight [10] (equation (6)) which poses as the reference equation in this work.
Conditions for latent drying of wet clay are based on the assumption that the drying process took place at room temperature. And under this condition, P_w and P_o are equal. Therefore,

$$P_w = P_o \qquad (34)$$

Substituting equation (34) into equation (31) and evaluating further,

$$C_o = C \qquad (35)$$

This indicates that at room temperature where $P_w = P_o$, $C_o = C$. This implies that at this temperature there was no loss of water during drying of the clay.

Substituting equation (35) into equation (33) and evaluating further,

$$j = 0 \qquad (36)$$

This indicates that there was no evaporation of water when $P_w = P_o$ and $C_o = C$.
It was discovered by Reed [1] that shrinkage sets in when both the evaporating surface and evaporating rate are constant. This however implies that at conditions; $P_w = P_o$, $C_o = C$ and $j = 0$, equation (33) becomes;

$$\Delta w = 0 \qquad (37)$$

This shows that no shrinkage occurred on the wet clay slab since no evaporation occurred as well let alone being constant. However, it is believed that latent drying occurred as a result of latent evaporation of water, since room temperature gives out some heat suspected to be latent heat of vapourization. It is also suspected that a latent temperature gradient existed between the drying clay mass and the surrounding atmosphere, hence permitting latent evaporation of water due to the role played by the latent heat of vapourization. It is therefore posited that this latent evaporation of water results to latent drying of wet clays.

Conclusion
Based on the foregoing, latent drying of wet clays occurs within a very short period of time (0.37s) during drying at conditions;
(1) $P_w = P_o$
(2) $C_o = C$
(3) $j = 0$
(4) $\Delta w = 0$
(5) Room temperature (drying temperature)

Correspondence to:
Chukwuka Ikechukwu Nwoye
Department of Materials and Metallurgical Engineering, Federal University of Technology, P.M.B 1526 Owerri. Imo State, Nigeria.
Cellular phone: 0803 800 6092
Email: chikeyn@yahoo.com

References.
[1]Reed J. Principles of Ceramic Processing, Wiley Interscience Publication, Canada 1988; 470-478.
[2]Barsoum M. Fundamentals of Ceramics. McGraw Hill Incorporated, Singapore 1997; 410
[3]Viewey F, Larrly P. Ceramic Processing Before Firing, John-Wiley & Sons, New York 1978;p3-8.
[4]Keey RB. Introduction to Industrial Drying Operations, Pergamon Press, Elmsford, New York 1978;132-157.
[5]Nwoye CI. Mathematical Model for Computational Analysis of Volume Shrinkage Resulting from I Air-Drying of Wet Clay Products. International Research Journal of Engineering Science and Technology 2008; 5(1):82-85.
[6]Nwoye CI, Iheanacho IO, Onyemaobi OO. Model for the Evaluation of Overall Volume Shrinkage in Molded Clay Products from Initial Air-Drying Stage to Completion of Firing. International Journal of Natural & Applied Science 2008; 4(2):234-238.
[7]Nwoye CI. Model for Calculating the Quantity of Water Lost by Evaporation during Oven Drying of Clay. Researcher Journal 2009; 1(3): 8-13.
[8]Nwoye CI, Okeke K, Obi M, Nwanyanwu U, Ofoegbu S. Model for Predictive Analysis of the Quantity of Water Evaporated during the Primary-Stage Processing of Bioceramic Material Sourced from Kaolin. Journal of Nature and Science 2009;7(4):79-84.
[9]Nwoye CI, Nwakwuo CC, Obi MC, Obasi GC, Onyemaobi OO. Model for Quantifying the Extent and Magnitude of Water Evaporated during Time Dependent Drying of Clay. New York Journal of Science 2009; 2(3):55-58.

6/16/2009