On k-Nearly Uniformly Convex Property in Generalized Cesáro Sequence Space Defined by Weighted Means

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Abstract: The main purpose of this paper is to show that the sequence space $ces[(a_n),(p_n),(q_n)]$ defined by Altay and Başar (2007) is k-nearly uniformly convex (k-NUC) for $k \geq 2$ when $\underset{n\to\infty}{Lim}\inf\, p_n > 1$.Therefore it is fully k-rotund (kR), NUC and has a drop property. [New York Science Journal 2010; 3(8):48-53]. (ISSN: 1554-0200).

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Introduction

Let (X, ||.||) be Banach space over the real numbers $\mathbb{R}$ and let B(X) (respec. S(X)) be the closed unit ball (resp. unit sphere) of X.

A point $x \in S(X)$ is an extreme point of B(X), if for any $y, z \in S(X)$, the equality $x = \dfrac{y+z}{2}$ implies $y = z$.

A Banach space X is said to be Rotund (R) if for every point of S(X) is an extreme point of B(X).Clarkson [1]who introduced the concept of uniform convexity.

A Banach space X is called uniformly convex (UC) if $\forall$ ε >0 $\exists\,\delta$ >0 such that for $x, y \in S(X)$, the inequality $\|x-y\| < \varepsilon$ implies that $\left\|\dfrac{x+y}{2}\right\| < \delta$.

(1.1) for any $x \notin B(X)$, the drop determined by x is the set

$$D(x, B(X)) = conv(\{x\} \cup B(X)). \qquad (1.2)$$

Rolewicz [12], basing on Daneš drop theorem [4], introduced the notation of drop property for Banach spaces.

A Banach space X has the drop property (D) if For every closed set C disjoint with B(X) $\exists\, x \in$C such that D(x, B(X)) $\cap$ C = {x}.
(1.3)

X is said to have the property (H), if for any sequence on the unit sphere of X, weak convergence coincides norm convergence. In [13], Rolewicz proved that if the Banach space X has the drop property (D), then X is reflexive. Montesinos [11] extended this result by showing that X has the drop property if and only if X is reflexive and has the property (H).A sequence $\{x_n\} \subset X$ is said to be ε -separated sequence for some $\varepsilon > 0$ if

$$sep(x_n) = \inf\{\|x_n - x_m\| : n \neq m\} > \varepsilon \quad . \qquad (1.4)$$

A Banach space X is called nearly uniformly convex (NUC) if $\forall\, \varepsilon > 0$ $\exists\, \delta \in (0,1)$ such that for every sequence $(x_n) \subseteq B(X)$ with $sep(x_n) \geq \varepsilon$, we have

$$conv(x_n) \cap (1-\delta)B(X) \neq \phi. \qquad (1.5)$$

Huff [6] proved that every NUC Banach spaces X is reflexive and it has property (H). Kutzarova [7] has defined k-nearly uniformly convex Banach spaces. Let $k \geq 2$ be an integer, a Banach space X is called k-nearly uniformly convex (k-NUC) if

$\forall\, \varepsilon > 0$ $\exists\, \delta$ >0 such that for any sequence $(x_n) \subset B(X)$ with $sep(x_n) \geq \varepsilon$ there are $n_1, n_2.n_3, \ldots, n_k \in \mathbb{N}$, where $\mathbb{N}$= {1, 2, 3,…}.

Such that $\left\| \frac{x_{n_1} + x_{n_2} + x_{n_3} + ... + x_{n_k}}{k} \right\| < \delta$.

(1.6) Clearly, k-NUC Banach spaces are NUC, however the opposite implication does not hold in general [7].

Fan and Gliksberg [5] have introduced

k-Rotund (kR) Banach spaces. A Banach space X is called fully k-rotund (kR) if for any sequence

$(x_n) \subset B(X)$

$\left\| \frac{x_{n_1} + x_{n_2} + x_{n_3} + ... + x_{n_k}}{k} \right\| \to 1$ as

$\min\{n_i : 1 \le i \le k] \to \infty$ implies that (x_n) is

convergent. It is well known that UC implies kR and kR implies (k+1)R, and kR spaces are reflexive and rotund. Byω, we denote the space of all real or complex sequences .

For a real vector space X, a function $\sigma : X \to [0,\infty]$ is called modular, if it satisfies the following conditions:

(i) $\sigma(x) = 0 \Leftrightarrow x = 0 \ \forall x \in X$,

(ii) $\sigma(\lambda x) = \sigma(x) \ \forall \ \lambda \in \mathbb{R}$ with $|\lambda| = 1$,

(iii) $\sigma(\lambda x + \beta y) \le \sigma(x) + \sigma(y) \ \forall x, y \in X$
$\forall \lambda, \beta \ge 0; \ \lambda + \beta = 1$.

Further, the modular σ is called convex if

(iv) $\sigma(\lambda x + \beta y) \le \lambda\sigma(x) + \beta\sigma(y) \ \forall x, y \in X$
$\forall \lambda, \beta \ge 0; \ \lambda + \beta = 1$. If σ is a modular on X, we define $X_\sigma = \left\{x \in X : \lim_{\lambda \to 0^-} \sigma(\lambda x) = 0\right\}$, (1.7)

$$X_\sigma^* = \{x \in X : \sigma(\lambda x) < \infty, \exists \lambda > 0\}.$$

It is clear that $X_\sigma \subseteq X_\sigma^*$. If σ is a convex

modular$\forall \ x \in X_\sigma$, we define

$$\| x \| = \inf\left\{\lambda > 0 : \sigma\left(\frac{x}{\lambda}\right) \le 1\right\}. \qquad (1.8)$$

Orlicz [10] proved that if σ is a convex modular on X, then $X_\sigma = X_\sigma^*$ and $\| \cdot \|$ is a norm on X_σ for which X_σ is a Banach space. The norm $\| \cdot \|$, defined as in (1.8), is called the Luxemburg norm.

A modular σ is said to satisfy the δ_2-condition $(\sigma \in \delta_2)$ if$\forall \ \varepsilon > 0$ $\exists$constants $K \ge 2$ and $a > 0$ such that $\sigma(2u) \le K\sigma(u) + \varepsilon$, (1.9)

$\forall \ u \in X_\sigma$ With $\sigma(u) \le a$ .If σ satisfies the δ_2-condition $\forall \ a > 0$ with $K \ge 2$ depending on a,

we say that σ satisfies the strong δ_2-condition

$(\sigma \in \delta_2^s)$.

The following known results are very important for our consideration.

Theorem1.1. [2]

If $\sigma \in \delta_2^s$, then$\forall \ L > 0$ and $\forall \ \varepsilon > 0 \ \exists \ \delta > 0$ such

that $|\sigma(u+v) - \sigma(u)| < \varepsilon$, (1.10)

$u, v \in X_\sigma$ With $\sigma(u) \le L$ and $\sigma(v) \le \delta$.

Proof. See [2, Lemma 2.1].

Theorem1.2. [2]

(1) If $\sigma \in \delta_2^s$, then$\forall \ x \in X_\sigma$, $\| x \| = 1$ if and only if $\sigma(x) = 1$.

(2) If $\sigma \in \delta_2^s$, then for any sequence (x_n) in X_σ, $\| x_n \| \to 0$ if and only if $\sigma(x_n) \to 0$.

Proof. See [2, Corollary 2.2 and Lemma 2.3].

Theorem 1.3.

If $\sigma \in \delta_2^s$, then$\forall \ \varepsilon \in (0,1) \ \exists \ \delta \in (0,1)$ such that

$\sigma(x) \le 1 - \varepsilon$ implies $\|x\| \le 1 - \delta$.

Proof. Suppose that the theorem does not hold, then $\exists \ \varepsilon > 0$ and (x_n) in X_σ such that $\sigma(x_n) \le 1 - \varepsilon$

, and $\frac{1}{2} \le \| x_n \| \xrightarrow{n\to\infty} 1$. Let $a_n = \frac{1}{\|x_n\|} - 1$.

Then $a_n \xrightarrow{n\to\infty} 0$ .Let $L = \sup_n \sigma(2x_n)$ .Since $\sigma \in \delta_2^s\ \exists K \geq 2$ such that $\sigma(2u) \leq K\sigma(u)+1$ (1.11)$\forall\, u \in X_\sigma$ with $\sigma(u) < 1$.By(1.11), we

have $\sigma(2x_n) \leq K\sigma(x_n)+1 < K+1 \,\forall\, n \in \mathbb{N}$. Hence $0 \leq L < \infty$, by theorem 1.2(1), we have

$$1 = \sigma\left(\frac{x_n}{\|x_n\|}\right) = \sigma(2a_n x_n + (1-a_n)x_n) \qquad (1.12)$$

$$\leq a_n\sigma(2x_n) + (1-a_n)\sigma(x_n) \leq$$

$$a_n L + (1-\varepsilon) \xrightarrow{n\to\infty} 1-\varepsilon$$

, which is a contradiction.

Altay and Başar (2007) defined the sequence space $ces[(a_n),(p_n),(q_n)]$ as

$$ces[(a_n),(p_n),(q_n)] = \left\{ x \in \omega : \sum_{n=1}^{\infty}\left(a_n \sum_{k=1}^{n} q_k |x_k|\right)^{p_n} < \infty \right\} \quad (1.13),$$

where $(a_n),(p_n)$ *and* (q_n) are sequences of

positive real numbers, $1 \leq p_n < \infty\ \forall n \in \mathbb{N}$. with the norm

$$\|x\| = \left[\sum_{n=1}^{\infty}\left(a_n \sum_{k=1}^{n} q_k\,|x_k|\right)^{p_n}\right]^{\frac{1}{H}} \qquad (1.14),$$

$H = \sup_n p_n$.

They also showed that the space

$ces[(a_n),(p_n),(q_n)]$ is a complete linear metric space paranormed

by $g(x) = \left[\sum_{n=1}^{\infty}\left(a_n \sum_{k=1}^{n} q_k\,|x_k|\right)^{p_n}\right]^{\frac{1}{H}}$ also V.Karakaya and N.Şimşek [16] proved that this space is a Banach space and posses Kadec-Klee (H).

Remarks:

(1)Taking $a_n = \dfrac{1}{\sum_{k=1}^{n} q_k}$, then $Ces((a_n),(p_n),(q_n)) = Ces((p_n),(q_n))$ the N¨orlund sequence spaces studied by [18].

(2)Taking $a_n = \dfrac{1}{n}; q_n = 1, \forall n \in \mathbb{N}$,

then $Ces((a_n),(p_n),(q_n)) = Ces(p_n)$ studied by

W. Sanhan and S. Suantai [15].

(3)Taking $a_n = \dfrac{1}{n},\ q_n = 1,\ p_n = p\ , \forall n \in \mathbb{N}$,

then $Ces((a_n),(p_n),(q_n)) = Ces_{\ p}$ studied by

Many authors see [8,9and14].

Throughout this paper, the sequence (p_n) is a bounded sequence of positive real numbers with $\liminf_{n\to\infty} p_n > 1$, and also

1) $H = \sup_n p_n$.

2) Let (p_k) be a bounded sequence of positive real numbers, we have $|a_k + b_k|^{p_k} \leq 2^{H-1}(|a_k|^{p_k} + |b_k|^{p_k})\ \forall\ k \in \mathbb{N}$.

2. Main results

<u>Proposition2.1.</u>

The functional σ is convex modular

on $ces[(a_n),(p_n),(q_n)]$ and for

any $x \in ces[(a_n),(p_n),(q_n)]$ the functional

σ on $ces[(a_n),(p_n),(q_n)]$ satisfies the following properties:

(i) If 0<r<1, then

(ii) $r^H \sigma\left(\dfrac{x}{r}\right) \leq \sigma(x)$ and $\sigma(rx) \leq r\sigma(x)$.

(ii) If r>1, then $\sigma(x) \leq r^H \sigma\left(\dfrac{x}{r}\right)$.

(iii) If r≥1, then $\sigma(x) \leq r\sigma(x) \leq \sigma(rx)$.

Proof. All assertions are clearly obtained by the definition and convexity of σ see [17] .

Proposition2.2.

For any $x \in ces[(a_n),(p_n),(q_n)]$, the following assertions are satisfied:

(i) If $\| x \| < 1$, then $\sigma(x) \le \| x \|$,

(ii) if $\| x \| > 1$, then $\sigma(x) \ge \| x \|$,

(iii) $\| x \| = 1$ if and only if $\sigma(x) = 1$.

Proof: It can be proved with standard techniques in a similar way as in [17].

Proposition2.3. $\forall L > 0$ and $\forall \varepsilon > 0 \; \exists \delta > 0$ such that $|\sigma(u+v) - \sigma(u)| < \varepsilon$ whenever $u, v \in ces[(a_n),(p_n),(q_n)]$ with $\sigma(u) \le L$ and $\sigma(v) \le \delta$

Proof: Since (p_n) is bounded, it is easy to see that $\sigma \in \delta_2^s$.Hence the proposition is obtained directly from theorem (1.1).

Proposition2.4. For any sequence $(x_n) \in ces[(a_n),(p_n),(q_n)], \| x_n \| \to 0$ if and only if $\sigma(x_n) \to 0$.

Proof: It follows directly from Theorem (1.2-2) since $\sigma \in \delta_2^s$.

Theorem2.5. $\forall x \in ces[(a_n),(p_n),(q_n)]$ and $\forall \varepsilon \in (0,1)$, $\exists \delta \in (0,1)$ such that

$\sigma(x) \le 1 - \varepsilon$ implies $\|x\| \le 1 - \delta$.

Proof: Since $\sigma \in \delta_2^s$, the theorem is obtained directly from theorem (1.3).

Theorem2.6. The space $ces[(a_n),(p_n),(q_n)]$ is k-NUC $\forall$ integer $k \ge 2$.

Proof:
Let $\varepsilon > 0$ and $(x_n) \in B(ces[(a_n),(p_n),(q_n)])$ with $sep(x_n) \ge \varepsilon$ .For each $m \in \mathbb{N}$, let

$x_n^m = (0,0,......,0, x_n(m), x_n(m+1),...)$ .Since for each $i \in \mathbb{N}$, $(x_n(i))_{n=1}^{\infty}$ is bounded, we have that $\forall\, i \in \mathbb{N}$, $(x_n(i))_{n=1}^{\infty}$ is bounded, by using the diagonal method, we can find a subsequence $(x_{n_j}(i))$ of (x_n) such that $(x_{n_j}(i))$ converges for each $i \in \mathbb{N}$, $1 \le i \le m$.Therefore, there exists an increasing sequence of positive integer (t_m) such that $sep((x_{n_j}^m)_{j>t_m}) \ge \varepsilon$ .Hence, there is a sequence of positive integers $(r_m)_{m=1}^{\infty}$ with $r_1 < r_2 < r_3 < ...$ such that $\|x_{r_m}^m\| \ge \frac{\varepsilon}{2} \; \forall\, m \in \mathbb{N}$. Then by proposition (2.4), we may assume that there exists $\eta > 0$ such

that $\sigma(x_{r_m}^m) \ge \eta \; \forall\, m \in \mathbb{N}$. (2.1)

Let $\alpha > 0$ be such that $1 < \alpha < \underset{n\to\infty}{Lim} \inf\; p_n$ .For fixed integer $k \ge 2$, let $\varepsilon_1 = (\frac{(k^{\alpha-1}-1)}{(k-1)k^{\alpha}})(\frac{\eta}{2})$, then by proposition (2.3) $\exists \delta > 0$

Such that $|\sigma(u+v) - \sigma(u)| < \varepsilon_1$. (2.2)

Whenever $\sigma(u) \le 1$ and $\sigma(v) \le \delta$ .Since by Proposition (2.2-1) $\sigma(x_n) \le 1 \forall \; n \in \mathbb{N} \; \exists$ positive integers $m_i (i = 1,2,3,....., k-1)$ with $m_1 < m_2 < m_3 < .... < m_{k-1}$ such that $\sigma(x_i^{m_i}) \le \delta$ and $\alpha \le p_j \; \forall \; j \ge m_{k-1}$ .Define $m_k = m_{k-1} + 1$. By (2.1), we have $\sigma(x_{r_{m_k}}^{m_k}) \ge \eta$. Let $s_i = i$ for $1 \le i \le k-1$,

and $s_k = r_{m_k}$ .Then in virtue of (2.1),(2.2), and Convexity of function $f_i(u) = |u|^{p_i} (i \in \mathbb{N})$, we have

$$\sigma\left(\frac{x_{s_1}+x_{s_2}+x_{s_3}+....+x_{s_k}}{k}\right)=$$

$$=\sum_{n=1}^{\infty}\left(a_n\sum_{i=1}^{n}q_i\left|\frac{x_{s_1}(i)+x_{s_2}(i)+x_{s_3}(i)+...+x_{s_k}(i)}{k}\right|\right)^{p_n}=$$

$$=\sum_{n=1}^{m_1}\left(a_n\sum_{i=1}^{n}q_i\left|\frac{x_{s_1}(i)+x_{s_2}(i)+x_{s_3}(i)+...+x_{s_k}(i)}{k}\right|\right)^{p_n}+$$

$$+\sum_{n=m_1+1}^{\infty}\left(a_n\sum_{i=1}^{n}q_i\left|\frac{x_{s_1}(i)+x_{s_2}(i)+x_{s_3}(i)+...+x_{s_k}(i)}{k}\right|\right)^{p_n}\leq$$

$$\sum_{n=1}^{m_1}\frac{1}{k}\sum_{j=1}^{k}\left(a_n\sum_{i=1}^{n}q_i\left|x_{s_j}(i)\right|\right)^{p_n}+\sum_{n=m_1+1}^{m_2}\left(a_n\sum_{i=1}^{n}q_i\left|\frac{x_{s_2}(i)+x_{s_3}(i)+...+x_{s_k}(i)}{k}\right|\right)^{p_n}+$$

$$+\sum_{n=m_2+1}^{\infty}\left(a_n\sum_{i=1}^{n}q_i\left|\frac{x_{s_3}(i)+...+x_{s_k}(i)}{k}\right|\right)^{p_n}+2\varepsilon_1\leq$$

$$\leq\sum_{n=1}^{m_1}\frac{1}{k}\sum_{j=1}^{k}\left(a_n\sum_{i=1}^{n}q_i\left|x_{s_j}(i)\right|\right)^{p_n}+\sum_{n=m_1+1}^{m_2}\frac{1}{k}\sum_{j=2}^{k}\left(a_n\sum_{i=1}^{n}q_i\left|x_{s_j}(i)\right|\right)^{p_n}$$

$$+\sum_{n=m_2+1}^{m_3}\frac{1}{k}\sum_{j=3}^{k}\left(a_n\sum_{i=1}^{n}q_i\left|x_{s_j}(i)\right|\right)^{p_n}+...+\sum_{n=m_{k-1}+1}^{m_3}\frac{1}{k}\sum_{j=k-1}^{k}\left(a_n\sum_{i=1}^{n}q_i\left|x_{s_j}(i)\right|\right)^{p_n}+$$

$$+\sum_{n=m_k+1}^{\infty}\left(a_n\sum_{i=1}^{n}q_i\left|\frac{x_{s_j}(i)}{k}\right|\right)^{p_n}+(k-1)\varepsilon_1\leq$$

$$\leq\frac{\sigma(x_{s_1})+\sigma(x_{s_2})+.....+\sigma(x_{s_{k-1}})}{k}+\frac{1}{k}\sum_{n=1}^{m_k}\left(a_n\sum_{i=1}^{n}q_i\left|x_{s_k}(i)\right|\right)^{p_n}+$$

$$+\sum_{n=m_k+1}^{\infty}\left(a_n\sum_{i=1}^{n}q_i\left|\frac{x_{s_k}(i)}{k}\right|\right)^{p_n}+(k-1)\varepsilon_1\leq$$

$$\leq\frac{k-1}{k}+\frac{1}{k}\sum_{n=1}^{m_k}\left(a_n\sum_{i=1}^{n}q_i\left|x_{s_k}(i)\right|\right)^{p_n}+\frac{1}{k^{\alpha}}\sum_{n=m_k+1}^{\infty}\left(a_n\sum_{i=1}^{n}q_i\left|x_{s_k}(i)\right|\right)^{p_n}+(k-1)\varepsilon_1\leq$$

$$\leq 1-\frac{1}{k}+\frac{1}{k}\left[1-\sum_{n=m_k+1}^{\infty}\left(a_n\sum_{i=1}^{n}q_i\left|x_{s_k}(i)\right|\right)^{p_n}\right]+\frac{1}{k^{\alpha}}\sum_{n=m_k+1}^{\infty}\left(a_n\sum_{i=1}^{n}q_i\left|x_{s_k}(i)\right|\right)^{p_n}+(k-1)\varepsilon_1$$

$$\leq 1+(k-1)\varepsilon_1-\left(\frac{k^{\alpha-1}-1}{k^{\alpha}}\right)\sum_{n=m_k+1}^{\infty}\left(a_n\sum_{i=1}^{n}q_i\left|x_{s_k}(i)\right|\right)^{p_n}$$

$$\leq 1+(k-1)\varepsilon_1-\left(\frac{k^{\alpha-1}-1}{k^{\alpha}}\right)\eta=1-\left(\frac{k^{\alpha-1}-1}{k^{\alpha}}\right)\left(\frac{\eta}{2}\right).$$

By theorem (2.5) $\exists\,\gamma>0$ such that

$\left\|\frac{x_{s_1}+x_{s_2}+x_{n_3}+...+x_{s_k}}{k}\right\|<1-\gamma$. Therefore,

$ces[(a_n),(p_n),(q_n)]$ is k-NUC.

Since k-NUC implies k R and k R implies R and reflexivity holds, and k-NUC implies NUC and NUC implies H-property and reflexivity holds, by theorem (2.6), the following results are obtained.

COROLLARY2.7. For $\underset{n\to\infty}{Lim}\inf\, p_n>1$, the space $ces[(a_n),(p_n),(q_n)]$ is k R, NUC, and has a drop property.

COROLLARY2.8. For $\underset{n\to\infty}{Lim}\inf\, p_n>1$, the space $ces[(a_n),(p_n)]$ is k-NUC.

COROLLARY2.9. For $\underset{n\to\infty}{Lim}\inf\, p_n>1$, the space $ces[(p_n)]$ is k-NUC.

COROLLARY2.10. For $1<p<\infty$, the space Ces_p is k-NUC.

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