On k-Nearly Uniformly Convex Property in Generalized Cesáro Sequence Space Defined by Weighted Means

N. Faried and A.A. Bakery

Department of Mathematics, Faculty of Science, Ain Shams University, Cairo, Egypt
awad_bakery@yahoo.com

Abstract: The main purpose of this paper is to show that the sequence space $ces[(a_n),(p_n),(q_n)]$ defined by Altay and Başar (2007) is k-nearly uniformly convex (k-NUC) for $k \geq 2$ when $\underset{n\to\infty}{Lim}\inf p_n > 1$. Therefore it is fully k-rotund (kR), NUC and has a drop property. [New York Science Journal 2010;3(8):48-53]. (ISSN: 1554-0200).

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Introduction

Let (X, ||.||) be Banach space over the real numbers $\mathbb{R}$ and let B(X) (respec. S(X)) be the closed unit ball (resp. unit sphere) of X.

A point $x \in S(X)$ is an extreme point of B(X), if for any $y, z \in S(X)$, the equality $x = \frac{y+z}{2}$ implies y=z.

A Banach space X is said to be Rotund (R) if for every point of S(X) is an extreme point of B(X).Clarkson [1]who introduced the concept of uniform convexity.

A Banach space X is called uniformly convex (UC) if $\forall\ \varepsilon > 0\ \exists \delta > 0$ such that for $x, y \in S(X)$, the inequality $\|x-y\| < \varepsilon$ implies that $\left\|\frac{x+y}{2}\right\| < \delta$.

(1.1) for any $x \notin B(X)$, the drop determined by x is the set

$$D(x, B(X)) = conv(\{x\} \cup B(X)). \qquad (1.2)$$

Rolewicz [12], basing on Daneš drop theorem [4], introduced the notation of drop property for Banach spaces.
A Banach space X has the drop property (D) if
For every closed set C disjoint with B(X) $\exists x \in$ C such that D(x, B(X)) $\cap$ C = {x}.
(1.3)

X is said to have the property (H), if for any sequence on the unit sphere of X, weak convergence coincides norm convergence. In [13], Rolewicz proved that if the Banach space X has the drop property (D), then X is reflexive. Montesinos [11]extended this result by showing that X has the drop property if and only if X is reflexive and has the property (H).A sequence $\{x_n\} \subset X$ is said to be ε -separated sequence for some $\varepsilon > 0$ if

$$sep(x_n) = \inf\{\|x_n - x_m\| : n \neq m\} > \varepsilon \ . \qquad (1.4)$$

A Banach space X is called nearly uniformly convex (NUC) if $\forall \varepsilon > 0\ \exists \delta \in (0,1)$ such that for every sequence $(x_n) \subseteq B(X)$ with $sep(x_n) \geq \varepsilon$, we have $conv(x_n) \cap (1-\delta)B(X) \neq \phi$. (1.5)

Huff [6] proved that every NUC Banach spaces X is reflexive and it has property (H). Kutzarova [7] has defined k-nearly uniformly convex Banach spaces. Let $k \geq 2$ be an integer, a Banach space X is called k-nearly uniformly convex (k-NUC) if

$\forall \varepsilon > 0\ \exists \delta > 0$ such that for any sequence $(x_n) \subset B(X)$ with $sep(x_n) \geq \varepsilon$ there are $n_1, n_2, n_3, \ldots, n_k \in \mathbb{N}$ such that

$$\left\|\frac{x_{n_1} + x_{n_2} + x_{n_3} + \ldots\ldots\ldots\ldots + x_{n_k}}{k}\right\| < \delta. \qquad (1.6)$$

Clearly, k-NUC Banach spaces are NUC, however the opposite implication does not hold in general [7].

Fan and Gliksberg [5] have introduced

k-Rotund (kR) Banach spaces. A Banach space X is called fully k-rotund (kR) if for any sequence

$(x_n) \subset B(X)$

$$\left\| \frac{x_{n_1} + x_{n_2} + x_{n_3} + \dots\dots\dots\dots + x_{n_k}}{k} \right\| \to 1 \text{ as}$$

$\min\{n_i : 1 \le i \le k\} \to \infty$ implies that (x_n) is convergent. It is well known that UC implies kR and kR implies (k+1)R, and kR spaces are reflexive and rotund. By ω, we denote the space of all real or complex sequences and the set of natural numbers by $\mathbb{N}$= {1, 2, 3,…}.

For a real vector space X, a function $\sigma : X \to [0,\infty]$ is called modular, if it satisfies the following conditions:

(i) $\sigma(x) = 0 \Leftrightarrow x = 0 \ \forall x \in X$,

(ii) $\sigma(\lambda x) = \sigma(x) \ \forall \lambda \in \mathbb{R}$ with $|\lambda| = 1$,

(iii) $\sigma(\lambda x + \beta y) \le \sigma(x) + \sigma(y) \ \forall x, y \in X$ $\forall \lambda, \beta \ge 0; \ \lambda + \beta = 1$.

Further, the modular σ is called convex if

(iv) $\sigma(\lambda x + \beta y) \le \lambda\sigma(x) + \beta\sigma(y) \ \forall x, y \in X$ $\forall \lambda, \beta \ge 0; \ \lambda + \beta = 1$. If σ is a modular on X, we define $X_\sigma = \{x \in X : \lim_{\lambda \to 0^-} \sigma(\lambda x) = 0\}$, (1.7)

$$X_\sigma^* = \{x \in X : \sigma(\lambda x) < \infty, \exists \lambda > 0\}.$$

It is clear that $X_\sigma \subseteq X_\sigma^*$. If σ is a convex modular $\forall x \in X_\sigma$, we define

$$\|x\| = \inf\left\{\lambda > 0 : \sigma\left(\frac{x}{\lambda}\right) \le 1\right\}. \qquad (1.8)$$

Orlicz [10] proved that if σ is a convex modular on X, then $X_\sigma = X_\sigma^*$ and $\|\cdot\|$ is a norm on X_σ for which X_σ is a Banach space. The norm $\|\cdot\|$, defined as in (1.8), is called the Luxemburg norm.

A modular σ is said to satisfy the δ_2 -condition $(\sigma \in \delta_2)$ if $\forall \varepsilon > 0$ $\exists$constants $K \ge 2$ and $a > 0$ such that $\sigma(2u) \le K\sigma(u) + \varepsilon$, (1.9)

$\forall \ u \in X_\sigma$ With $\sigma(u) \le a$ .If σ satisfies the δ_2 -condition $\forall a > 0$ with $K \ge 2$ depending on a, we say that σ satisfies the strong δ_2 -condition $(\sigma \in \delta_2^s)$.

The following known results are very important for our consideration.

Theorem1.1. [2]

If $\sigma \in \delta_2^s$, then $\forall L > 0$ and $\forall \varepsilon > 0 \ \exists \delta > 0$ such that $|\sigma(u+v) - \sigma(u)| < \varepsilon$, (1.10)

$u, v \in X_\sigma$ With $\sigma(u) \le L$ and $\sigma(v) \le \delta$.

Proof. See [2, Lemma 2.1].

Theorem1.2. [2]

(1) If $\sigma \in \delta_2^s$, then $\forall x \in X_\sigma$, $\|x\| = 1$ if and only if $\sigma(x) = 1$.

(2) If $\sigma \in \delta_2^s$, then for any sequence (x_n) in X_σ, $\|x_n\| \to 0$ if and only if $\sigma(x_n) \to 0$.

Proof. See [2, Corollary 2.2 and Lemma 2.3].

<u>Theorem1.3.</u>

If $\sigma \in \delta_2^s$, then $\forall \varepsilon \in (0,1) \ \exists \delta \in (0,1)$ such that $\sigma(x) \le 1 - \varepsilon$ implies $\|x\| \le 1 - \delta$.

Proof. Suppose that the theorem does not hold, then $\exists \varepsilon > 0$ and (x_n) in X_σ such that $\sigma(x_n) \le 1 - \varepsilon$

, and $\frac{1}{2} \le \|x_n\| \xrightarrow{n\to\infty} 1$. Let $a_n = \frac{1}{\|x_n\|} - 1$.

Then $a_n \xrightarrow{n\to\infty} 0$ .Let $L = \sup_n \sigma(2x_n)$ .Since $\sigma \in \delta_2^s \ \exists K \ge 2$ such that $\sigma(2u) \le K\sigma(u) + 1$ (1.11) $\forall u \in X_\sigma$ with $\sigma(u) < 1$ .By(1.11), we have $\sigma(2x_n) \le K\sigma(x_n) + 1 < K + 1 \ \forall n \in \mathbb{N}$. Hence, $0 \le L < \infty$ .By theorem1.2(1), we have $1=\sigma\left(\frac{x_n}{\|x_n\|}\right)=\sigma(2a_nx_n+(1-a_n)x_n)$ (1.12)

$\leq a_n\sigma(2x_n)+(1-a_n)\sigma(x_n)\leq$

$a_nL+(1-\varepsilon)\xrightarrow{n\to\infty}1-\varepsilon$

, which is a contradiction.

Altay and Başar (2007) defined the sequence space $ces[(a_n),(p_n),(q_n)]$ as

$ces[(a_n),(p_n),(q_n)]=$

$$\left\{x\in\omega:\sum_{n=0}^{\infty}\left(a_n\sum_{k=0}^{n}q_k|x_k|\right)^{p_n}<\infty\right\} \quad (1.13),$$

where $(a_n),(p_n)$ *and* (q_n) are sequences of positive real numbers, $1\leq p_n<\infty$ $\forall n\in\mathbb{N}$. with the norm

$$\|x\|=\left[\sum_{n=1}^{\infty}\left(a_n\sum_{k=1}^{n}q_k|x_k|\right)^{p_n}\right]^{\frac{1}{H}} \quad (1.14),$$

$H=\sup_n p_n$.

They also showed that the space

$ces[(a_n),(p_n),(q_n)]$ is a complete linear metric space paranormed

by $g(x)=\left[\sum_{n=1}^{\infty}\left(a_n\sum_{k=1}^{n}q_k|x_k|\right)^{p_n}\right]^{\frac{1}{H}}$ also

V.Karakaya and N.Şimşek [16] proved that this space is a Banach space and posses Kadec-Klee (H).

Remarks:

(1)Taking $a_n=\dfrac{1}{\sum_{k=1}^{n}q_k}$, then

$Ces((a_n),(p_n),(q_n))=$ $Ces((p_n),(q_n))$ the N¨orlund sequence spaces studied by [18].

(2)Taking $a_n=\dfrac{1}{n}$; $q_n=1,\forall n\in\mathbb{N}$,

then $Ces((a_n),(p_n),(q_n))=Ces(p_n)$ studied by W. Sanhan and S. Suantai [15].

(3)Taking $a_n=\dfrac{1}{n}$, $q_n=1$, $p_n=p$,$\forall n\in\mathbb{N}$,

then $Ces((a_n),(p_n),(q_n))=Ces_p$ studied by Many authors see [8,9and14].

Throughout this paper, the sequence (p_n) is a bounded sequence of positive real numbers with $\lim\limits_{n\to\infty}\inf p_n>1$, and also

1) $H=\sup_n p_n$.

2) Let (p_k) be a bounded sequence of positive real numbers, we have $|a_k+b_k|^{p_k}\leq 2^{H-1}(|a_k|^{p_k}+|b_k|^{p_k})$ $\forall k\in\mathbb{N}$.

2. Main results

Proposition2.1.

The functional σ is convex modular on $ces[(a_n),(p_n),(q_n)]$ and for any $x\in ces[(a_n),(p_n),(q_n)]$ the functional σ on $ces[(a_n),(p_n),(q_n)]$ satisfies the following properties:

(i) If 0<r<1, then

(ii) $r^H\sigma\left(\frac{x}{r}\right)\leq\sigma(x)$ and $\sigma(rx)\leq r\sigma(x)$.

(ii) If r>1, then $\sigma(x)\leq r^H\sigma\left(\frac{x}{r}\right)$.

(iii) If r≥1, then $\sigma(x)\leq r\sigma(x)\leq\sigma(rx)$.

Proof. All assertions are clearly obtained by the definition and convexity of σ see [17] .

Proposition2.2.

For any $x\in ces[(a_n),(p_n),(q_n)]$, the following assertions are satisfied:

(i) If $\| x \| < 1$, then $\sigma(x) \leq \| x \|$,

(ii) if $\| x \| > 1$, then $\sigma(x) \geq \| x \|$,

(iii) $\| x \| = 1$ if and only if $\sigma(x) = 1$.

Proof: It can be proved with standard techniques in a similar way as in [17].

Proposition2.3. $\forall L > 0$ and $\forall \varepsilon > 0 \, \exists \, \delta > 0$ such that $|\sigma(u+v) - \sigma(u)| < \varepsilon$ whenever $u, v \in ces[(a_n),(p_n),(q_n)]$ with $\sigma(u) \leq L$ and $\sigma(v) \leq \delta$

Proof: Since (p_n) is bounded, it is easy to see that $\sigma \in \delta_2^s$. Hence the proposition is obtained directly from theorem (1.1).

Proposition2.4. For any sequence $(x_n) \in ces[(a_n),(p_n),(q_n)], \| x_n \| \to 0$ if and only if $\sigma(x_n) \to 0$.

Proof: It follows directly from Theorem (1.2-2) since $\sigma \in \delta_2^s$.

Theorem2.5. $\forall x \in ces[(a_n),(p_n),(q_n)]$ and $\forall \varepsilon \in (0,1)$, $\exists \, \delta \in (0,1)$ such that $\sigma(x) \leq 1 - \varepsilon$ implies $\| x \| \leq 1 - \delta$.

Proof: Since $\sigma \in \delta_2^s$, the theorem is obtained directly from theorem (1.3).

Theorem2.6. The space $ces[(a_n),(p_n),(q_n)]$ is k-NUC $\forall$ integer $k \geq 2$.

Proof:
Let $\varepsilon > 0$ and $(x_n) \in B(ces[(a_n),(p_n),(q_n)])$ with $sep(x_n) \geq \varepsilon$. For each $m \in \mathbb{N}$, let $x_n^m = (0, 0, \ldots\ldots, 0, x_n(m), x_n(m+1), \ldots)$. Since for each $i \in \mathbb{N}$, $(x_n(i))_{n=1}^{\infty}$ is bounded, we have that $\forall i \in \mathbb{N}$, $(x_n(i))_{n=1}^{\infty}$ is bounded, by using the diagonal method, we can find a subsequence $(x_{n_j}(i))$ of (x_n) such that $(x_{n_j}(i))$ converges for each $i \in \mathbb{N}$, $1 \leq i \leq m$. Therefore, there exists an increasing sequence of positive integer (t_m) such that $sep((x_{n_j}^m)_{j > t_m}) \geq \varepsilon$. Hence, there is a sequence of positive integers $(r_m)_{m=1}^{\infty}$ with $r_1 < r_2 < r_3 < \ldots$ such that $\| x_{r_m}^m \| \geq \frac{\varepsilon}{2} \, \forall m \in \mathbb{N}$. Then by proposition (2.4), we may assume that there exists $\eta > 0$ such that $\sigma(x_{r_m}^m) \geq \eta \; \forall m \in \mathbb{N}$. (2.1)

Let $\alpha > 0$ be such that $1 < \alpha < \liminf_{n \to \infty} p_n$. For fixed integer $k \geq 2$, let $\varepsilon_1 = \left(\frac{(k^{\alpha - 1} - 1)}{(k-1)k^{\alpha}}\right)\left(\frac{\eta}{2}\right)$, then by proposition (2.3) $\exists \, \delta > 0$

Such that $|\sigma(u+v) - \sigma(u)| < \varepsilon_1$. (2.2)

whenever $\sigma(u) \leq 1$ and $\sigma(v) \leq \delta$. Since by proposition (2.2-1) $\sigma(x_n) \leq 1 \, \forall \; n \in \mathbb{N} \; \exists$ positive integers $m_i (i = 1,2,3,\ldots\ldots,k-1)$ with $m_1 < m_2 < m_3 < \ldots\ldots.. < m_{k-1}$ such that $\sigma(x_i^{m_i}) \leq \delta$ and $\alpha \leq p_j \; \forall \; j \geq m_{k-1}$. Define $m_k = m_{k-1} + 1$. By (2.1), we have $\sigma(x_{r_{m_k}}^{m_k}) \geq \eta$. Let $s_i = i$ for $1 \leq i \leq k-1$, and $s_k = r_{m_k}$. Then in virtue of (2.1),(2.2), and Convexity of function $f_i(u) = |u|^{p_i}$ $(i \in \mathbb{N})$, we have

$$\sigma\left(\frac{x_{s_1} + x_{s_2} + x_{s_3} + \ldots + x_{s_k}}{k}\right) = \sum_{n=1}^{\infty}\left(a_n \sum_{i=1}^{n} q_i \left|\frac{x_{s_1}(i) + x_{s_2}(i) + x_{s_3}(i) + \ldots + x_{s_k}(i)}{k}\right|\right)^{p_n} =$$

$$= \sum_{n=1}^{m_1}\left(a_n \sum_{i=1}^{n} q_i \left|\frac{x_{s_1}(i) + x_{s_2}(i) + x_{s_3}(i) + \ldots + x_{s_k}(i)}{k}\right|\right)^{p_n} +$$

$$+ \sum_{n=m_1+1}^{\infty}\left(a_n \sum_{i=1}^{n} q_i \left|\frac{x_{s_1}(i) + x_{s_2}(i) + x_{s_3}(i) + \ldots + x_{s_k}(i)}{k}\right|\right)^{p_n} \leq$$

$$\sum_{n=1}^{m_1}\frac{1}{k}\sum_{j=1}^{k}\left(a_n\sum_{i=1}^{n}q_i\left|x_{s_j}(i)\right|\right)^{p_n}+\sum_{n=m_1+1}^{m_2}\left(a_n\sum_{i=1}^{n}q_i\left|\frac{x_{s_2}(i)+x_{s_3}(i)+\ldots+x_{s_k}(i)}{k}\right|\right)^{p_n}+$$

$$+\sum_{n=m_2+1}^{\infty}\left(a_n\sum_{i=1}^{n}q_i\left|\frac{x_{s_3}(i)+\ldots+x_{s_k}(i)}{k}\right|\right)^{p_n}+2\varepsilon_1\le$$

$$\le\sum_{n=1}^{m_1}\frac{1}{k}\sum_{j=1}^{k}\left(a_n\sum_{i=1}^{n}q_i\left|x_{s_j}(i)\right|\right)^{p_n}+\sum_{n=m_1+1}^{m_2}\frac{1}{k}\sum_{j=2}^{k}\left(a_n\sum_{i=1}^{n}q_i\left|x_{s_j}(i)\right|\right)^{p_n}$$

$$+\sum_{n=m_2+1}^{m_3}\frac{1}{k}\sum_{j=3}^{k}\left(a_n\sum_{i=1}^{n}q_i\left|x_{s_j}(i)\right|\right)^{p_n}+\ldots+\sum_{n=m_{k-1}+1}^{m_3}\frac{1}{k}\sum_{j=k-1}^{k}\left(a_n\sum_{i=1}^{n}q_i\left|x_{s_j}(i)\right|\right)^{p_n}+$$

$$+\sum_{n=m_k+1}^{\infty}\left(a_n\sum_{i=1}^{n}q_i\left|\frac{x_{s_j}(i)}{k}\right|\right)^{p_n}+(k-1)\varepsilon_1\le$$

$$\le\frac{\sigma(x_{s_1})+\sigma(x_{s_2})+\ldots.+\sigma(x_{s_{k-1}})}{k}+\frac{1}{k}\sum_{n=1}^{m_k}\left(a_n\sum_{i=1}^{n}q_i\left|x_{s_k}(i)\right|\right)^{p_n}+$$

$$+\sum_{n=m_k+1}^{\infty}\left(a_n\sum_{i=1}^{n}q_i\left|\frac{x_{s_k}(i)}{k}\right|\right)^{p_n}+(k-1)\varepsilon_1\le$$

$$\le\frac{k-1}{k}+\frac{1}{k}\sum_{n=1}^{m_k}\left(a_n\sum_{i=1}^{n}q_i\left|x_{s_k}(i)\right|\right)^{p_n}+\frac{1}{k^{\alpha}}\sum_{n=m_k+1}^{\infty}\left(a_n\sum_{i=1}^{n}q_i\left|x_{s_k}(i)\right|\right)^{p_n}+(k-1)\varepsilon_1\le$$

$$\le1-\frac{1}{k}+\frac{1}{k}\left[1-\sum_{n=m_k+1}^{\infty}\left(a_n\sum_{i=1}^{n}q_i\left|x_{s_k}(i)\right|\right)^{p_n}\right]+\frac{1}{k^{\alpha}}\sum_{n=m_k+1}^{\infty}\left(a_n\sum_{i=1}^{n}q_i\left|x_{s_k}(i)\right|\right)^{p_n}+(k-1)\varepsilon_1$$

$$\le1+(k-1)\varepsilon_1-\left(\frac{k^{\alpha-1}-1}{k^{\alpha}}\right)\sum_{n=m_k+1}^{\infty}\left(a_n\sum_{i=1}^{n}q_i\left|x_{s_k}(i)\right|\right)^{p_n}$$

$$\le1+(k-1)\varepsilon_1-\left(\frac{k^{\alpha-1}-1}{k^{\alpha}}\right)\eta=1-\left(\frac{k^{\alpha-1}-1}{k^{\alpha}}\right)\left(\frac{\eta}{2}\right)$$

By theorem (2.5) $\exists\,\gamma>0$ such that

$$\left\|\frac{x_{s_1}+x_{s_2}+x_{n_3}+\ldots+x_{s_k}}{k}\right\|<1-\gamma.$$ Therefore, $ces[(a_n),(p_n),(q_n)]$ is k-NUC.

Since k-NUC implies k R and k R implies R and reflexivity holds, and k-NUC implies NUC and NUC implies H-property and reflexivity holds, by theorem (2.6), the following results are obtained.

COROLLARY2.7. For $\liminf_{n\to\infty} p_n>1$, the space $ces[(a_n),(p_n),(q_n)]$ is k R, NUC, and has a drop property.

COROLLARY2.8. For $\liminf_{n\to\infty} p_n>1$, the space $ces[(a_n),(p_n)]$ is k-NUC.

COROLLARY2.9. For $\liminf_{n\to\infty} p_n>1$, the space $ces[(p_n)]$ is k-NUC.

COROLLARY2.10. For $1<p<\infty$, the space Ces_p is k-NUC.

Corresponding author

N. Faried
Department of Mathematics, Faculty of Science, Ain Shams University, Cairo, Egypt
E-mail: n_faried@hotmail.com

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