The Difference Sequence Space Defined on Musielak-Orlicz Function

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Abstract: The idea of difference sequence spaces was introduced by Kizmaz [4]. Recently, Subramanian [12] studied the difference sequence space $\ell_M(\Delta)$ defined on Orlicz function M. In this paper we introduce new sequence spaces that we call Musielak-Orlicz difference sequence space and denote it by $\ell_M(\Delta)$, the difference paranormed Musielak-Orlicz sequence space $\ell_M(\Delta, p)$, where $M = (M_k)$ is a sequence of Orlicz functions, and study some inclusion relations and completeness of this spaces. [New York Science Journal 2010; 3(8):54-59]. (ISSN: 1554-0200).

Key words: Musielak-Orlicz function, paranorm, difference sequence.

Introduction

Orlicz [9] used the idea of Orlicz function to construct the space (L^M). Lindentrauss and Tzafriri [5] investigated Orlicz sequence spaces in more detail, and they proved that every Orlicz sequence space ℓ_M contains a subspace isomorphic to $\ell_p (1 \le p < \infty)$.

Subsequently different classes of sequence spaces defined by Parashar and Ghoudhary [10], Murasaleen et al. [6] Bekates and Altin [1], Tripathy et al. [13], Rao and Subramanian [2] and many others. Orlicz sequence spaces are the special cases of Orlicz spaces studied in Ref [3].

Recall ([3], [9]) an Orlicz function is a function $M : [0,\infty) \to [0,\infty)$ which is continuous, non-decreasing and convex with $M(0) = 0$, $M(x) > 0$ for $x > 0$, and $M(x) \to \infty$ as $x \to \infty$.

If convexity of Orlicz function M is replaced by $M(x+y) \le M(x) + M(y)$ then this function is called modulus function, introduced by Nakano and further discussed by Ruckle [11] and Maddox [7]. An Orlicz function M is said to satisfy Δ_2–condition for all values of u, if there exists a constant K>0, such that $M(2u) \le KM(u) (u \ge 0)$. The Δ_2–condition is equivalent to $M(\ell u) \le K\ell M(u)$, for all values of u and for $\ell > 1$. By ω, we shall denote the space of all real or complex sequences. The sets of natural numbers and real numbers will denote by $\mathbb{N}$= {1, 2,3, …}, $\mathbb{R}$ respectively.

A linear topological space X over $\mathbb{R}$ is said to be a paranormed space if there is a sub additive function $g : X \to \mathbb{R}$ such that $g(\theta) = 0$, $g(-x) = g(x)$ and for any sequence (x_n) in X such that $g(x_n - x) \xrightarrow{n-\infty} 0$, and any sequence (α_n) in $\mathbb{R}$ such that $|\alpha_n - \alpha| \xrightarrow{n-\infty} 0$, we get $g(\alpha_n x_n - \alpha x) \xrightarrow{n-\infty} 0$.

Lindentrauss and Tzafriri [5] used the idea of Orlicz function to construct Orlicz sequence space

$$\ell_M = \left\{ x \in \omega : \sum_{k=1}^{\infty} M\left(\frac{|x_k|}{\rho}\right) < \infty, \text{for some } \rho > 0 \right\}$$

. The space ℓ_M with the norm

$\| x \| = \inf\left\{\rho > 0 : \sum_{k=1}^{\infty} M\left(\frac{|x_k|}{\rho}\right) \leq 1\right\}$ becomes a Banach space which is called an Orlicz sequence space. For $M(t) = t^p, 1 \leq p < \infty$, the space ℓ_M coincide with the classical sequence space ℓ_p.

The idea of difference sequence was first introduced by Kizmaz [4] write

$\Delta x_k = x_k - x_{k+1}$, for k=1,2,3,..., $\Delta : \omega \to \omega$ be the difference defined by $\Delta x = (\Delta x_k)_{k=1}^{\infty}$, and $M : [0,\infty) \to [0,\infty)$ be an Orlicz function; or a modulus function.

Let ℓ be the sequence of absolutely convergent series. Define a sequence space.

$\ell(\Delta) = \{x = (x_k) : \Delta x \in \ell\}$.The sequence space

$$\ell_M(\Delta) = \left\{x \in \omega \sum_{k=1}^{\infty} M\left(\frac{|\Delta x_k|}{\rho}\right) < \infty, \text{for some } \rho > 0\right\}$$

, with the norm

$$\| x \| = \inf\left\{\rho > 0 : \sum_{k=1}^{\infty} M\left(\frac{|\Delta x_k|}{\rho}\right) \leq 1\right\}$$

, becomes a Banach space which is called an Orlicz difference sequence space $\ell_M(\Delta, M)$, see [12].

A sequence $M = (M_k)$ of Orlicz functions

$M_k \ \forall k \in \mathbb{N}$ is called a Musielak- Orlicz function, for a given Musielak-Orlicz function M. The function $I_M : \omega \to [0,\infty]; I_M(x) = \sum_{k=1}^{\infty} M_k(x_k)$;

$\forall x \in \omega$ is convex modular.

The Musielak-Orlicz function space ℓ_M generated by $M = (M_k)$ is defined by

$$\ell_M = \left\{x \in \omega : \sum_{k=1}^{\infty} M_k\left(\frac{|x_k|}{\rho}\right) < \infty, \exists \rho > 0\right\},$$

and ℓ_M with the norm

$\| x \| = \inf\left\{\rho > 0 : \sum_{k=1}^{\infty} M_k\left(\frac{|x_k|}{\rho}\right) \leq 1\right\}$ is a Banach space seeing [8].

We define the following new sequence space

Definition: Musielak-Orlicz difference sequence space $\ell_M(\Delta)$ is

$$\ell_M(\Delta) = \left\{x \in \omega : \sum_{k=1}^{\infty} M_k\left(\frac{|\Delta x_k|}{\rho}\right) < \infty, \exists \rho > 0\right\}$$

, where $M = (M_k)$ is a sequence of Orlicz functions. With the norm

$$\| x \| = \inf\left\{\rho > 0 : \sum_{k=1}^{\infty} M_k\left(\frac{|\Delta x_k|}{\rho}\right) \leq 1\right\}.$$

If $M_k = M \ \forall k \in \mathbb{N}$, then $\ell_M(\Delta)$ reduces to Orlicz difference sequence Space studied by Subramanian [12].

Theorem (1): The space $\ell_M(\Delta)$, where $M = (M_k)_{k=1}^{\infty}$ is a sequence of Orlicz functions is a Banach space with the norm

$$\| x \| = \inf\left\{\rho > 0 : \sum_{k=1}^{\infty} M_k\left(\frac{|\Delta x_k|}{\rho}\right) \leq 1\right\}.$$

Proof:

Let $x^{(i)}$ be any Cauchy sequence in $\ell_M(\Delta)$, where $x^{(i)} = (x_k^{(i)}) = (x_1^{(i)}, x_2^{(i)}, \ldots) \in \ell_M(\Delta) \ \forall i \in \mathbb{N}$. Let $r, x_0 > 0$ be fixed, then for each $\frac{\varepsilon}{r x_0} > 0$, there exist a positive integer N such that $\| x^{(i)} - x^{(j)} \| < \frac{\varepsilon}{r x_0} \ \forall i, i \geq N$.

Using the definition of norm we get

$$\sum_{k=1}^{\infty} M_k\left(\frac{\Delta x_k^{(i)}-\Delta x_k^{(j)}}{\| x^{(i)}-x^{(j)}\|_\Delta}\right)\le 1 \ \forall i,j\ge N$$

$$\Rightarrow M_k\left(\frac{|\Delta x_k^{(i)}-\Delta x_k^{(j)}|}{\| x^{(i)}-x^{(i)}\|_\Delta}\right)\le 1 \ \ \forall k\in\mathbb{N}$$

, and $\forall i,j\ge N$. Hence we can find $r>0$ with $M_k\left(\frac{rx_0}{k}\right)>1\ \forall k\in\mathbb{N}$, such that

$$M_k\left(\frac{|\Delta x_k^{(i)}-\Delta x_k^{(j)}|}{\| x^{(i)}-x^{(j)}\|_\Delta}\right)\le M_k\left(\frac{rx_0}{k}\right)$$

.Since M_k is non-decreasing $\forall k\in\mathbb{N}$. This implies that

$$\frac{|\Delta x_k^{(i)}-\Delta x_k^{(j)}|}{\| x^{(i)}-x^{(j)}\|_\Delta}\le\frac{rx_0}{k}\Rightarrow$$

$$|\Delta x_k^{(i)}-\Delta x_k^{(j)}|\le\frac{rx_0}{k}\| x^{(i)}-x^{(j)}\|_\Delta<\frac{rx_0}{k}\frac{\varepsilon}{rx_0}=\frac{\varepsilon}{k}$$

.Therefore $\forall\ \varepsilon(0<\varepsilon<1)$ then $\exists$ a positive integer N such that

$$|(\Delta x_1^{(i)}-\Delta x_1^{(j)})+..........+(\Delta x_1^{(i)}-\Delta x_1^{(j)})|\le$$

$$|\Delta x_1^{(i)}-\Delta x_1^{(j)}|+..........+|\Delta x_k^{(i)}-\Delta x_k^{(j)}|\le k\frac{\varepsilon}{k}$$

$$\Rightarrow|(\Delta x_1^{(i)}-\Delta x_1^{(j)})|+..........+|(\Delta x_k^{(i)}-\Delta x_k^{(j)})|\le\varepsilon$$

.

Since $|\Delta x_k^{(i)}-\Delta x_k^{(j)}|\le\{|\Delta x_1^{(i)}-\Delta x_1^{(j)}|+......+|x_k^{(i)}-\Delta x_k^{(j)}|\}$, we get $|\Delta x_k^{(i)}-\Delta x_k^{(j)}|\le\varepsilon\ \forall$i, j $\ge N$.

Therefore $(\Delta x_k^{(j)})_{j=1}^{\infty}$ is a Cauchy sequence in $\mathbb{R}$, for each fixed k. Using the continuity of $M_k\ \forall\ k\in\mathbb{N}$, we can find

that $\sum_{k=1}^{N} M_k\left(\frac{|\Delta x_k^{(i)}-Lim_{j\to\infty}\Delta x_k^{(j)}|}{\rho}\right)\le 1$. Thus

$$\sum_{k=1}^{N} M_k\left(\frac{|\Delta x_k^{(i)}-\Delta x_k|}{\rho}\right)\le 1\ \forall i\ge N.$$

Taking infimum of such ρ's we get

$$\inf\left\{\rho>0:\sum_{n=1}^{N} M_k\left(\frac{|\Delta x_k^{(i)}-\Delta x|}{\rho}\right)\le 1\right\}<\varepsilon$$

$\forall\, i\ge N$, since $\Delta x^{(i)}\in\ell_M(\Delta)$ and M_k is continuous $\forall\, k\in\mathbb{N}$ then $\Delta x\in\ell_M(\Delta)$. This completes the proof.

Theorem (2): Let $M=(M_k)$ be a Musielak-modulus function which satisfies Δ_2-condition, then $\ell(\Delta)\subset\ell_M(\Delta)$.

Proof: Let $x\in\ell(\Delta)\Rightarrow\sum_{k=1}^{\infty}|\Delta x_k|\le N$, since M_k is non-decreasing $\forall\, k\in\mathbb{N}$

$$\Rightarrow\left(M_k\left(\sum_{k=1}^{\infty}\frac{|\Delta x_k|}{\rho}\right)\right)\le\left(M_k\left(\frac{N}{\rho}\right)\right)\le K\ell M_k(N).$$

By Δ_2 -condition, we get $x\in\ell_M(\Delta)$.

Paranormed sequence spaces:

Let $p=(p_k)$ be any sequence of positive real numbers, then in the same way we can also define the following sequence spaces for a Musielak–Orlicz function M as ℓ extended to $\ell(p)$

$$\ell_M(\Delta,p)=\left\{x\in\omega:\sum_{k=1}^{\infty}\left(M_k\left(\frac{|\Delta x_k|}{\rho}\right)\right)^{p_k}<\infty,\exists\,\rho>0\right\}$$

Note: If $p_k=$ p $\forall\ k\in\mathbb{N}$, then

$$\ell_M(\Delta,p)=\ell_M(\Delta).$$

Theorem (3): $\ell_M(\Delta,p)$ is a complete paranormed space with

$$g^*(x)=\inf\left\{\rho^{\frac{P_k}{H}}:\left[\sum_{k=1}^{\infty}\left(M_k\left(\frac{|\Delta x_k|}{\rho}\right)\right)^{P_k}\right]^{\frac{1}{H}}\le 1\right\}$$

. For $1\le p_k<\infty\ \forall\ k\in\mathbb{N}$,

$$H=\max\{1,\sup_n P_n\}.$$

Proof: Let $x^{(i)}$ be any Cauchy sequence in $\ell_M(\Delta, p)$, where

$x^{(i)} = (x_1^{(i)}, x_2^{(i)}, x_3^{(i)} ..) \in \ell_M(\Delta, p)\ \forall i \in \mathbb{N}$. Let $r, x_0 > 0$ is fixed. Then $\forall\ \frac{\varepsilon}{rx_o} > 0\ \exists$a positive integer N such that

$g^*(x^{(i)} - x^{(j)}) < \frac{\varepsilon}{rx_o}\ \forall i, j \geq N$ .Using the definition of paranorm we get

$$\left[\sum_{k=1}^{\infty}\left(M_k\left(\frac{|\Delta x_k^{(i)} - \Delta x_k^{(j)}|}{g^*(x^{(i)} - x^{(j)})}\right)\right)^{p_k}\right]^{\frac{1}{H}} \leq 1,$$

Since $1 \leq p_k \leq \infty$, $\forall k \in \mathbb{N}$. It follows that

$$M_k\left(\frac{|\Delta x_k^{(i)} - \Delta x_k^{(j)}|}{g^*(x^{(i)} - x^{(j)})}\right) \leq 1, \forall k \geq 1 \text{ and } \forall i, j \geq N.$$

Hence we can find r>0 $\forall k \in \mathbb{N}$ with $M_k\left(\frac{rx_0}{k}\right) > 1$ such that $M_k\left(\frac{|\Delta x_k^{(i)} - \Delta x_k^{(j)}|}{g^*(x^{(i)} - x^{(j)})}\right) \leq M_k\left(\frac{rx_0}{k}\right)$.

Since M_k is non-decreasing $\forall k \in \mathbb{N}$ We get

$$\frac{|\Delta x_k^{(i)} - \Delta x_k^{(j)}|}{g^*(x^{(i)} - x^{(j)})} \leq \frac{rx_0}{k}$$

$$\Rightarrow |\Delta x_k^{(i)} - \Delta x_k^{(j)}| \leq \frac{rx_0}{k} g^*(x^{(i)} - x^{(j)}) \leq \frac{rx_0}{k}\frac{\varepsilon}{rx_0} = \frac{\varepsilon}{k}$$

. Therefore for each $0 < \varepsilon < 1$ then there exist a positive integer N such that

$$|(\Delta x_1^{(i)} - \Delta x_1^{(j)}) + ... + (\Delta x_1^{(i)} - \Delta x_1^{(j)})| \leq$$
$$|\Delta x_1^{(i)} - \Delta x_1^{(j)}| + ... + |\Delta x_k^{(i)} - \Delta x_k^{(j)}| \leq k\frac{\varepsilon}{k}$$

Since

$$|(\Delta x_k^{(i)} - \Delta x_k^{(j)}) | \leq \left|\Delta x_1^{(i)} - \Delta x_1^{(j)}\right| + ... + \left|\Delta x_k^{(i)} - \Delta x_k^{(j)}\right|$$

we get $|\Delta x_k^{(i)} - \Delta x_k^{(j)}| \leq \varepsilon$, $\forall k \in \mathbb{N}$.

Therefore $(\Delta x_k^{(j)})_{j=1}^{\infty}$ be a Cauchy sequence in $\mathbb{R}$, for fixed k. Using the continuity of $M_k\ \forall k \in \mathbb{N}$, we can find that

$$\left[\sum_{k=1}^{N}\left(M_k\left(\frac{|\Delta x_k^{(i)} - \lim_{j\to\infty} \Delta x_k^{(j)}}{\rho}\right)\right)^{P_k}\right]^{\frac{1}{H}} \leq 1$$

$$\Rightarrow \left[\sum_{k=1}^{N}\left(M_k\left(\frac{|\Delta x_k^{(i)} - \Delta x|}{\rho}\right)\right)^{P_k}\right]^{\frac{1}{H}} \leq 1.$$

Taking infimum of such ρ 's we get

$$\inf\left\{\rho^{\frac{P_k}{H}} : \left[\sum_{k=1}^{N}\left[M_k\left(\frac{|\Delta x_k^{(i)} - \Delta x|}{\rho}\right)\right]^{P_k}\right]^{\frac{1}{H}} \leq 1\right\} < \varepsilon$$

$\forall i \geq N$, and $j \to \infty$.

Since $(x^{(i)}) \in \ell_M(\Delta, p)$ and M_k is continuous $\forall\ k \in \mathbb{N}$ it follows that $x \in \ell_M(\Delta, p)$.

Theorem (4): Let $0 < p_k < q_k < \infty\ \forall\ k \in \mathbb{N}$**,** then $\ell_M(\Delta, p) \subseteq \ell_M(\Delta, q)$.

Proof: Let $x \in \ell_M(\Delta, p)$

$$\Rightarrow \sum_{k=1}^{\infty} M_k\left[\left(\frac{|\Delta x_k|}{\rho}\right)\right]^{P_k} < \infty \text{, then}$$

$M_k\left(\frac{|\Delta x_k|}{\rho}\right) \leq 1\ \forall\ k \in \mathbb{N}$. For sufficiently large k, since M_k is non-decreasing. Hence we get

$$\sum_{k=1}^{\infty}\left[M_k\left(\frac{|\Delta x_k|}{\rho}\right)\right]^{q_k} \leq \sum_{k=1}^{\infty}\left[M_k\left(\frac{|\Delta x_k|}{\rho}\right)\right]^{P_k} < \infty$$

$$\Rightarrow \sum_{k=1}^{\infty}\left[M_k\left(\frac{|\Delta x_k|}{\rho}\right)\right]^{q_k} < \infty.$$

$$\Rightarrow x \in \ell_M(\Delta, q).$$

Theorem (5):

(a) Let $0 < \inf_k \; p_k \le p_k \le 1 \; \forall \; k \in \mathbb{N}$.

Then $\ell_M(\Delta, p) \subseteq \ell_M(\Delta)$

(b) Let $1 \le p_k \le \sup_k \; p_k < \infty \; \forall \; k \in \mathbb{N}$. Then.

Proof:

(a) For $x \in \ell_M(\Delta, p)$, then

$$\sum_{k=1}^{\infty}\left[M_k\left(\frac{|\Delta x_k|}{\rho}\right)\right]^{p_k} < \infty \Rightarrow M_k\left(\frac{|\Delta x_k|}{\rho}\right) \le 1$$

For sufficiently large k, since

$0 < \inf \; p_k \le p_k \le 1 \forall \; k \in \mathbb{N}$

$$\Rightarrow \sum_{k=1}^{\infty} M_k\left(\frac{|\Delta x_k|}{\rho}\right) \le \sum_{k=1}^{\infty}\left(M_k\left(\frac{|\Delta x_k|}{\rho}\right)^{P_k}\right)$$

$$\Rightarrow \sum_{k=1}^{\infty} M_k\left(\frac{|\Delta x_k|}{\rho}\right) < \infty \Rightarrow x \in \ell_M(\Delta)$$

(b) For $P_k \ge 1 \;\; \forall \; k \in \mathbb{N}$ and $\sup \; p_k < \infty$ and

$x \in \ell_M(\Delta)$ we get $\sum_{k=1}^{\infty} M_k\left(\frac{|\Delta x_k|}{\rho}\right) < \infty \Rightarrow$

$M_k\left(\frac{|\Delta x_k|}{\rho}\right) \le 1$. For sufficiently large k,

since $1 \le p_k \le \sup p_k < \infty \; \forall \; k \in \mathbb{N}$, we get

$$\sum_{k=1}^{\infty}\left(M_k\left(\frac{|\Delta x_k|}{\rho}\right)\right)^{P_k} \le \sum_{k=1}^{\infty} M_k\left(\frac{|\Delta x_k|}{\rho}\right) < \infty$$

$$\Rightarrow \sum_{k=1}^{\infty}\left(M_k\left(\frac{|\Delta x_k|}{\rho}\right)\right)^{P_k} < \infty . \Rightarrow x \in \ell_M(\Delta, p).$$

Theorem (6): Let $0 \le p_k \le q_k \; \forall \; k \in \mathbb{N}$

and $\left(\frac{q_k}{p_k}\right)$ be bounded,

then $\ell_M(\Delta, q) \subset \ell_M(\Delta, p)$.

Proof: For $x \in \ell_M(\Delta, q)$ (i.e.)

$$\sum_{k=1}^{\infty}\left[M_k\left(\frac{|\Delta x_k|}{\rho}\right)\right]^{q_k} < \infty \text{ and}$$

$$t_k = \sum_{k=1}^{\infty}\left[M_k\left(\frac{|\Delta x_k|}{\rho}\right)\right]^{q_k} \text{ and } \lambda_k = \frac{q_k}{p_k}.$$

Since $p_k \le q_k \; \forall \; k \in \mathbb{N}$

therefore $0 \le \lambda_k \le 1 \; \forall \; k \in \mathbb{N}$.Take $0 < \lambda < \lambda_k$

$\forall \; k \in \mathbb{N}$. Define $u_k = t_k (t_k \ge 1)$;

$u_k = 0(t_k < 1)$ and $v_k = 0(t_k \ge 1)$,

$u_k = t_k (t_k < 1) . t_k = u_k + v_k$. (i.e.)

$t_k^{\lambda_k} = u_k^{\lambda_k} + v_k^{\lambda_k}$ .Now it follows that

$u_k^{\lambda_k} \le u_k \le t_k$ and $v_k^{\lambda_k} \le v_k^{\lambda}$ (1).

(i.e.) $\sum_{k=1}^{\infty} t_k^{\lambda_k} = \sum_{k=1}^{\infty} (u_k + v_k)^{\lambda_k}$

$$\Rightarrow \sum_{k=1}^{\infty} t_k^{\lambda_k} \le \sum_{k=1}^{\infty} u_k^{\lambda_k} + \sum_{k=1}^{\infty} v_k^{\lambda_k}.$$

$$\Rightarrow \sum_{k=1}^{\infty} t_k^{\lambda_k} \le \sum_{k=1}^{\infty} t_k + \sum_{k=1}^{\infty} v_k^{\lambda}.$$

By using equation (1), we

$$\text{get} \sum_{k=1}^{\infty}\left[M_k\left(\frac{|\Delta x_k|}{\rho}\right)\right]^{q_k \lambda_k} \le \sum_{k=1}^{\infty}\left[M_k\left(\frac{|\Delta x_k|}{\rho}\right)\right]^{q_k}$$

$$\Rightarrow \sum_{k=1}^{\infty}\left[M_k\left(\frac{|\Delta x_k|}{\rho}\right)\right]^{p_k} \le \sum_{k=1}^{\infty}\left[M_k\left(\frac{|\Delta x_k|}{\rho}\right)\right]^{q_k},$$

then $\ell_M(\Delta, q) \subset \ell_M(\Delta, p)$.

Theorem (7): Let $1 \le p_k \le \sup_k \; p_k < \infty \; \forall \; k \in \mathbb{N}$.

Then $\ell_M(\Delta, p)$ where $M = (M_k)$ be a Musielak-modulus function is a linear set over the set of complex numbers.

Proof: is easy so omitted.

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