The Difference Sequence Space Defined on Musielak-Orlicz Function

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Abstract: The idea of difference sequence spaces was introduced by Kizmaz [4]. Recently, Subramanian [12] studied the difference sequence space $\ell_M(\Delta)$ defined on Orlicz function M. In this paper we introduce new sequence spaces that we call Musielak-Orlicz difference sequence space and denote it by $\ell_M(\Delta)$, the difference paranormed Musielak-Orlicz sequence space $\ell_M(\Delta, p)$, where $M = (M_k)$ is a sequence of Orlicz functions, and study some inclusion relations and completeness of this spaces. [New York Science Journal 2010;3(8):54-]. (ISSN: 1554-0200).

Key words: Musielak-Orlicz function, paranorm, difference sequence.

Introduction

Orlicz [9] used the idea of Orlicz function to construct the space (L^M). Lindentrauss and Tzafriri [5] investigated Orlicz sequence spaces in more detail, and they proved that every Orlicz sequence space ℓ_M contains a subspace isomorphic to $\ell_p (1 \le p < \infty)$.

Subsequently different classes of sequence spaces defined by Parashar and Ghoudhary [10], Murasaleen et al. [6] Bekates and Altin [1], Tripathy et al. [13], Rao and Subramanian [2] and many others. Orlicz sequence spaces are the special cases of Orlicz spaces studied in Ref [3].

Recall ([3], [9]) an Orlicz function is a function $M : [0, \infty) \to [0, \infty)$ which is continuous, non-decreasing and convex with $M(0) = 0$, $M(x) > 0$ for $x > 0$, and $M(x) \to \infty$ as $x \to \infty$.

If convexity of Orlicz function M is replaced by $M(x+y) \le M(x) + M(y)$ then this function is called modulus function, introduced by Nakano and further discussed by Ruckle [11] and Maddox [7].An Orlicz function M is said to satisfy Δ_2–condition for all values of u, if there exists a constant K>0,such that $M(2u) \le KM(u) (u \ge 0)$. The Δ_2–condition is equivalent to $M(lu) \le KlM(u)$, for all values of u and for l >1. Byω, we shall denote the space of all real or complex sequences. The sets of natural numbers and real numbers will denote by $\mathbb{N} = \{1, 2, 3, \ldots\}$, $\mathbb{R}$ respectively.

A linear topological space X over $\mathbb{R}$ is said to be a paranormed space if there is a sub additive function $g : X \to \mathbb{R}$ such that $g(\theta) = 0$, $g(-x) = g(x)$ and for any sequence (x_n) in X such that $g(x_n - x) \xrightarrow{n-\infty} 0$, and any sequence (α_n) in $\mathbb{R}$ such that $|\alpha_n - \alpha| \xrightarrow{n-\infty} 0$, we get $g(\alpha_n x_n - \alpha x) \xrightarrow{n-\infty} 0$.

Lindentrauss and Tzafriri [5] used the idea of Orlicz function to construct Orlicz sequence space

$$\ell_M = \left\{ x \in \omega : \sum_{k=1}^{\infty} M\left(\frac{|x_k|}{\rho} \right) < \infty, \text{for some } \rho > 0 \right\}$$

. The space ℓ_M with the norm

$$\| x \| = \inf \left\{ \rho > 0 : \sum_{k=1}^{\infty} M\left(\frac{|x_k|}{\rho} \right) \le 1 \right\}$$

becomes a Banach space which is called an Orlicz sequence space. For $M(t) = t^p, 1 \le p < \infty$, the space ℓ_M coincide with the classical sequence space ℓ_p.

The idea of difference sequence was first introduced by Kizmaz [4] write

$\Delta x_k = x_k - x_{k+1}$, for k=1,2,3,..., $\Delta : \omega \to \omega$ be the difference defined by $\Delta x = (\Delta x_k)_{k=1}^{\infty}$, and $M : [0,\infty) \to [0,\infty)$ be an Orlicz function; or a modulus function.

Let l be the sequence of absolutely convergent series. Define a sequence space.

$l(\Delta) = \{x = (x_k) : \Delta x \in l\}$.The sequence space

$$l_M(\Delta) = \left\{ x \in \omega \sum_{k=1}^{\infty} M\left(\frac{|\Delta x_k|}{\rho}\right) < \infty, \text{ for some } \rho > 0 \right\}$$

, with the norm

$$\| x \| = \inf \left\{ \rho > 0 : \sum_{k=1}^{\infty} M\left(\frac{|\Delta x_k|}{\rho}\right) \le 1 \right\}$$

becomes a Banach space which is called an Orlicz difference sequence space $l_M(\Delta, M)$, see [12].

A sequence $M = (M_k)$ of Orlicz

functions $M_k \; \forall k \in \mathbb{N}$ is called a Musielak- Orlicz function, for a given Musielak-Orlicz function M. The function

$$I_M : \omega \to [0,\infty]; I_M(x) = \sum_{k=1}^{\infty} M_k(x_k); \; \forall x \in \omega$$

is convex modular.

The Musielak-Orlicz function space l_M generated by $M = (M_k)$ is defined by

$$l_M = \left\{ x \in \omega : \sum_{k=1}^{\infty} M_k\left(\frac{|x_k|}{\rho}\right) < \infty, \exists \rho > 0 \right\},$$

and l_M with the norm

$$\| x \| = \inf \left\{ \rho > 0 : \sum_{k=1}^{\infty} M_k\left(\frac{|x_k|}{\rho}\right) \le 1 \right\} \text{ is a}$$

Banach space seeing [8].

We define the following new sequence space

Definition: Musielak-Orlicz difference sequence space $l_M(\Delta)$ is

$$l_M(\Delta) = \left\{ x \in \omega : \sum_{k=1}^{\infty} M_k\left(\frac{|\Delta x_k|}{\rho}\right) < \infty, \exists \rho > 0 \right\}$$

, where $M = (M_k)$ is a sequence of Orlicz

functions. With the norm

$$\| x \| = \inf \left\{ \rho > 0 : \sum_{k=1}^{\infty} M_k\left(\frac{|\Delta x_k|}{\rho}\right) \le 1 \right\}.$$

If $M_k = M \; \forall k \in \mathbb{N}$, then $l_M(\Delta)$ reduces to Orlicz difference sequence Space studied by Subramanian [12].

Theorem(1): The space $l_M(\Delta)$, where

$M = (M_k)_{k=1}^{\infty}$ is a sequence of Orlicz functions is a Banach space with the norm

$$\| x \| = \inf \left\{ \rho > 0 : \sum_{k=1}^{\infty} M_k\left(\frac{|\Delta x_k|}{\rho}\right) \le 1 \right\}.$$

Proof:

Let $x^{(i)}$ be any Cauchy sequence in $l_M(\Delta)$, where $x^{(i)} = (x_k^{(i)}) = (x_1^{(i)}, x_2^{(i)}, \ldots) \in l_M(\Delta) \; \forall i \in \mathbb{N}$. Let $r, x_0 > 0$ be fixed, then for each $\frac{\varepsilon}{r x_0} > 0$,

there exist a positive integer N such

that $\| x^{(i)} - x^{(j)} \| < \frac{\varepsilon}{r x_0} \forall \text{i}, \text{i} \ge \text{N}$.

Using the definition of norm we get

$$\sum_{k=1}^{\infty} M_k\left(\frac{\Delta x_k^{(i)} - \Delta x_k^{(j)}}{\| x^{(i)} - x^{(j)} \|_\Delta}\right) \le 1 \; \forall i, j \ge N$$

$$\Rightarrow M_k\left(\frac{|\Delta x_k^{(i)} - \Delta x_k^{(j)}|}{\| x^{(i)} - x^{(i)} \|_\Delta}\right) \le 1 \quad \forall k \in \mathbb{N}$$

, and $\forall i, j \ge N$. Hence we can find $r > 0$ with $M_k\left(\frac{r x_0}{k}\right) > 1 \; \forall k \in \mathbb{N}$, such that

$$M_k\left(\frac{|\Delta x_k^{(i)}-\Delta x_k^{(j)}|}{\| x^{(i)}-x^{(j)}\|_\Delta}\right)\le M_k\left(\frac{rx_0}{k}\right)$$

.Since M_k is non-decreasing $\forall k\in\mathbb{N}$. This implies that

$$\frac{|\Delta x_k^{(i)}-\Delta x_k^{(j)}|}{\| x^{(i)}-x^{(j)}\|_\Delta}\le\frac{rx_0}{k}\Rightarrow$$

$$|\Delta x_k^{(i)}-\Delta x_k^{(j)}|\le\frac{rx_0}{k}\| x^{(i)}-x^{(j)}\|_\Delta<\frac{rx_0}{k}\frac{\varepsilon}{rx_0}=\frac{\varepsilon}{k}$$

.Therefore $\forall\ \varepsilon(0<\varepsilon<1)$ then$\exists$ a positive integer N such that

$$|(\Delta x_1^{(i)}-\Delta x_1^{(j)})+\ldots\ldots\ldots+(\Delta x_1^{(i)}-\Delta x_1^{(j)})|\le$$

$$|\Delta x_1^{(i)}-\Delta x_1^{(j)}|+\ldots\ldots\ldots+|\Delta x_k^{(i)}-\Delta x_k^{(j)}|\le k\frac{\varepsilon}{k}$$

$$\Rightarrow|(\Delta x_1^{(i)}-\Delta x_1^{(j)})|+\ldots\ldots\ldots+|(\Delta x_k^{(i)}-\Delta x_k^{(j)})|\le\varepsilon$$

.

Since $|\Delta x_k^{(i)}-\Delta x_k^{(j)}|\le\{|\Delta x_1^{(i)}-\Delta x_1^{(j)}|+\ldots\ldots+|x_k^{(i)}-\Delta x_k^{(j)}|\}$, we get $|\Delta x_k^{(i)}-\Delta x_k^{(j)}|\le\varepsilon\ \forall$i, j $\ge N$.

Therefore $(\Delta x_k^{(j)})_{j=1}^{\infty}$ is a Cauchy sequence in $\mathbb{R}$, for each fixed k. Using the continuity of $M_k\ \forall\ k\in\ \mathbb{N}$, we can find

that $\sum_{k=1}^{N}M_k\left(\frac{|\Delta x_k^{(i)}-Lim_{j\to\infty}\Delta x_k^{(j)}|}{\rho}\right)\le 1$. Thus

$$\sum_{k=1}^{N}M_k\left(\frac{|\Delta x_k^{(i)}-\Delta x_k|}{\rho}\right)\le 1\ \forall i\ge N.$$

Taking infimum of such ρ's we get

$$\inf\left\{\rho>0:\sum_{n=1}^{N}M_k\left(\frac{|\Delta x_k^{(i)}-\Delta x|}{\rho}\right)\le 1\right\}<\varepsilon$$

$\forall i\ge N$, since $\Delta x^{(i)}\in l_M(\Delta)$ and M_k is continuous $\forall\, k\in\mathbb{N}$ then $\Delta x\in l_M(\Delta)$. This completes the proof.

Theorem(2): Let $M=(M_k)$ be a Musielak-modulus function which satisfies Δ_2 condition, then $l(\Delta)\subset l_M(\Delta)$.

Proof: Let $x\in l(\Delta)\Rightarrow\sum_{k=1}^{\infty}|\Delta x_k|\le N$, since M_k is non-decreasing $\forall\, k\in\mathbb{N}$

$$\Rightarrow\left(M_k\left(\sum_{k=1}^{\infty}\frac{|\Delta x_k|}{\rho}\right)\right)\le\left(M_k\left(\frac{N}{\rho}\right)\right)\le K l M_k(N)$$

.By Δ_2 condition, therefore $x\in l_M(\Delta)$.

Paranormed sequence spaces:

Let $p=(p_k)$ be any sequence of positive real numbers, then in the same way we can also define the following sequence spaces for a Musielak–Orlicz function M as l extended to $l(p)$

$$l_M(\Delta,p)=\left\{x\in\omega:\sum_{k=1}^{\infty}\left(M_k\left(\frac{|\Delta x_k|}{\rho}\right)\right)^{p_k}<\infty,\exists\rho>0\right\}$$

Note: If $p_k=$ p $\forall\ k\in\mathbb{N}$,

then $l_M(\Delta,p)=l_M(\Delta)$.

Theorem (3): $l_M(\Delta,p)$ is a complete paranormed space with

$$g^*(x)=\inf\left\{\rho^{\frac{P_k}{H}}:\left[\sum_{k=1}^{\infty}\left(M_k\left(\frac{|\Delta x_k|}{\rho}\right)\right)^{P_k}\right]^{\frac{1}{H}}\le 1\right\}$$

. For $1\le p_k<\infty\ \ \forall\ k\in\mathbb{N}$,

$H=\max\{1,\sup_n P_n\}$.

Proof: Let $x^{(i)}$ be any Cauchy sequence in $l_M(\Delta,p)$, where

$x^{(i)}=(x_1^{(i)},x_2^{(i)},x_3^{(i)}..)\in l_M(\Delta,p)\ \forall i\in\mathbb{N}$. Let $r,x_0>0$ is fixed. Then $\forall\ \frac{\varepsilon}{rx_o}>0$ $\exists$a positive integer N such that

$g^*(x^{(i)}-x^{(j)}) < \frac{\varepsilon}{rx_o}$ $\forall i, j \geq N$ .Using the definition of paranorm we get

$$\left[\sum_{k=1}^{\infty}\left(M_k\left(\frac{|\Delta x_k^{(i)}-\Delta x_k^{(j)}|}{g^*(x^{(i)}-x^{(j)})}\right)\right)^{p_k}\right]^{\frac{1}{H}} \leq 1,$$

Since $1 \leq p_k \leq \infty$, $\forall k \in \mathbb{N}$. It follows that

$M_k\left(\frac{|\Delta x_k^{(i)}-\Delta x_k^{(j)}|}{g^*(x^{(i)}-x^{(j)})}\right) \leq 1$, $\forall k \geq 1$ and $\forall i, j \geq N$.

Hence we can find r>0 $\forall k \in \mathbb{N}$ with $M_k\left(\frac{rx_0}{k}\right) > 1$ such that $M_k\left(\frac{|\Delta x_k^{(i)}-\Delta x_k^{(j)}|}{g^*(x^{(i)}-x^{(j)})}\right) \leq M_k\left(\frac{rx_0}{k}\right)$.

Since M_k is non-decreasing $\forall k \in \mathbb{N}$ We get

$$\frac{|\Delta x_k^{(i)}-\Delta x_k^{(j)}|}{g^*(x^{(i)}-x^{(j)})} \leq \frac{rx_0}{k}$$

$$\Rightarrow |\Delta x_k^{(i)}-\Delta x_k^{(j)}| \leq \frac{rx_0}{k} g^*(x^{(i)}-x^{(j)}) \leq \frac{rx_0}{k}\frac{\varepsilon}{rx_0} = \frac{\varepsilon}{k}$$

. Therefore for each $0 < \varepsilon < 1$ then there exist a positive integer N such that

$$\left|(\Delta x_1^{(i)}-\Delta x_1^{(j)})+\ldots\ldots+(\Delta x_k^{(i)}-\Delta x_k^{(j)})\right| \leq$$

$$|(\Delta x_1^{(i)}-\Delta x_1^{(j)})|+\ldots\ldots+|(\Delta x_k^{(i)}-\Delta x_k^{(j)})| \leq k\frac{\varepsilon}{k}$$

.Since

$|(\Delta x_x^{(i)}-\Delta x_x^{(j)})| \leq \left|\Delta x_1^{(i)}-\Delta x_1^{(j)}\right| + \ldots\ldots + \left|\Delta x_k^{(i)}-\Delta x_k^{(j)}\right|$

we get

$$|\Delta x_k^{(i)}-\Delta x_k^{(j)}| \leq \varepsilon, \ \forall k \in \mathbb{N}$$

Therefore $(\Delta x_k^{(j)})_{j=1}^{\infty}$ be a Cauchy sequence in $\mathbb{R}$, for fixed k. Using the continuity of M_k $\forall k \in \mathbb{N}$, we can find that

$$\left[\sum_{k=1}^{N}\left(M_k\left(\frac{|\Delta x_k^{(i)}-\lim_{j\to\infty}\Delta x_k^{(j)}|}{\rho}\right)\right)^{P_k}\right]^{\frac{1}{H}} \leq 1$$

$$\Rightarrow \left[\sum_{k=1}^{N}\left(M_k\left(\frac{|\Delta x_k^{(i)}-\Delta x|}{\rho}\right)\right)^{P_k}\right]^{\frac{1}{H}} \leq 1.$$

Taking infimum of such ρ 's we get

$$\inf\left\{\rho^{\frac{P_k}{H}} : \left[\sum_{k=1}^{N}\left[M_k\left(\frac{|\Delta x_k^{(i)}-\Delta x|}{\rho}\right)\right]^{P_k}\right]^{\frac{1}{H}} \leq 1\right\} < \varepsilon$$

$\forall i \geq N$, and $j \to \infty$.

Since $(x^{(i)}) \in l_M(\Delta, p)$ and M_k is continuous $\forall k \in \mathbb{N}$ it follows that $x \in l_M(\Delta, p)$.

Theorem (4): Let $0 < p_k < q_k < \infty$ $\forall k \in \mathbb{N}$, then $l_M(\Delta, p) \subseteq l_M(\Delta, q)$.

Proof: Let $x \in l_M(\Delta, p)$

$\Rightarrow \sum_{k=1}^{\infty} M_k\left[\left(\frac{|\Delta x_k|}{\rho}\right)\right]^{P_k} < \infty$, then

$M_k\left(\frac{|\Delta x_k|}{\rho}\right) \leq 1$ $\forall k \in \mathbb{N}$. For sufficiently large k, since M_k is non-decreasing. Hence we get

$$\sum_{k=1}^{\infty}\left[M_k\left(\frac{|\Delta x_k|}{\rho}\right)\right]^{q_k} \leq \sum_{k=1}^{\infty}\left[M_k\left(\frac{|\Delta x_k|}{\rho}\right)\right]^{P_k} < \infty$$

$$\Rightarrow \sum_{k=1}^{\infty}\left[M_k\left(\frac{|\Delta x_k|}{\rho}\right)\right]^{q_k} < \infty .$$

$$\Rightarrow x \in l_M(\Delta, q) .$$

Theorem(5):

(a) Let $0 < \inf_k p_k \leq p_k \leq 1$ $\forall k \in \mathbb{N}$.

Then $l_M(\Delta, p) \subseteq l_M(\Delta)$

(b) Let $1 \leq p_k \leq \sup_k p_k < \infty$ $\forall k \in \mathbb{N}$. Then

$l_M(\Delta) \subseteq l_M(\Delta, p)$.

Proof:

(a) For $x \in l_M(\Delta, p)$, then

$$\sum_{k=1}^{\infty}\left[M_k\left(\frac{|\Delta x_k|}{\rho}\right)\right]^{p_k} < \infty \Rightarrow M_k\left(\frac{|\Delta x_k|}{\rho}\right) \leq 1$$

For sufficiently large k, since

$0 < \inf\ p_k \leq p_k \leq 1 \forall k \in \mathbb{N}$

$$\Rightarrow \sum_{k=1}^{\infty} M_k\left(\frac{|\Delta x_k|}{\rho}\right) \leq \sum_{k=1}^{\infty}\left(M_k\left(\frac{|\Delta x_k|}{\rho}\right)^{P_k}\right)$$

$$\Rightarrow \sum_{k=1}^{\infty} M_k\left(\frac{|\Delta x_k|}{\rho}\right) < \infty \Rightarrow x \in l_M(\Delta)$$

(b) For $P_k \geq 1\ \forall k \in \mathbb{N}$ and $\sup\ p_k < \infty$ and

$x \in l_M(\Delta)$ we get $\sum_{k=1}^{\infty} M_k\left(\frac{|\Delta x_k|}{\rho}\right) < \infty \Rightarrow$

$M_k\left(\frac{|\Delta x_k|}{\rho}\right) \leq 1$. For sufficiently large k,

since $1 \leq p_k \leq \sup\ p_k < \infty \forall k \in \mathbb{N}$, we get

$$\sum_{k=1}^{\infty}\left(M_k\left(\frac{|\Delta x_k|}{\rho}\right)\right)^{P_k} \leq \sum_{k=1}^{\infty} M_k\left(\frac{|\Delta x_k|}{\rho}\right) < \infty$$

$$\Rightarrow \sum_{k=1}^{\infty}\left(M_k\left(\frac{|\Delta x_k|}{\rho}\right)\right)^{P_k} < \infty. \Rightarrow x \in l_M(\Delta, p).$$

Theorem(6): Let $0 \leq p_k \leq q_k \forall k \in \mathbb{N}$ and $\left(\frac{q_k}{p_k}\right)$ be bounded, then $l_M(\Delta, q) \subset l_M(\Delta, p)$.

Proof: Let $x \in l_M(\Delta, q)$

(i.e.)

$$\sum_{k=1}^{\infty}\left[M_k\left(\frac{|\Delta x_k|}{\rho}\right)\right]^{q_k} < \infty \text{ and}$$

$$t_k = \sum_{k=1}^{\infty}\left[M_k\left(\frac{|\Delta x_k|}{\rho}\right)\right]^{q_k} \text{ and } \lambda_k = \frac{q_k}{p_k}.\text{Since}$$

$p_k \leq q_k \forall k \in \mathbb{N}$

therefore $0 \leq \lambda_k \leq 1 \forall k \in \mathbb{N}$.Take $0 < \lambda < \lambda_k$ $\forall k \in \mathbb{N}$. Define $u_k = t_k (t_k \geq 1)$;

$u_k = 0 (t_k < 1)$ and $v_k = 0 (t_k \geq 1)$,

$u_k = t_k (t_k < 1)$. $t_k = u_k + v_k$. (i.e.)

${t_k}^{\lambda_k} = {u_k}^{\lambda_k} + {v_k}^{\lambda_k}$ .Now it follows that

${u_k}^{\lambda_k} \leq u_k \leq t_k$ and ${v_k}^{\lambda_k} \leq {v_k}^{\lambda}$ (1).

(i.e.) $\sum_{k=1}^{\infty} {t_k}^{\lambda_k} = \sum_{k=1}^{\infty} (u_k + v_k)^{\lambda_k}$

$$\Rightarrow \sum_{k=1}^{\infty} {t_k}^{\lambda_k} \leq \sum_{k=1}^{\infty} {u_k}^{\lambda_k} + \sum_{k=1}^{\infty} v_k^{\lambda_k}.$$

$$\Rightarrow \sum_{k=1}^{\infty} {t_k}^{\lambda_k} \leq \sum_{k=1}^{\infty} t_k + \sum_{k=1}^{\infty} v_k^{\lambda}.$$

By using equation (1), we

$$\text{get} \sum_{k=1}^{\infty}\left[M_k\left(\frac{|\Delta x_k|}{\rho}\right)\right]^{q_k \lambda_k} \leq \sum_{k=1}^{\infty}\left[M_k\left(\frac{|\Delta x_k|}{\rho}\right)\right]^{q_k}$$

$$\Rightarrow \sum_{k=1}^{\infty}\left[M_k\left(\frac{|\Delta x_k|}{\rho}\right)\right]^{p_k} \leq \sum_{k=1}^{\infty}\left[M_k\left(\frac{|\Delta x_k|}{\rho}\right)\right]^{q_k},$$

then $l_M(\Delta, q) \subset l_M(\Delta, p)$.

Theorem(7): Let $1 \leq p_k \leq \sup_k p_k < \infty \forall k \in \mathbb{N}$, then $l_M(\Delta, p)$ where $M = (M_k)$ be a Musielak-modulus function is a linear set over the set of complex numbers.

Proof: is easy so omitted.

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