MODELING AN INTEGRATED MULTI-DISTRIBUTOR SUPPLY CHAIN WITH PRODUCTION MEAN PROBLEM

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Abstract: One of the most important challenges in a Single-Vendor multiple-Distributor Multiple-Buyers supply chain is deciding about production mean amount. Production mean determines yield rate of the vendor that could affect production lot size and specify number of shipments between vendor, distributor and buyers. In this paper, an integrated Single-Vendor multiple-Distributor Multiple-branch problem formulated. In this model vendor allowed to deliver product lots to distributor in an unequal-sized shipments manner. Respectively distributor has been allowed to deliver products to each customer base on its demand. Outgoing items will be inspected and will be reprocessed if couldn't satisfy lower specification limits. This model expected to lower reprocessing cost due to the deviation from the optimum target value**.** Because of nonlinearity that exists on the total cost function due to fraction of conforming items produced and also ratio of yield rate to demand rate, we suggested a step by step numerical solution algorithm to finding the model optimal solution.

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1. Introduction

Eliminating all of the production variations is not possible but quality control procedures are effective in order to decreasing products variation (Montgomery, 2005)[1]. In the literature so many tools used to decrease variation in a production process. Targeting that particularly utilized in container-filling processes is one of them (i.e., cosmetic, drug, and health-care industries) (Duffuaa and Siddiqui, 2003)[2]. Containers, in like this processes, are filled with materials base on lower specification limits (Roan et al., 2000)[3]. However, there is a trade-off between expenses of a very tight process specifications and products reprocessing (Al-Sultan and Pulak, 2000)[4]. Recently, researchers formulated numbers of analytical models that combines targeting with inventory or production problems (Gong et al., 1988[3]; Al-Fawzan and Hariga, 2002[5]; Williams et al., 2000[6]; Hariga and Al-Fawzan, 2005[32]). Supply chain management development force firms to concentrate on managing inventories over entire supply chain (Ben-Daya et al., 2008[9]). In the literature, the single-vendor single-buyer problem as the building block of the wider supply chain models studied but there is not any special direction to multiple-buyer state of the supply chain problem.

In this model vendor allowed to deliver product lots to distributor in an unequal-sized shipments manner.

Respectively distributor has been allowed to deliver products to each customer base on its demand. Outgoing items will be inspected and will be reprocessed if couldn't satisfy lower specification limits. Literature review discussed in the next section. The notation, problem statement and assumptions introduced in Sections 3 and 4, respectively. In Section 5, modeling framework is presented. Vendor-distributor-buyer models are developed in Section 6. In Section 7, solution procedure illustrated and finally section 8 concludes the paper.

2. Literature review

Goyal (1977) [7] presented a lot-for-lot policy with infinite production rate model. Banerjee (1986) [8] developed this model by take finite production rate. An equal-sized shipments policy was developed by Goyal (1988) [10]. Goyal (1995) [11] formulated a different shipment policy model where the shipment size increases geometrically. Hill (1997) [12] modified Goyal (1995) [11] model by considering a growth factor. Goyal and Nebebe (2000) [13] formulated a equal size shipment policy. The optimal solution illustrated by Hill(1999)[14]. Chang et al. (2004)[15] researched on lead and Pan and Yang (2002)[16] developed an integrated model with controllable lead times. Ouyang et al. (2004)[17] take into study stochastic lead time. Hoque and Goyal (2006)[18] developed a heuristic solution procedure. Hill and Omar (2006)[19] developed previous model regarding to holding costs. Zhou and Wang (2007)[20] indicated a fixed factor equal to the ratio of the production rate. Hunter and Kartha (1977)[21], Bisgaard et al. (1984)[22], Golhar (1987)[23], Golhar and Pollock (1988)[24] and Boucher and Jafari (1991)[25]. Other researchers including Gibra (1974)[36], Arcelus and Banerjee (1985)[26], Rahim and Banerjee (1988)[27], Al-Sultan and Al-Fawzan (1997)[28], studied targeting problem. Rahim and Al-Sultan (2000)[29] considered both the production mean and variance of the process. Shao et al. (2000)[30] developed strategies for determining the optimal production mean. Lee and Elsayed (2002)[31] formulated the optimum production mean and screening limits. Gong et al. (1988)[3] integrated inventory and targeting issues. Al-Fawzan and Hariga (2002)[5] developed the previous model when it is time dependent. Williams et al. (2000)[6] analyzed alternatives for a container-filling process. Hariga and Al-Fawzan (2005)[32] formulated integrated optimal production cycle time and target value model. Darwish (2004 a,b)[33],[34] formulated an integrated single-vendor single-buyer problem. Lee et al. (2007)[35] take into study the targeting problem for a multiple products. Recently, Darwish (2009)[37] extended its previous preliminary model and proposed two integrated and hierarchical model for targeting problem.

3. Notation

The following notation has been used in developing the proposed model:

D demand rate

r production rate

$\hat{W}$ distributors warehouse rate

X a random variable represents the amount of raw material an item receives

L lower specification limit

p fraction of conforming items produced

λ yield rate of process ($\lambda = rp$)

ρ ratio of yield rate to demand rate ($\rho = k/D$)

Ň number of corporate branches

Ɖ number of corporate distributors

A_v vendor setup cost

A_d distributor ordering cost

A_b branches ordering cost

A_r ordering cost of raw material

h_v holding cost for the vendor per item per unit time

h_d holding cost for the distributor per item per unit time

h_b holding cost for the buyer per item per unit time

h_r holding cost for the raw material per unit per unit time

R per-item reprocessing cost

b fixed production cost $(b > 0)$

α value-added factor (α ≥ 1)

C unit material cost

q_i size of ith shipment

Q production lot size

T inventory cycle length $(T = \frac{Q}{D})$

N number of cycles $(N = \frac{1}{T})$

n number of shipments

T_p production cycle length $(T_p = Q/\lambda)$

TC_1 average total cost per unit time when $D = \hat{W}$

TC_2 average total cost per unit time when $D > \hat{W}$

TC_3 average total cost per unit time when $D < \hat{W}$

ε demand rate probability when $P(D = \hat{W})$

β demand rate probability when $P(D < \hat{W})$

γ demand rate probability when $P(D > \hat{W})$

Φ back order cost per item per unit time

4. Problem statement and assumptions

In this model vendor orders raw materials from supplier and uses in lots of Q. X represents the amount of raw material that an item receives. L represents lower specification limit. X considers to be better whenever it is greater. Each item categorized as standard if X is greater or equal to L. Otherwise it requires reprocessing operation. Performance variable X according what that stated by M.A. Darwish(2009)[37] assumed to have a normal distribution function with mean μ and constant variance σ^2. Φ represents standard normal distribution density function of conforming items.

$$p = P[X \geq L] = P\left[Z \geq \frac{L-\mu}{\sigma}\right] = \int_{\frac{L-\mu}{\sigma}}^{\infty} \Phi(z)dz$$

Production rate is r and production yield rate is $\lambda = rp$. Raw materials inventory is increased by the sum of recovered raw material $r(1-p)\,T_p$ and the amount of raw material that vendor receives from supplier $r(\mu-(1-p))\,T_p$. The shipment policy that developed by Goyal(1995) has been used in this model. Base on this policy, n unequal-sized shipments delivers to distributor. Size of ith shipment (q_i) is equal to

$$q_i = q_{i-1}\left(\frac{\lambda}{D}\right), i = 2,3, \ldots, n.$$

$$q_i = q_1\left(\frac{\lambda}{D}\right)^{i-1}, i = 2,3, \ldots, n.$$

$$Q = \sum_{i=1}^{n} q_i = q_1 \frac{\rho^n - 1}{\rho - 1}$$

The inventory cycle length is : $T=\frac{q_1}{D}\frac{\rho^n-1}{\rho-1}$ and $\rho = \lambda/D$

5. Modeling Framework

The costs in this model include vendor, distributor and inventory control of raw material, production and also customer loss costs. These costs described as bellows:

5.1. Corporate branches costs

Branches incur ordering and holding costs. Shipment takes time Q_i/D per time and in this time branche's average inventory is $Q_i/2$.

Average inventory per cycle = $\sum_{i=1}^{n} Q_i^{\,2}/2D$

Average cost of branch inventory per cycle = $HC_b = h_b \sum_{i=1}^{n} \frac{q_i^2}{2D} = \frac{h_b q_1^{\,2}}{2D}\frac{\rho^{2n}-1}{\rho^2-1}$

Average total cost of the branches per unit time = $\check{N}\left(\frac{nA_b+HC_b}{T}\right) = \check{N}\left(\frac{nA_bD}{q_1}\frac{\rho-1}{\rho^n-1} + \frac{h_b q_1}{2}\frac{\rho^n+1}{\rho+1}\right)$

5.2. Vendor's costs

Vendor also has setup and holding costs. For each production run vendor has setup cost A_v. As stated by Darwish(2009)[37] vendor costs is as belows:

Average vendor inventory cost per cycle = $HC_v = h_v \sum_{i=1}^{n} \frac{q_i^2}{2\lambda} = \frac{h_v q_1^{\,2}}{2\lambda}\frac{\rho^{2n}-1}{\rho^2-1}$

Average vendor total cost per unit time = C_v = $\left(\frac{A_vD}{q_1}\frac{\rho-1}{\rho^n-1} + \frac{h_v q_1}{2\rho}\frac{\rho^n+1}{\rho+1}\right)$

5.3. Inventory control costs related to raw materials

Inventory control costs related to raw materials includes ordering cost and holding cost also. Like what that stated by M.A. Darwish(2009) [37]

Average total cost of inventory control of raw materials = $C_r = \frac{D}{q_1}\frac{\rho-1}{\rho^n-1}\left(A_r + \frac{h_r\mu}{2r p^2}Q^2\right)$

5.4. Direct production costs

We assumed that direct production cost is a linear function and exactly like what that stated by M.A. Darwish [37] it is as belows:

Per cycle cost of producing Q items = $r\int_0^{T_P} g(\mu)dt$

Reprocessing cost of per item = R

Reprocessing cost of items in per cycle = $R(1-p).r.T_p$

Amount of raw materials in vendors warehouse at $= 0$: $I_r(0)$

Amount of raw materials that vendor receives from supplier in one cycle = $I_r(0) - r(1-p)\,T_p$

Raw material acquisition cost per cycle = $C = C(\,I_r(0) - r(1-p)\,T_p\,)$ = $\frac{CQ}{p}(\mu - (1-p))$

Total production cost per unit time = $C_r = \frac{D}{q_1}\frac{\rho-1}{\rho^n-1}\left(\frac{(b+c((\alpha+1)\mu-(1-p)))}{p}Q + \frac{1-p}{p}RQ\right)$

5.5. Distributor costs

The distributors cost consists of inventory holding cost, ordering cost, customer loss costs. Distributor output rate (Ŵ) assumed to be a linear function of production rate and directly depends to vendor's output. Customer demand has three statuses as below:

1. *P(D = Ŵ)* = ε
2. *P(D < Ŵ)* = β
3. *P(D > Ŵ)* = γ

While: $0 \le$ ε, β, γ$\le$ 1 and ε + β + γ = 1. In each of these three different customer demand statuses, distributors have different type of costs that it takes into study next.

5.5.1. *Average total cost of the distributors per unit time* = $D\left(\frac{nA_d}{T}\right)$, *P(D = Ŵ)* = ε

5.5.2. *Average total cost of the distributors per unit time* = $D\left(\frac{nA_d}{T}\right) + \Phi\left(\frac{D}{N} - \left(\frac{q_i^2}{2D}\right)\right)$, *P(D < Ŵ)* = β

5.5.3. *Average demand from distributor in each cycle* = $\frac{D}{N}$

Average distributor inventory per cycle = $\sum_{i=1}^{n}\frac{q_i^2}{2D}$

Average cost of distributor inventory per cycle = $HC_d = h_d\left(\frac{D}{N} - \sum_{i=1}^{n}\frac{q_i^2}{2D}\right) = h_d\frac{D}{N} - h_d\frac{q_1^{\,2}}{2D}\frac{\rho^{2n}-1}{\rho^2-1}$

Distributor ordering cost per cycle = n. A_d

Average total cost of the distributors per unit time = $Ð\left(\frac{n.A_d + \left(h_d\frac{D}{N} - h_d\frac{q_1^2}{2D}\frac{\rho^{2n}-1}{\rho^2-1}\right)}{T}\right)$, $P(D > \hat{W}) = \gamma$

6. Total Cost Function

According what that stated before on section 5, state based customers demand lead to a state based total cost function that each cost function could be formulated as below.

$$TC_1 = Ň\left(\frac{nA_bD}{q_1}\frac{\rho-1}{\rho^n-1} + \frac{h_bq_1}{2}\frac{\rho^n+1}{\rho+1}\right) + \left(\frac{A_vD}{q_1}\frac{\rho-1}{\rho^n-1} + \frac{h_vq_1}{2\rho}\frac{\rho^n+1}{\rho+1}\right) + \frac{D}{q_1}\frac{\rho-1}{\rho^n-1}\left(A_r + \frac{h_r\mu}{2rp^2}Q^2\right) + \frac{D}{q_1}\frac{\rho-1}{\rho^n-1}\left(\frac{b+c((\alpha+1)\mu-(1-p))}{p}Q + \frac{1-p}{p}RQ\right) + Ð\left(\frac{nA_d}{T}\right)$$

$$TC_2 = Ň\left(\frac{nA_bD}{q_1}\frac{\rho-1}{\rho^n-1} + \frac{h_bq_1}{2}\frac{\rho^n+1}{\rho+1}\right) + \left(\frac{A_vD}{q_1}\frac{\rho-1}{\rho^n-1} + \frac{h_vq_1}{2\rho}\frac{\rho^n+1}{\rho+1}\right) + \frac{D}{q_1}\frac{\rho-1}{\rho^n-1}\left(A_r + \frac{h_r\mu}{2rp^2}Q^2\right) + \frac{D}{q_1}\frac{\rho-1}{\rho^n-1}\left(\frac{b+c((\alpha+1)\mu-(1-p))}{p}Q + \frac{1-p}{p}RQ\right) + Ð\left(\frac{nA_d}{T}\right) + \Phi\left(\frac{D}{N} - \left(\frac{q_1^2}{2D}\right)\right)$$

$$TC_3 = Ň\left(\frac{nA_bD}{q_1}\frac{\rho-1}{\rho^n-1} + \frac{h_bq_1}{2}\frac{\rho^n+1}{\rho+1}\right) + \left(\frac{A_vD}{q_1}\frac{\rho-1}{\rho^n-1} + \frac{h_vq_1}{2\rho}\frac{\rho^n+1}{\rho+1}\right) + \frac{D}{q_1}\frac{\rho-1}{\rho^n-1}\left(A_r + \frac{h_r\mu}{2rp^2}Q^2\right) + \frac{D}{q_1}\frac{\rho-1}{\rho^n-1}\left(\frac{b+c((\alpha+1)\mu-(1-p))}{p}Q + \frac{1-p}{p}RQ\right) + Ð\left(\frac{n.A_d + \left(h_d\frac{D}{N} - h_d\frac{q_1^2}{2D}\frac{\rho^{2n}-1}{\rho^2-1}\right)}{T}\right)$$

7. Solution algorithm

The decision variables in this model are the productions mean (μ), number of vendor's shipments (n) and size of first shipments. We assumed that shipments are equal. The probability function for ε, β and γ are the same and in this model we could take into study the optimistic or pessimistic customers demand rate and its effect on the entire supply chain costs by sensitivity analysis process. Note that p and ρ that used in the section 6 are not constants also but functions of the production mean (μ). The objective of this model is to minimize Z in which

$Z = \varepsilon.TC_1 + \beta.TC_2 + \gamma TC_3$ While $\varepsilon + \beta + \gamma = 1$ and $0 \leq$ ε, β, γ$\leq$ 1. Z is a linear function of TC_1, TC_2 and TC_3 and therefore if we could prove that TC functions are convex, therefore the convexity of Z will be concluded. We can easily prove that TC functions that has been used in this model are convex functions in q_1 because $\frac{\partial^2 TC_1}{\partial q_1^2}$, $\frac{\partial^2 TC_2}{\partial q_1^2}$ and $\frac{\partial^2 TC_3}{\partial q_1^2} > 0$. Therefore for given values of $\varepsilon, \beta, \gamma$, n and μ, the value of q_1 which minimizes Z could be obtained by differentiating Z function with respect to q_1 and setting its results to 0. Then the average total cost per unit time of the integrated scenario based model (Z^*) would be obtained by setting amount of q_1 in the Z function. Because of nonlinearity of Z that exists due to p and ρ, the optimal solution could be found numerically. Steps that listed below could be use to finding an optimal solution.

Step 1: Set $n = 0$ and $Z_{min} \to \infty$

Step 2: Increase n by 1

Step 3: Determine the value of μ which minimizes Z^*

Step 4: If $Z^* < Z_{min}$, set $Z_{min} = Z^*$, $n^* = n$ and $\mu^* = \mu$. Go to step 2.

Step 5: Stop.

8. Conclusion

Multiple branch problem and targeting problem has been integrated in this paper. Optimal process mean, production lot size and number of deliveries regarded in this model. We assumed that demand rate of the customers are base on a status based manner. Also we assumed that distributors are isolated from each of other distributors. Because of nonlinearity that exists on the total cost function due to fraction of conforming items produced and also ratio of yield rate to demand rate, we suggested a step by step numerical solution algorithm to finding the model optimal solution. Relationship between distributors could be considered as a future study on this paper.

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