Flexible Manufacturing System Selection Using of Logarithmic Fuzzy Preference Programming and ELECTRE Methods

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Abstract: Selection of a flexible manufacturing system (FMS) is a challenging task because of the insufficient experience and data about this still-evolving technology. Further, the large investment involved makes the selection process critical. The purpose of this paper is applying a new integrated method to flexible manufacturing system selection. Proposed approach is based on Logarithmic fuzzy preference programming and ELECTRE (Elimination Et Choix Traduisant la REalite) methods. LFPP method is used in determining the weights of the criteria by decision makers and then rankings of flexible manufacturing systems are determined by ELECTRE method. A numerical example demonstrates the application of the proposed method.
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1. INTRODUCTION

Flexible manufacturing systems (FMS) have extensively been studied over the past fifteen years. Selection of FMS is a challenging task because of the insufficient experience and data about this still-evolving technology. Further, the large investment involved makes the selection process critical.An FMS is an integrated manufacturing system that consists of one or several work stations linked by a computerized inventory system, making it possible for jobs to follow diverse routes through the production system. An advantage of FMS is that it can simultaneously meet several goals: small batch sizes, high quality standards and efficiency of the production process. Boththe industrial and the academic community (Kuula, 1993., Buzacott et al, 1986., Jaikumar, 1986., Ranta et al, 1988) have been interested in the design of flexible manufacturing systems. The rest of the paper is organized as follows: The following section presents a concise treatment of the basic concepts of fuzzy set theory. Section 3 presents the methodology of Logarithmic fuzzy preference programming and ELECTRE. The application of the proposed framework to FMS selection is addressed in Section 4. Finally, conclusions are provided in Section 5.

2. FUZZY SET THEORY

Fuzzy set theory was first developed in 1965 by Zadeh; he was attempting to solve fuzzy phenomenon problems, including problems with uncertain, incomplete, unspecific, or fuzzy situations. Fuzzy set theory is more advantageous than traditional set theory when describing set concepts in human language. It allows us to address unspecific and fuzzy characteristics by using a membership function that partitions a fuzzy set into subsets of members that ‘‘incompletely belong to” or ‘‘incompletely do not belong to” a given subset.

2.1. FUZZY NUMBERS

We order the Universe of Discourse such that U is a collection of targets, where each target in the Universe of Discourse is called an element. Fuzzy number $\widetilde{A}$ is mapped onto U such that a random $x \rightarrow U$ is appointed a real number, $\mu_{\widetilde{A}}(x) \rightarrow [0,1]$. If another element in U is greater than x, we call that element under A.
The universe of real numbers R is a triangular fuzzy number (TFN)$\widetilde{A}$, which means that for $x \in R, \mu_{\widetilde{A}}(x) \in [0,1]$, and

$$\mu_{\widetilde{A}}(x) = \begin{cases} (x-L)/(M-L), & L \le x \le M, \\ (U-x)/(U-M), & M \le x \le U, \\ 0, \quad \text{otherwise}, \end{cases}$$

Note that $\widetilde{A} = (L, M, U)$, where L and U represent fuzzy probability between the lower and upper boundaries, respectively, as in Fig. 1. Assume two fuzzy numbers $\widetilde{A}_1 = (L_1, M_1, U_1)$, and $\widetilde{A}_2 = (L_2, M_2, U_2)$; then,

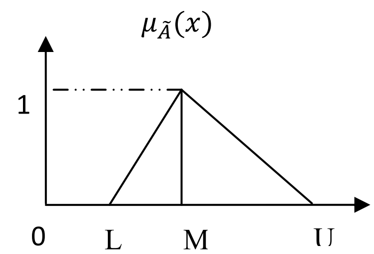

Fig. 1: Triangular fuzzy number

(1) $\tilde{A}_1 \oplus \tilde{A}_2 = (L_1, M_1, U_1) \oplus (L_2, M_2, U_2) = (L_1 + L_2, M_1 + M_2, U_1 + U_2)$
(2) $\tilde{A}_1 \otimes \tilde{A}_2 = (L_1, M_1, U_1) \otimes (L_2, M_2, U_2) = (L_1 L_2, M_1 M_2, U_1 U_2), L_i > 0, M_i > 0, U_i > 0$
(3) $\tilde{A}_1 - \tilde{A}_2 = (L_1, M_1, U_1) - (L_2, M_2, U_2) = (L_1 - L_2, M_1 - M_2, U_1 - U_2)$
(4) $\tilde{A}_1 \div \tilde{A}_2 = (L_1, M_1, U_1) \div (L_2, M_2, U_2) = \left(\frac{L_1}{L_2}, \frac{M_1}{M_2}, \frac{U_1}{U_2}\right), L_i > 0, M_i > 0, U_i > 0$
(5) $\tilde{A}_1^{-1} = (L_1, M_1, U_1)^{-1} = \left(\frac{1}{U_1}, \frac{1}{M_1}, \frac{1}{L_1}\right), L_i > 0, M_i > 0, U_i > 0$

2.2. FUZZY LINGUISTIC VARIABLES

The fuzzy linguistic variable is a variable that reflects different aspects of human language. Its value represents the range from natural to artificial language. When the values or meanings of a linguistic factor are being reflected, the resulting variable must also reflect appropriate modes of change for that linguistic factor. Moreover, variables describing a human word or sentence can be divided into numerous linguistic criteria, such as equally important, moderately important, strongly important, very strongly important, and extremely important. For the purposes of the present study, the 5-point scale (equally important, moderately important, strongly important, very strongly important and extremely important) is used.

3. RESEARCH METHODOLOGY

In this paper, the weights of each criterion are calculated using LFPP. After that, ELECTRE is utilized to rank the alternatives. Finally, we select the best FMS based on these results.

3.1. The LFPP-based nonlinear priority method

In this method for the fuzzy pairwise comparison matrix, Wang et al (2011) took its logarithm by the following approximate equation:

$\ln \tilde{a} = (\ln l_{ij}, \ln m_{ij}, \ln lu_{ij})$,
i,j = 1….,n (6)

That is, the logarithm of a triangular fuzzy judgment a_{ij} can still be seen as an approximate triangular fuzzy number, whose membership function can accordingly be defined as

$$\mu_{ij}\left(\ln\left(\frac{w_i}{w_j}\right)\right) = \begin{cases} \dfrac{\ln\left(\frac{w_i}{w_j}\right) - \ln l_{ij}}{\ln m_{ij} - \ln l_{ij}}, & \ln\left(\frac{w_i}{w_j}\right) \le \ln m_{ij}, \\ \dfrac{\ln u_{ij} - \ln\left(\frac{w_i}{w_j}\right)}{\ln u_{ij} - \ln m_{ij}}, & \ln\left(\frac{w_i}{w_j}\right) \ge \ln m_{ij}, \end{cases}$$

(7)

Where $\mu_{ij}\left(\ln\left(\frac{w_i}{w_j}\right)\right)$ is the membership degree of $\ln\left(\frac{w_i}{w_j}\right)$ belonging to the approximate triangular fuzzy judgment $\ln \tilde{a} = (\ln l_{ij}, \ln m_{ij}, \ln lu_{ij})$. It is very natural that we hope to find a crisp priority vector to maximize the minimum membership degree λ= min $\{\mu_{ij}\left(\ln\left(\frac{w_i}{w_j}\right)\right) \mid i=1,\dots,n-1; j=i+1,\dots, n\}$. The resultant model can be constructed (Wang et al, 2011) as

Maximize λ
Subject to

$$\begin{Bmatrix} \mu_{ij} \left(\ln\left(\frac{w_i}{w_j}\right)\right) \geq \lambda, i = 1, \dots, n-1; j = i+1, \dots, n, \\ w_i \geq 0, i = 1, \dots, n, \end{Bmatrix} \quad (8)$$

Or as

Maximize 1- λ

$$\text{Subject to} \begin{Bmatrix} \ln w_i - \ln w_j - \lambda \ln\left(\frac{m_{ij}}{l_{ij}}\right) \geq \ln l_{ij}, i = 1, \dots, n-1; j = i+1, \dots, n, \\ -\ln w_i + \ln w_j - \lambda \ln\left(\frac{u_{ij}}{m_{ij}}\right) \geq -\ln u_{ij}, i = 1, \dots, n; j = i+1, \dots, n, \end{Bmatrix} \quad (9)$$

It is seen that the normalization constraint $\sum_{i=1}^{n} w_i = 1$ is not included in the above two equivalent models. This is because the models will become computationally complicated if the normalization constraint is included. Before normalization, without loss of generality, we can assume $w_i \geq 1$ for all $i = 1, \dots, n$ such that $\ln w_i \geq 0$ for $i = 1, \dots, n$. Note that the nonnegative assumption for $\ln w_i \geq 0$ (i = 1,. . . ,n) is not essential. The reason for producing a negative value for λ is that there are no weights that can meet all the fuzzy judgments in $\tilde{A}$ within their support intervals. That is to say, not all the inequalities $\ln w_i - \ln w_j - \lambda \ln\left(\frac{m_{ij}}{l_{ij}}\right) \geq \ln l_{ij}$ or $-\ln w_i + \ln w_j - \lambda \ln\left(\frac{u_{ij}}{m_{ij}}\right) \geq -\ln u_{ij}$ can hold at the same time. To avoid k from taking a negative value, Wang et al (2011) introduced nonnegative deviation variables δ_{ij} and η_{ij} for $i = 1, \dots, n-1; j = i+1, \dots, n,$ such that they meet the following inequalities:

$$\ln w_i - \ln w_j - \lambda \ln\left(\frac{m_{ij}}{l_{ij}}\right) \geq \ln l_{ij}, i = 1, \dots, n-1; j = i+1, \dots, n$$

$$-\ln w_i + \ln w_j - \lambda \ln\left(\frac{u_{ij}}{m_{ij}}\right) \geq -\ln u_{ij}, i = 1, \dots, n; j = i+1, \dots, n \quad (10)$$

It is the most desirable that the values of the deviation variables are the smaller the better. Wang et al (2011) thus proposed the following LFPP-based nonlinear priority model for fuzzy AHP weight derivation:

$$\text{Minimize} \quad J = (1-\lambda)^2 + M.\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}(\delta_{ij}^2 + \eta_{ij}^2)$$

$$\text{Subject to} \begin{Bmatrix} x_i - x_j - \lambda \ln\left(\frac{m_{ij}}{l_{ij}}\right) + \delta_{ij} \geq \ln l_{ij}, i = 1, \dots, n-1; j = i+1, \dots, n, \\ -x_i + x_j - \lambda \ln\left(\frac{u_{ij}}{m_{ij}}\right) + \eta_{ij} \geq -\ln u_{ij}, i = 1, \dots, n; j = i+1, \dots, n, \\ \lambda, x_i \geq 0, i = 1, \dots, n \\ \delta_{ij}, \eta_{ij} \geq 0, i = 1, \dots, n-1; j = i+1, \dots, n \end{Bmatrix} \quad (11)$$

Where $x_i = \ln w_i$ for i = 1,. . . ,n and M is a specified sufficiently large constant such as $M = 10^3$. The main purpose of introducing a big constant M into the above model is to find the weights within the support intervals of fuzzy judgments without violations or with as little violations as possible.

3.2. The ELECTRE Method

The ELECTRE (Elimination Et Choix Traduisant la REalite´) method originated fromRoy in the late 1960s. The ELECTRE method is based on the study of outranking relations and uses concordance and discordance indexes to analyze the outranking relations among the alternatives. Concordance and discordance indexes can be viewed asmeasurements of satisfaction and dissatisfaction that a decision-maker chooses one alternative over the other.

Suppose a MCDM problem has *m* alternatives (A_1, A_2, . . . ,A_m), and *n* decision criteria/attributes (C_1,C_2, . . . ,C_n). Each alternative is evaluated with respect to then criteria/attributes. All the values/ratings assigned to the alternatives with respect toeach criterion form a decision matrix denoted by $X=(x_{ij})_{m\times n}$. Let W= (w_1,

w_2, . . . ,w_n) be the relative weight vector about the criteria, satisfying $\sum_{j=1}^{n} w_j = 1$. Then the ELECTRE method can be summarized as follows (Yoon and Hwang 1995).

1. Normalize the decision matrix X= $(x_{ij})_{m\times n}$ by calculating r_{ij}, which represents the normalized criteria/attribute value/rating,

$r_{ij}= \frac{\frac{1}{x_{ij}}}{\sqrt{\sum_{i=1}^{m}\frac{1}{x_{ij}^2}}}$ for the minimization objective, where i = 1, 2, . . . ,m and j = 1, 2, . . . , n, (12)

$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}}$ for the minimization objective, where i = 1, 2, . . . ,m and j = 1, 2, . . . , n, (13)

2. Calculate the weighted normalized decision matrix V= $(v_{ij})_{m\times n}$
$v_{ij}= r_{ij}\times w_j$, where i = 1, 2, . . . ,m and j = 1, 2, . . . , n,

where w_j is the relative weight of the *j*th criterion or attribute, and $\sum_{j=1}^{n} w_j = 1$.

3. Determine the concordance and discordance sets. For each pair of alternatives A_p and A_q(p, q=1, 2, . . . ,m and p≠q) the set of criteria is divided into two distinct subsets. If alternative A_p is preferred to alternative A_q for all criteria, the concordance set is composed. This can be written as

$$C (p,q) = \{j \mid V_{pj}>V_{qj}\} \quad (14)$$

Where V_{pj} is the weighted normalized rating of alternative A_p with respect to the *j*th criterion. In other words, C (p, q) is the collection of attributes where A_p is better than or equal to A_q. The complement of C (p, q), the discordance set, contains all criteria for which A_p is worse than Aq. This can be written as

$$D (p,q) = \{j \mid V_{pj}<V_{qj}\} \quad (15)$$

4. Calculate the concordance and discordance indexes. The concordance index of C (p, q) is defined as

$$C_{pq} = \sum_{j^*} w_{j^*} \quad (16)$$

where j* are attributes contained in the concordance set C(p, q). The discordance index D (p, q) represents the degree of disagreement in ($A_p \rightarrow A_q$) and can be defined as

$$D_{pq} = \frac{\sum_{j^+} |V_{pj^+} - V_{qj^+}|}{\sum_j |V_{pj} - V_{qj}|} \quad (17)$$

where j^+ are attributes contained in the discordance set D(p, q) and v_{ij} is the weighted normalized evaluation of alternatives i on criterion j. Outranking relationship. The method defines that A_p outranks A_q when $C_{pq} \geq C$ and $D_{pq} \leq D$, where C and D are the averages of C_{pq} and D_{pq}, respectively.

4. A NUMERICAL APPLICATION OF PROPOSED APPROACH

The criteria for this example are taken from Shamsuzzaman et al (2003). These criteria are including: Flexibility (C_1), Cost (C_2), Risk (C_3), Production rate (C_4), System utilization (C_5) and Throughput time (C_6).In addition, there are six alternatives include A_1, A_2, A_3, A_4, A_5 and A_6.In this paper, the weights of criteria are calculated by using LFPP, and these calculated weight values are used as ELECTRE inputs. Then, after ELECTRE calculations, evaluation of the alternatives and selection of Flexible Manufacturing System is realized.

Logarithmic Fuzzy Preference Programming:

In LFPP, firstly, we should determine the weights of each criterion by utilizing pair-wise comparison matrices. We compare each criterion with respect to other criteria. You can see the pair-wise comparison matrix for Flexible Manufacturing System criteria in Table 1.

Table 1. Inter-criteria comparison matrix

	C_1	C_2	C_3	C_4	C_5	C_6
C_1	(1,1,1)	(3,4,5)	(1,2,3)	(2,3,4)	(3,4,5)	(2,3,4)
C_2	(1/5,1/4,1/3)	(1,1,1)	(1/4,1/3,1/2)	(1/3,1/2,1)	(1,2,3)	(3,4,5)
C_3	(1/3,1/2,1)	(2,3,4)	(1,1,1)	(1,2,3)	(2,3,4)	(1/3,1/2,1)
C_4	(1/4,1/3,1/2)	(1,2,3)	(1/3,1/2,1)	(1,1,1)	(1/2,3/2,5/2)	(2,3,4)
C_5	(1/5,1/4,1/3)	(1/3,1/2,1)	(1/4,1/3,1/2)	(2/5,2/3,2)	(1,1,1)	(1,2,3)
C_6	(1/4,1/3,1/2)	(1/5,1/4,1/3)	(1,2,3)	(1/4,1/3,1/2)	(1/3,1/2,1)	(1,1,1)

After forming the model (11) for the comparison matrix and solving this model using of Genetic algorithms, the weight vector is obtained as follow:

w^t= (0.301755, 0.206396, 0.188336, 0.135751, 0.08891, 0.078852)^T

ELECTRE:

The weights of the criteria are calculated by LFPP up to now, and then these values can be used in ELECTRE. So, the ELECTRE methodology must be started at the second step. Thus, weighted normalized decision matrix can be prepared. This matrix can be seen from Table 2.

Table 2. The weighted normalized decision matrix

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	0.05	0.04	0.02	0.01	0.01	0.01
A_2	0.05	0.03	0.02	0.02	0.02	0.01
A_3	0.06	0.03	0.04	0.03	0.02	0.02
A_4	0.06	0.04	0.02	0.02	0.01	0.01
A_5	0.06	0.03	0.03	0.03	0.02	0.01
A_6	0.04	0.05	0.05	0.03	0.01	0.01

In the next step, according to ELECTRE methodology, we obtain the concordance and discordance indexes that are show in Table 3 and Table 4.

Table 3. Concordance indexes (C_{pq})

	A_1	A_2	A_3	A_4	A_5	A_6
A_1		0.206396	0.206396	0	0.206396	0.390665
A_2	0.793604		0	0.167762	0.374158	0.469516
A_3	0.793604	1		0.491849	0.864249	0.605268
A_4	1.00	0.508151	0.508151		0.508151	0.390665
A_5	0.793604	0.625842	0	0.491849		0.605268
A_6	0.609335	0.530484	0.394732	0.609335	0.394732	

Table 4. Discordance indexes (D_{pq})

	A_1	A_2	A_3	A_4	A_5	A_6
A_1		1	1	1	1	1
A_2	0.996292		1	0.987174	1	1
A_3	0.529104	0		0.427511	0	0.914291
A_4	0	0.277586	1		0.788971	1
A_5	0.659379	0.315037	1	1		1
A_6	0.30367	0.472277	1	0.715037	0.841566	

After that we obtain the $C_{pq} \geq \bar{C}$ & $D_{pq} \leq \bar{D}$ that show in Table 5.

Table 5. $C_{pq} \geq \bar{C}$ & $D_{pq} \leq \bar{D}$

	A_1	A_2	A_3	A_4	A_5	A_6
A_1		0	0	0	0	0
A_2	0		0	0	0	0
A_3	1	1		1	0	0
A_4	1	1	0		0	0
A_5	1	1	0	0		0
A_6	1	1	0	1	0	

The ELECTRE results are shown in Table 6 as follow:

Table 6. The result of ELECTRE method

Alternative	Ranking
A_3 & A_6	1
A_2	2
A_4	3
A_1 & A_2	4

According to result, if the best one is needed to be selected, then the alternative A_3 or A_6 must be chosen.

5. CONCLUSIONS

Selection of a flexible manufacturing system (FMS) is a challenging task because of the insufficient experience and data about this still-evolving technology. Further, the large investment involved makes the selection process critical. This paper illustrates an application of LFPP along with ELECTRE in selecting FMS. Fuzzy set theory is incorporated to overcome the vagueness in the preferences. Two steps LFPP and ELECTRE methodology is structured here that LFPP uses ELECTRE result weights as input weights. According to this methodology, A_6 and A_3 are selected as the best FMS.

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