New Approach for the Synthesis of Planar 4-Bar Mechanisms for 2 Coupler-Positions Generation

Galal A. Hassaan[1] , Mohammed A. Al-Gamil[1] and Maha M. Lashin[2]

[1]Mechanical Design & Production Department, Faculty of Engineering, Cairo University
[2]Mechanical Engineering Department, Shoubra Faculty of Engineering, Banha University
maha_lashin@yahoo.com

Abstract: Analytical synthesis of mechanisms is a useful tool towards computer-aided machinery design and production. 4-bar planar mechanisms have wide applications in industry and thus receive more attention from machinery design researchers. The proposed approach relies on forming a mathematical model for the mechanism position incorporating the 2 coupler positions. The model consists of 6 nonlinear equations considering the transmission angle of the mechanism in the 2 coupler positions. A case study is presented as a justification for the proposed approach. Exact coupler positions are attained with transmission angles not more than 7 % of the optimum value of 90 degrees.

[Galal A. Hassaan, Mohammed A. Al-Gamil and Maha M. Lashin. **New Approach for the Synthesis of Planar 4-Bar Mechanisms for 2 Coupler-Positions Generation.** *N Y Sci J* 2012;5(10):86-90]. (ISSN: 1554-0200). http://www.sciencepub.net/newyork. 14

Key words: Planar, Coupler-Positions

1. Introduction

Mechanisms represent the skeleton of machinery. Successful synthesis of mechanisms leads to a successful machine design. On the other hand classical mechanism synthesis techniques lead to mechanisms satisfying some kinematic requirements such as stroke, time ratio, specific link positions, specific function generation etc.

Mechanism synthesis techniques ranges from simple graphical techniques going through analytical approaches with many assumptions and trials to sophisticated techniques using optimization application.

The subject of mechanism synthesis has occupied the attention of researchers over decades. Only some publications are reviewed over the last decade to highlight some of the efforts focused on mechanism synthesis. Simionescu and Beale (2002) studied the optimal synthesis of 5-bar suspension system used in automotives. Figliolini and Angeles (2002) proposed an algorithm for the synthesis of conjugate Geneva mechanism with curved slots. Lebedov (2003) developed a vector method for the analysis of guidance and transmission mechanisms applied to 4-bar mechanisms. Balli and Chand (2003) studied the synthesis of a planar 7-bar mechanism with variable topology using complex number dyadic approach for motion, path and function generation.

Laribi et al (2004) used a combined genetic algorithm-fuzzy logic method to solve the path generation problem. Sen and Joshi (2004) presented a methodology for the synthesis of link geometry for interference free planar all revolute joints mechanisms. Russel and Sodhi (2005) presented a method for the designing of slider-crank mechanisms to achieve multi-phase motion generation applications by adjustable planar 4-bar motion generators. Bustos and others (2005) proposed the use of genetic algorithms with a finite-element-based error function for the synthesis of 1DOF mechanisms. Wu and Chen (2005) used an adjustable link to synthesize exactly any input-output relationship using a planar 4-bar mechanism.

Kyung and Sacks (2006) presented a parameter synthesis algorithm for planar higher pair mechanical systems. Sunkari and Schmidt (2006) used a synthesis algorithm for a planar mechanism based on McKay-type algorithm. Varbanov et al 2006) studied the application of an expert system in the design of planar mechanisms. Hongying et al (2007) presented a computerized method using coupler-angle function curve to approximately synthesize a 4-bar path mechanism. Ding and Huang (2007) proposed 2 basic loop operations for the topological structure analysis of kinematic chains with some applications. Gregorio (2007) studied the singularity analysis of 1DOF planar mechanisms by giving geometric conditions for any type of singularity.

Hwang and Fan (2008) used polynomial equations for the acceleration pole in the synthesis of slider-crank mechanisms. Chen and Angeles (2008) introduced a family of linkages for motion generation and presented a synthesis method to one linkage visiting up to 11 poses exactly. Shen et al (2008) investigated the synthesis of a 4-bar mechanism with rolling contacts for motion and function generation. Litvin et al (2009) proposed a new approach for the generation of functions based on the application of multi-gear drive. Pennock and Israr (2009) investigated the kinematics of an adjustable 6-bar linkage using a novel technique. Du et al (2009) developed methods for robust assessment and robust mechanism synthesis when random and internal variables are involved.

Wei and Dai (2010) investigated the geometric and kinematic analysis of a 7-bar mechanism. Parlaktas et al (2010) presented a novel method for the analysis and design of a certain type of geared 4-bar mechanism with collinear input and output shafts. Huang and Zhang (2010) presented a method for robust tolerance design of function generation mechanisms with joint clearance. Nie et al (2011) proposed a method to the kinematic configuration analysis of kinematic chains with R-pairs. Soong and Chang (2011) proposed a technique for the exact function generation problems of 4-bar linkages using variable length driving links. Tanik (2011) studied the transmission angle of compliant slider-crank mechanisms via two theorems.

Ding et al. (2012) studied the analysis of planar 1DOF chains and created their atlas database. Kim and Yoo (2012) applied a unified synthesis approach to planar 4-bar mechanisms for the purpose of function generation. Lu et al (2012) used the contracted graph technique to derive topology graphs for type synthesis of closed mechanisms.

Nomenclatures

$f_1, f_2, \ldots, f_6$: nonlinear mechanism functions.
r_1, r_2, r_3, r_4: lengths of links 1, 2, 3 and 4.
r_{1n}, r_{3n}, r_{4n}: normalized lengths of links 1, 3 and 4.
x_{A1}, y_{A1}: coordinates of point A_1.
x_{A2}, y_{A2}: coordinates of point A_2.
$x_1, x_2, \ldots, x_6$: mechanism unknown parameters.
μ_1: mechanism transmission angle in the first coupler position.
μ_2: mechanism transmission angle in the second coupler position.
θ_1: orientation of link 1 (frame).
$\theta_{21}, \theta_{31}, \theta_{41}$: orientation of links 2, 3 and 4 in the first mechanism position.
$\theta_{22}, \theta_{32}, \theta_{42}$: orientation of links 2, 3 and 4 in the second mechanism position.

Methodology

- The proposed methodology is applied to standard 4-bar mechanisms having fixed lengths. The approach is applied as follows:
- The desired 2 positions of the coupler are assigned in the motion plane.
- Closed loops are formed for the mechanism in the 2 positions.
- 2 equations are written for each loop in the x and y directions.
- 2 equations are written for the 2 transmission angles (one per mechanism position).
- The 6 equations are written in a normalized form by dividing each link dimension by r_2.
- The equations are written such that the right hand side is zero.
- The model in its final form consists of 6 nonlinear equations in 6 unknowns.
- The model is solved using MATLAB for the mechanism unknowns.

3.1. Requirements:

It is desired to have a coupler of a known length in 2 positions: A_1B_1 and A_2B_2 with known orientations θ_{31} and θ_{32} as in Fig.1.

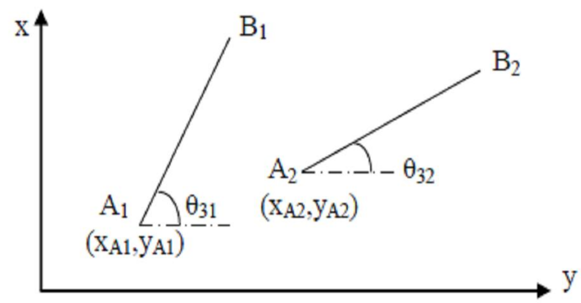


Fig.1: Positions and Orientation of 4-Bar Mechanism

3.2. Mechanism:

- A 4-bar mechanism as shown in Fig.2.

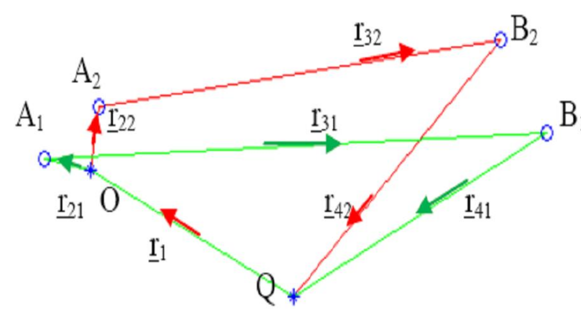


Fig.2: 4-Bar Mechanism in 2 Positions.

- 2 polygons are closed which are required for displacement analysis in each mechanism position.

3.3. Analysis:

- The 2 coupler positions are: A_1B_1 and A_2B_2.
- Polygon 1: OA_1B_1QO. The displacement equation across the polygon is:

$$\underline{r}_1 + \underline{r}_{21} + \underline{r}_{31} + \underline{r}_{41} = 0$$

- Working with the vectors components in the x-direction ; $\sum r_x = 0$ gives:

$$r_1\cos\theta_1 + r_2\cos\theta_{21} + r_3\cos\theta_{31} + r_4\cos\theta_{41} = 0 \quad (1)$$

- Working with the vectors components in the y-direction ; $\sum r_y = 0$ gives:

$r_1\sin\theta_1 + r_2 \sin\theta_{21} + r_3 \sin\theta_{31} + r_4 \sin\theta_{41} = 0$ (2)

- Polygon 2: OA_2B_2QO. The displacement equation across the polygon is:

$\underline{r}_1 + \underline{r}_{22} + \underline{r}_{32} + \underline{r}_{42} = 0$

- Working with the vectors components in the x-direction ; $\sum r_x = 0$ gives:

$r_1\cos\theta_1 + r_2 \cos\theta_{22} + r_3 \cos\theta_{32} + r_4 \cos\theta_{42} = 0$ (3)

- Working with the vectors components in the y-direction ; $\sum r_y = 0$ gives:

$r_1\sin\theta_1 + r_2 \sin\theta_{22} + r_3 \sin\theta_{32} + r_4 \sin\theta_{42} = 0$ (4)

- Unknowns in Eqs.1-4: r_1, r_2, , r_4, θ_1, θ_{21}, θ_{41}, θ_{22} and θ_{42}.
- Number of unknowns: 8.
- Number of equations so far: 4.
- The number of design parameters is reduced through:
  1. Assigning the ground length, r_1.
  2. Using normalized dimensions by referring all the dimensions to r_2.

  In this case, the unknown design parameters are: $x_1 = r_{4n}$, $x_2 = \theta_1$, $x_3 = \theta_{21}$, $x_4 = \theta_{41}$, $x_5 = \theta_{22}$, and $x_6 = \theta_{42}$.
- The number of unknowns is reduced to 6.
- Two more equations may be written for the transmission angle in the 2 positions of the mechanism.
- The transmission angle is related to links 3 and 4 orientation angles through:

$\mu_1 = \theta_{41} - \pi - \theta_{31}$ (5)

and $\mu_2 = \theta_{42} - \pi - \theta_{32}$ (6)

- Now, equations: 1 – 6 are written in the normalized form as:

$f_1 = r_{1n}\cos x_2 + \cos x_3 + r_{3n}\cos\theta_{31} + x_1\cos x_4$ (7)

$f_2 = r_{1n}\sin x_2 + \sin x_3 + r_{3n}\sin\theta_{31} + x_1\sin x_4$ (8)

$f_3 = r_{1n}\cos x_2 + \cos x_5 + r_{3n}\cos\theta_{32} + x_1\cos x_6$ (9)

$f_4 = r_{1n}\sin x_2 + \sin x_5 + r_{3n}\sin\theta_{32} + x_1\sin x_6$ (10)

$f_5 = \mu_1 - x_4 + \pi + \theta_{31}$ (11)

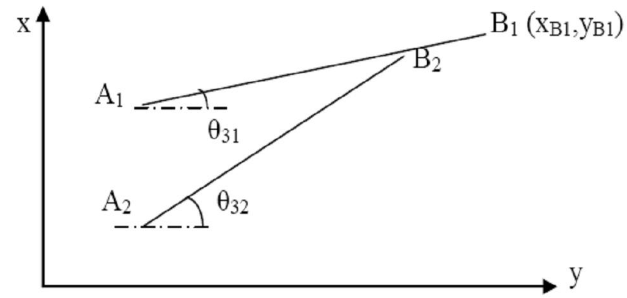
$f_6 = \mu_2 - x_6 + \pi + \theta_{32}$ (12)

3.4. Mechanism Design:

- The design equations are equations 7-12 (6 equations).
- The equations are nonlinear in 6 unknowns.
- The 6 equations are in the form: $f = 0$
- The 6 equations may be solved using the MATLAB command "fsolve" or any other numerical technique.
- The coordinates of B_1 will be use to locate the fixed pivot Q in the xy-plane.

3.5. Case Study

It is required a 4-bar planar mechanism to move the coupler AB from position A_1B_1 to A_2B_2 shown in Fig.3.



$r_3 = AB = 200$ mm $\theta_{31} = 20^o$, $\theta_{31} = 43^o$

$x_{A1} = 258.5$, $y_{A1} = 115.5$, $x_{A2} = 261$, $y_{A2} = 35$ mm

Fig.3: A 4-Bar Planar mechanism Movies

3.5.1. Mechanism Synthesis:

A MATLAB code is written to solve Eqs.7-12 satisfying there right hand side which is zero for the 6 equations.

- Code inputs (guessed values):
  $r_{3n} = 5$, $r_{1n} = 6$, $\theta_{31} = 20^o$, $\theta_{32} = 43^o$, $\mu_1 = \mu_2 = 90^o$
- Code output:

3.5565	(r_{4n})
3.0556	(θ_1)
1.5531	(θ_{21})
5.0827	(θ_{41})
-1.4013	(θ_{22})
5.3528	(θ_{42})

- Values of the nonlinear functions:

0.0255 -0.0899 -0.0273 0.0881 -0.0213 0.1101

- Mechanism dimensions:

Coupler length: $r_3 = 200$ mm (required)
Crank length: $r_2 = r_3/r_{3n} = 200/5 = 40$ mm
Rocker length: $r_4 = r_{4n} x r_2 = 142.26$ mm
Ground length: $r_1 = r_{1n} x r_2 = 240$ mm
Ground angle: $\theta_1 = 175^o$
Crank orientation: $\theta_{21} = 89^o$
$\theta_{22} = 279.7^o$
Rocker orientation: $\theta_{41} = 291.2^o$
$\theta_{42} = 306.7^o$

The designed mechanism in its 2 positions is shown in Fig.4.

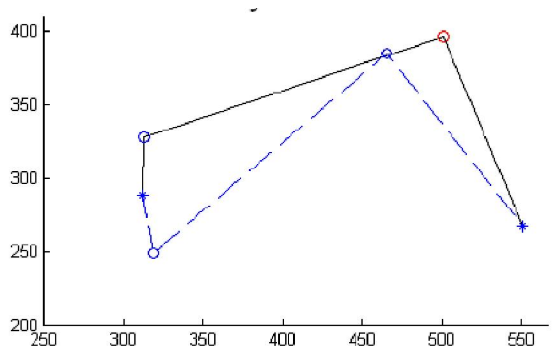


Fig.4: Two Positions of 4-Bar Mechanism

- Coordinates x_{B1} and y_{B1} are used to locate Q in the xy-plane.
- Transmission angles (μ) of the designed mechanism:

  In the first position:

  $\mu_1 = 91.0^o$

  In the second position:

  $\mu_2 = 83.7^o$

- Mechanism type:

  $L_{min} = 40$ mm (crank)

  $L_{max} = 240$ mm

  $L_a = 200$ mm

  $L_b = 142.26$ mm

  $L_{min} + L_{max} = 280$ mm

  $L_a + L_b = 342.2$ mm

  This means that: $L_{min} + L_{max} < L_a + L_b$ and the crank has the minimum length.

Therefore, the designed mechanism is a crank-rocker Grashof mechanism.

4. Discussions

- The proposed approach is very accurate and reliable in synthesizing 4-bar planar mechanisms for 2 specific positions of its coupler.
- The assumptions are only one dimension (r_1) giving easy and straight forward design of the 4-bar mechanism.
- In a previous work, such mechanism requires 3 free choices to design the 4-bar mechanism for 2 positions.
- The coupler traces exactly the desired 2-positions.
- The deviation of the transmission angle of the mechanism from the ideal value of 90^o is:
  - 1.1 % error in the first coupler position.
  - 7.0 % error in the second coupler position.

Corresponding author
Maha M. Lashin
Mechanical Engineering Department, Shoubra Faculty of Engineering, Banha University
maha_lashin@yahoo.com

5. References:

1. Figliolini G.& J. Angeles, "Synthesis of conjugate Geneva mechanisms with curved slots" Mechanism and Machine Theory, Vol. 37, No. 10, October 2002, pp. 1043-1061.
2. SimionescuP. and D. Beale, "Synthesis and analysis of the five-link rear suspension system used in automobiles", ibid, Vol. 37, No. 9, September 2002, pp. 815-832.
3. . Lebedov P., "Vector method for the synthesis of mechanisms ", ibid, Vol. 38, No. 3, March 2003, pp. 265-276.
4. BalliS. and S. Chand, "Synthesis of a planar seven-link mechanism with variable topology for motion between two dead-center positions ", ibid, Vol. 38, No. 11, November 2003, pp. 1271-1287.

5Laribi M. *et al.,* "A combined genetic algorithm–fuzzy logic method (GA–FL) in mechanisms synthesis", ibid, Vol. 39, No. 7, July 2004, pp. 717-735.

6. SenD. and V. Joshi, "Issues in geometric synthesis of mechanisms ", ibid, Vol. 39, No. 12, December 2004, pp. 1321-1330.
7. Russel K. and R. Sodhi, "On the design of design of slider-crank mechanisms", ibid, Vol. 100, No.2, August 2005, pp. 131-143.
8. BustosI. *et al.*, "Kinematical synthesis of 1 DOF mechanisms using finite elements and genetic algorithms", Finite Elements in Analysis & Design, Vol.41, No.15, September 2005, pp.1441-1463.
9. Wu J. and C. Chen, "Mathematical model and its simulation of exactly mechanism synthesis", Applied Mathematics & Computations, Vo.160, No.2, January 2005, pp.3.9-316.
10. Kyung M. and E. Sacks, "Robust parameter synthesisfor planarhigher pair mechanical systems ", Computer-Aided Design, Vol. 38, No. 5, May 2006, pp. 518-530.

11Sankari R. and L. Schmidt, "Structural synthesis of planar kinematic chains by adapting a Mckay-type algorithm ", Mechanism and Machine Theory, Vol. 41, No.9, September 2006, pp. 1021-1030.

12.. Varbanov H. *et al.* "mechanisms design ", Expert Systems with Applications, Vol. 31, No. 3, October 2006, pp. 558-569.

13. Hongying Y. *et al.* ,Study on a new computer path synthesis method of a four-bar linkage", Mechanism and Machine Theory, Vol. 42, No. 4, April 2007, pp. 383-392.
14. DingH. and Z. Huang, "A new theory for the topological structure analysis of kinematic chains and its applications", ibid, Vol. 42, No. 10, October 2007, pp. 1264-1279.
15Gregorio R., "A novel geometric and analytic technique for the singularity analysis of one-dof planar mechanisms ", ibid, Vol. 42, No. 11, November 2007, pp. 1462-1483
16.. HwangW. and Y. Fan, "Polynomial equations for the loci of the acceleration pole of a slider crank mechanism ", ibid, Vol. 43, No. 2, February 2008, pp.123-137
17. C. Chen and J. Angeles, "A novel family of linkages for advanced motion synthesis ibid, Vol. 43, No. 7, July 2008, pp. 882-890.
18. Sheu J. *et al.,* "Kinematic synthesis of a four-link mechanism with rolling contacts for motion and function generation ", Mathematical & Computer Modeling, Vol. 48, No. 5–6, September 2008, pp. 805-817.
19.. Litvin F. *et al.*, "Tandem design of mechanisms for function generation and output speed variation ", Computer Methods in Applied Mechanics & Engineering, Vol. 198, No. 5–8, 15 January 2009, pp. 860-876.
20. Pennock G. and A. Israr, "Kinematic analysis and synthesis of an adjustable six-bar linkage ", Mechanism and Machine Theory, Vol. 44, No. 2, February 2009, pp. 306-323.
21. DuX. *et al.* "Robust mechanism synthesis with random and interval variables ", ibid, Vol. 44, No. 7, July 2009, pp.1321-1337
22.. Wei G. and J. Dai, "Geometric and kinematic analysis of a seven-bar three-fixed-pivoted compound-joint mechanism ", ibid, Vol. 45, No. 2, February 2010, pp. 170-184.
23. Parlaktas V. *et al.* "On the synthesis of a geared four-barmechanism", ibid, Vol. 45, No. 8, August 2010, pp. 1142-1152.
24. Huang X.and Y. Zhang, "Robust tolerance design for function generation mechanisms with joint clearances ", ibid, Vol. 45, No. 9, September 2010, pp. 1286-1297.
25. NieS. *et al.* "Kinematic configuration analysis of planar mechanisms based on basic kinematic chains ", ibid, Vol. 46, No. 10, October 2011, pp. 1327-1334.
26.. Soong R. and S. Chang, "Synthesis of function-generation mechanisms using variable length driving links ", ibid, Vol. 46, No. 11, November 2011, pp. 1696-1706.
27. E. Tanik, "Transmission angle in compliant slider-crank mechanism ", ibid, Vol. 46, No. 11, November 2011, pp. 1623-1632.
28. H. Ding *et al.* "Synthesis of the whole family of planar 1-DOF kinematic chains and creation of their atlas database ", ibid, Vol. 47, January 2012, pp. 1-15.
29.. Kim and H. Yoo, "Unified synthesis of a planar four-bar mechanism for function generation using a spring-connected arbitrarily sized block model ", ibid, Vol. 49, March 2012, pp. 141-156.
30. LuY. *et al.* "Derivation of valid contracted graphs from simpler contracted graphs for type synthesis of closed mechanisms ", ibid, Vol. 31. G. Sandor and A. Erdman, "Advanced mechanism design: analysis and synthesis", Prentice Hall, Vol.2, 1984, p.94.

8/8/2012