Integration of Fuzzy Prioritization Method and TOPSIS for Strategy Ranking

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Abstract: Choosing among strategic alternatives is usually a difficult task. Managers encounter this difficulty because they lack perfect foresight. They must choose a strategy today, whose success depends on future conditions, without knowing exactly what the future looks like. The purpose of this paper is applying a new integrated method for Strategy Ranking. Proposed approach is based on Fuzzy Prioritization Method and TOPSIS. Fuzzy Prioritization Method is used in determining the weights of the criteria by decision makers and then ranking of Strategies are determined by TOPSIS method. In this paper a numerical example demonstrates the application of the proposed method. [Ali Mohaghar, Hossein Bazargani, Abdol Hossein Jafarzadeh, Mohammad Hosein Soleimani-Sarvestani. **Integration of Fuzzy Prioritization Method and TOPSIS for Strategy Ranking**. *N Y Sci J* 2013;6(5):29-33]. (ISSN: 1554-0200). http://www.sciencepub.net/newyork. 6

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1. INTRODUCTION

The business environment is becoming more competitive and complex. Various factors have increased competitive pressures in markets and made relationships between environmental factors more complex (e.g., global markets, faster technology transfers). Weaker firms are being driven from markets, with the more aggressive survivors having significant resources to exploit any strategic opportunity. In such an environment a competitive advantage is the ability to quickly and correctly interpret changes in the environment and determine what strategies, if any, the firm should take (Weigelt et al, 1988). Choosing among strategic alternatives is usually a difficult task. Managers encounter this difficulty because they lack perfect foresight. They must choose a strategy today, whose success depends on future conditions, without knowing exactly what the future looks like (Farzipoor Saen et al, 2009). Corner (1991) surveyed multi-attribute decision analysis applications in operations research literature and found many of the applications to address strategic decisions. Wind (1980) applied the Analytic Hierarchy Process (AHP) to the portfolio decision of a firm whose management is concerned with the determination of the desired target portfolio and allocation of resources among its components. Wind (1987) presented an application for corporate strategy for evaluating strategic options on multiple and interdependent objectives to ensure effective utilization of resources. Hastings (1996) provided a method for ranking strategy on quantitative, qualitative and intangible criteria based on AHP. Farzipoor (2009) used a Mathematical Programming for ranking strategy. Farzipoor (2009) used a super-efficiency analysis for strategy ranking. This paper is organized as follows. Section 3 presents the methodology. The application of the proposed method is addressed in Section 3. Finally, conclusions are provided in Section 4.

2. RESEARCH METHODOLOGY

In this paper, the weights of each criterion are calculated using Fuzzy Prioritization Method. After that, TOPSIS is utilized to rank the alternatives. Finally, we rank the Strategies based on these results. Before explaining about fuzzy prioritization method, it has been described fuzzy sets and fuzzy numbers as follow.

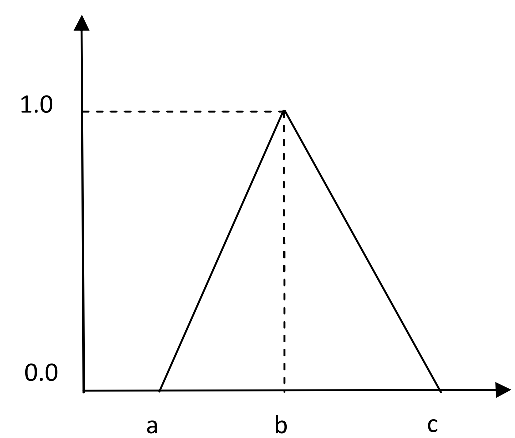

Fig 1. A triangular fuzzy number $\tilde{A}$.

2.1. Fuzzy sets and fuzzy numbers

Fuzzy set theory, which was introduced by Zadeh (1965) to deal with problems in which a source of vagueness is involved, has been utilized for incorporating imprecise data into the decision framework. A fuzzy set $\tilde{A}$ can be defined mathematically by a membership function $\mu_{\tilde{A}}(X)$, which assigns each element x in the universe of discourse X a real number in the interval [0,1]. A triangular fuzzy number $\tilde{A}$ can be defined by a triplet (a, b, c) as illustrated in Fig 1.

The membership function $\mu_{\tilde{A}}(X)$ is defined as

$$\mu_{\tilde{A}}(x)=\begin{cases}\frac{x-l}{m-l} & l \le x \le m\\ \frac{x-u}{m-u} & m \le x \le u\\ 0 & oterwise\end{cases} \quad (1)$$

Where l, m, and u are also considered as the lower bound, the mean bound, and the upper bound, respectively. The triangular fuzzy number $\tilde{N}$ is often represented as (l,m,u). According to Table 1, criteria compare with each other. After pairwise comparisons, are finished at a level, a fuzzy reciprocal judgment matrix $\tilde{A}$ can be established as

$$\tilde{A}=\{\tilde{a}_{ij}\}=\begin{bmatrix}\tilde{a}_{11} & \tilde{a}_{12} & \cdots & \tilde{a}_{1n}\\ \tilde{a}_{21} & \tilde{a}_{22} & \cdots & \tilde{a}_{2n}\\ \vdots & \vdots & \ddots & \vdots\\ \tilde{a}_{n1} & \tilde{a}_{n2} & \cdots & \tilde{a}_{nn}\end{bmatrix} \quad (2)$$

Table 1. Linguistic variables for important of each criteria

linguistic variables	triangular fuzzy numbers
very low	(0.00,0.00,0.00)
low	(0.10,0.20,0.30)
medium low	(0.20,0.35,0.50)
medium	(0.40,0.50,0.60)
medium high	(0.50,0.65,0.80)
high	(0.70,0.80,0.90)
very high	(0.80,1.00,1.00)

Where n is the number of the related elements at this level, and a_{ij} =1/ a_{ij}. Basic arithmetic operations on triangular fuzzy numbers $A_1 = (l_1,m_1,u_1)$, where $l_1 \le m_1 \le u_1$, and $A_2 = (l_2,m_2,u_2)$, where $l_2 \le m_2 \le u_2$, can be shown as follows:

Addition:

$$A_1 \oplus A_2 = (l_1 + l_2, m_1 + m_2, u_1 + u_2) \quad (3)$$

Subtraction:

$$A_1 \ominus A_2 = (l_1 - u_2, m_1 - m_2, u_1 - l_2) \quad (4)$$

Multiplication: if K is a scalar

$$K \otimes A_1 = \begin{cases}(kl_1, km_1, ku_1), & k > 0\\ (ku_1, km_1, kl_1), & k < 0\end{cases}$$

$$A_1 \otimes A_2 \approx (l_1 l_2, m_1 m_2, u_1 u_2), \quad \text{if} \quad l_1 \ge 0, \; l_2 \ge 0 \quad (5)$$

$$\text{Division: } A_1 \oslash A_2 \approx \left(\frac{l_1}{u_2}, \frac{m_1}{m_2}, \frac{u_1}{l_2}\right), \quad \text{if} \quad l_1 \ge 0, \quad l_2 \ge 0 \quad (6)$$

Although multiplication and division operations on triangular fuzzy numbers do not necessarily yield a triangular fuzzy number, triangular fuzzy number approximations can be used for many practical applications (Kaufmann and Gupta, 1988). Triangular fuzzy numbers are appropriate for quantifying the vague information about most decision problems including Facility location selection. The primary reason for using triangular fuzzy numbers can be stated as their intuitive and computational-efficient representation (Karsak, 2002). A linguistic variable is defined as a variable whose values are not numbers, but words or sentences in natural or artificial language. The concept of a linguistic variable appears as a useful means for providing approximate characterization of phenomena that are too complex or ill-defined to be described in conventional quantitative terms (Zadeh, 1975).

2.2. Fuzzy Prioritization Method (FPM)

Fuzzy prioritization method is described by Wang et al (2006) as follow: suppose that a fuzzy judgment matrix is constructed as Eq. (2) in a prioritization problem, where n elements are taken into account. Among this judgment matrix A, the triangular fuzzy number a_{ij} is expressed as (l_{ij},m_{ij},u_{ij}), i and j=1,2,…,n, where l_{ij}, m_{ij}, and u_{ij} are the lower bound, the mean bound, and the upper bound of this fuzzy triangular set, respectively. Furthermore, we assume that $l_{ij}<m_{ij}<u_{ij}$ when i≠j. If i=j, then $a_{ij}= a_{ji}$ = (1, 1, 1). Therefore, an exact priority vector w = $(w_1, w_2,\dots,w_n)^T$ derived from A must satisfy the fuzzy inequalities:

$$l_{ij} \lesssim \frac{w_i}{w_j} \lesssim m_{ij} \quad (7)$$

Where $w_i > 0$, $w_j > 0$, i≠j, and the symbol $\lesssim$ means "fuzzy less or equal to".To measure the degree of satisfaction for different crisp ratios w_i/w_j with regard to the double side inequality (7), a function can be defined as:

$$\mu_{ij}\left(\frac{w_i}{w_j}\right)=\begin{cases}\frac{m_{ij}-(w_i/w_j)}{m_{ij}-l_{ij}} & 0<\frac{w_i}{w_j}\le m_{ij}\\ \frac{(w_i/w_j)-m_{ij}}{u_{ij}-m_{ij}} & ,\ \frac{w_i}{w_j}>m_{ij}\end{cases} \quad (8)$$

Where i≠j. Being different from the membership function (1) of triangular fuzzy numbers, the function value of μ_{ij} (w_i/w_j) may be larger than one, and is linearly decreasing over the interval $(0,m_{ij}]$ and linearly increasing over the interval $[m_{ij},\infty)$, as shown in Fig. 2. The less value of μ_{ij} (w_i/w_j) indicates that the exact ratio w_i/w_j is more acceptable.

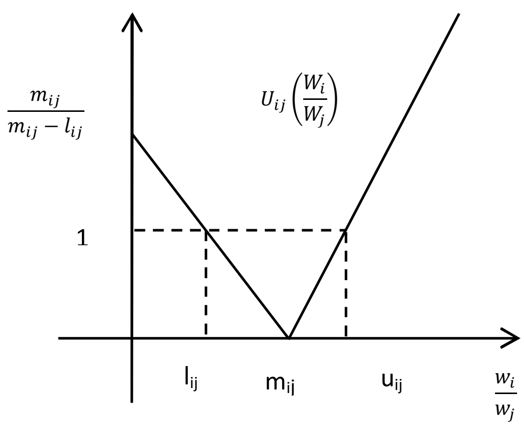

Fig 2. Function for measuring the satisfaction degree of w_i/w_j

To find the solution of the priority vector $(w_1,w_2,\dots,w_n)^T$, the idea is that all exact ratios w_i/w_j should satisfy n(n-1) fuzzy comparison judgments (l_{ij},m_{ij},u_{ij}) as possible as they can, i and j=1,2,…,n, i≠j. Therefore, in this study, the crisp priorities assessment is formulated as a constrained optimization problem:

$$\text{Min } J(w_1, w_2, \dots, w_n)$$

$$= \min \sum_{i=1}^{n}\sum_{j=1}^{n}\left[m_{ij}\left(\frac{w_i}{w_j}\right)\right]$$

$$= \min \sum_{i=1}^{n}\sum_{j=1}^{n}\left[\begin{array}{l}\delta\left(m_{ij}-\frac{w_i}{w_j}\right)\left(\frac{m_{ij}-(w_i/w_j)}{m_{ij}-l_{ij}}\right)^P\\ +\delta\left(\frac{w_i}{w_j}-m_{ij}\right)\left(\frac{(w_i/w_j)-m_{ij}}{u_{ij}-m_{ij}}\right)^P\end{array}\right]$$

Subject to

$$\sum_{k=1}^{n} w_k = 1, w_k>0,\ k=1,2,\dots,n.$$

Where i≠j ,P ϵ N , and

$$\delta(x)=\begin{cases}0 & ,x<0\\ 1 & ,x\ge 0\end{cases} \quad (9)$$

The power index P is fixed, and chosen by decision makers in a specific MCDM problem. A larger P is suggested, e.g. 10, as illustrated briefly in the next section. The function J $(w_1,w_2,\dots,w_n)$ is non-differentiable. In some cases, decision-makers are unable or unwilling to give all pairwise comparison judgments of n elements. However, provided that the known fuzzy set of pairwise comparisons involves n elements, such as $F=\{a_{ij}\}=\{a_{12}, a_{13}, \dots, a_{n1}\}$ or $\{a_{21}, a_{31}, \dots, a_{n1}\}$, the solution of priority vector $(w_1,w_2,\dots,w_n)^T$ will be still able to be derived based on the optimization problem above. In order to measure the consistency degree of the fuzzy comparison judgment matrix A as Eq. (2), an index γ can be defined after the optimal crisp priority vector $(w_1^*,w_2^*,\dots,w_n^*)^T$ is obtained:

$$\gamma=\exp\left\{-\max\left\{\mu_{ij}\left(\frac{w_i^*}{w_j^*}\right)\middle|\, i,j=1,2,\dots,n, i\ne j\right\}\right\} \quad (10)$$

Where $\mu_{ij}(w_i^*/w_j^*)$ is the function of (8). The value of γ satisfies $0<\gamma\le 1$ always. If it is larger than e^{-1}=0.3679, all exact ratios satisfy the crisp inequalities $i_{ij}\le w_i^*/w_j^*\le m_{ij}$, i and j=1,2,…,n, i≠j, and the corresponding fuzzy judgment matrix has good consistency. γ=1 indicates that the fuzzy judgment matrix is completely consistent. In conclusion, the fuzzy judgment matrix with a larger γ value is more consistent. For solving this optimization problem that has non-linear constraints, we used the genetic algorithm.

2.3. TOPSIS Method

The TOPSIS method is proposed in Chen and Hwang (1992), with reference to Hwang and Yoon (1981). The basic principle is that the chosen alternative should have the shortest distance from the ideal solution that maximizes the benefit and also minimizes the total cost, and the farthest distance from the negative-ideal solution that minimizes the benefit and also maximizes the total cost (Opricovic and Tzeng, 2003).

The TOPSIS method consists of the following steps:

Step 1: Calculate the normalized decision matrix. The normalized value r_{ij} is calculated as

$r_{ij} = X_{ij} / \sqrt{\sum_{i=1}^{n} X_{ij}^2}, \forall i, j$ (11)

Step 2: Calculate the weighted normalized decision matrix. The weighted normalized value v_{ij} is calculated as

$v_{ij} = w_j r_{ij}, \forall i,j$ (12)

Where w_j is the weight of the *j*th criterion, and

$\sum_{i=1}^{m} w_j = 1$

Step 3: Determine the ideal and negative-ideal solution.

$A^* = \{v_1^*, \ldots, v_m^*\} = \{(\max_i v_{ij} \mid j \epsilon C_h), (\min_i v_{ij} \mid j \epsilon C_c)\}$ (13)

$A^- = \{v_1^-, \ldots, v_m^-\} = \{(\min_i v_{ij} \mid j \epsilon C_h), (\max_i v_{ij} \mid j \epsilon C_c)\}$ (14)

where C_b is associated with benefit criteria and C_c is associated with cost criteria.

Step 4: Calculate the separation measures, using the m-dimensional Euclidean distance. The separation of each alternative from the ideal solution is given as

$S_i^* = \sqrt{\sum_{j=1}^{m} (v_{ij} - v_j^*)^2}, \forall i$ (15)

Similarity, the separation from the negative-ideal solution is given as

$S_i^- = \sqrt{\sum_{j=1}^{m} (v_{ij} - v_j^-)^2}, \forall i$ (16)

Step 5: Calculate the relative closeness to the ideal solution. The relative closeness of the alternative A_i with respect to A^* is defined as

$RC_I^* = \frac{S_i^-}{S_i^* + S_i^-}, \forall i$ (17)

Step 6: Rank the preference order.

The index values of RC_I^* lie between 0 and 1. The larger index value means the closer to ideal solution for alternatives.

3. NUMERICAL EXAMPLE

The criteria for this example are taken from Farzi poor (2009). These criteria are including: Total Cost (C_1), Risk (C_2), Net Present Value (C_3), and Payback time (C_4). After forming the decision hierarchy for strategy ranking problem, the criteria to be used in evaluation process are assigned weights by using FPM method. Geometric means of these values are found to obtain the pairwise compassion matrix on which there is a consensus (Table 2).

Table 2. Fuzzy comparison matrix

	C_1	C_2	C_3	C_4
C_1	(1.00,1.00,1.00)	(2.00,3.00,4.00)	(6.00,6.33,7.00)	(0.20,2.13,6.00)
C_2	(0.25,0.36,0.50)	(1.00,1.00,1.00)	(1.00,3.25,6.00)	(0.14,0.83,2.00)
C_3	(0.14,0.16,0.17)	(0.17,0.50,1.00)	(1.00,1.00,1.00)	(0.20,1.18,3.00)
C_4	(0.17,3.39,6.00)	(0.50,2.83,5.00)	(0.33,0.85,7.00)	(1.00,1.00,1.00)

After that we formulate the fuzzy comparison matrix as a constrained optimization problem and we solve this optimization problem using Genetic algorithm. In order to employ Genetic algorithm, we use the MATLAB toolbox. Some settings that are used: Population Size equal to 100, the number of direct transfer to the next generation (Elite count) equal to 2, crossover fraction equal to 0.8 and the stopping conditions are described as follow: transfer from 100 generation and a lack of improvement in 50 generation. The results obtained from solving optimization problem using of Genetic algorithm are presented in Table 3.

Table 3.The weight of criteria

W_1	W_2	W_3	W_4
0.35380	01522	0.1616	0.32802

After calculating the weights, we formed the decision matrix of TOPSIS and then by using Eqs. (11) and (12), the weighted normalized decision matrix is obtained, as presented in Table 4.

Table 4: The weighted normalized decision matrix

	C_1	C_2	C_3	C_4
A_1	0.651	0.590	0.666	0.379
A_2	0.391	0.579	0.583	0.758
A_3	0.651	0.563	0.466	0.531
A_4	0.423	0.430	0.114	0.615
A_5	0.631	0.332	0.218	0.517

After developing the weighted normalized decision matrix, the final ranking procedure should determine the ideal solution and negative-ideal solutions by using Eqs. (13) and (14). In particular, the ideal

solution and negative-ideal solution are determined as follows:
$A^{*} = \{0.268, 0.139, 0.131, 0.071\}$ and
$A^{-} = \{0.161, 0.133, 0.092, 0.035\}$

Table 5: Final ranking of strategy

Rank	strategy	S_I^*	S_I^-	RC_I^*
1	A_1	0.09	0.19	0.70
2	A_2	0.09	0.18	0.66
3	A_3	0.13	0.18	0.59
5	A_4	0.22	0.07	0.24
4	A_5	0.14	0.15	0.53

By using Eqs. (15) and (16), the computed distances of each strategy from ideal solution (S_i^*) and negative-ideal solution (S_i^-) are presented in Table 5. Based on their relative closeness to the ideal solution obtained by using Eq. (17), the final step of the TOPSIS method consists of ranking strategies. According to Table 15, the A_1 is the best strategy among other strategies.

4. CONCLUSION

Strategy selection has long been recognized as a multi-criteria problem. The joint consideration of multiple criteria complicates the selection decision, even in the case of experienced managers, because competing strategies have different levels of success under multiple criteria. In this paper, a two step FPM and TOPSIS methodology is structured here that TOPSIS uses FPM result weights as input weights. According to this methodology, the first strategy (A_1) is selected as the best strategy.

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