Flexible Manufacturing System Selection Based on VIKOR and Fuzzy Prioritization Method

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Abstract: Selection of a flexible manufacturing system (FMS) is a challenging task because of the insufficient experience and data about this still-evolving technology. Further, the large investment involved makes the selection process critical. The purpose of this paper is applying a new integrated method to flexible manufacturing system selection. Proposed approach is based on Fuzzy Prioritization Method and VIKOR (VlseKriterijumska Optimizacija I Kompromisno Resenje). Fuzzy Prioritization Method is used in determining the weights of the criteria by decision makers and then ranking of alternative are determined by VIKOR method. In this paper a numerical example demonstrates the application of the proposed method.

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1. INTRODUCTION

Flexible manufacturing systems (FMS) have extensively been studied over the past fifteen years. Selection of FMS is a challenging task because of the insufficient experience and data about this still-evolving technology. Further, the large investment involved makes the selection process critical. An FMS is an integrated manufacturing system that consists of one or several work stations linked by a computerized inventory system, making it possible for jobs to follow diverse routes through the production system. An advantage of FMS is that it can simultaneously meet several goals: small batch sizes, high quality standards and efficiency of the production process. Boththe industrial and the academic community (Kuula, 1993., Buzacott et al, 1986., Jaikumar, 1986., Ranta et al, 1988) have been interested in the design of flexible manufacturing systems. The rest of the paper is organized as follows: The following section presents a concise treatment of the basic concepts of fuzzy set theory. Section 3 presents the methodology of Fuzzy Prioritization Method and VIKOR. The application of the proposed framework to FMS selection is addressed in Section 4. Finally, conclusions are provided in Section 5.

2. Fuzzy sets and fuzzy numbers

Fuzzy set theory, which was introduced by Zadeh (1965) to deal with problems in which a source of vagueness is involved, has been utilized for incorporating imprecise data into the decision framework. A fuzzy set $\tilde{A}$ can be defined mathematically by a membership function $\mu_{\tilde{A}}(X)$, which assigns each element x in the universe of discourse X a real number in the interval [0,1]. A triangular fuzzy number $\tilde{A}$can be defined by a triplet (a, b, c) as illustrated in Fig 1.

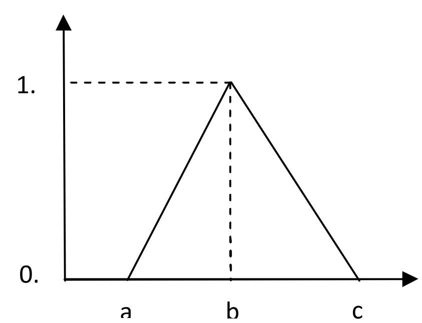

Fig1. A triangular fuzzy number $\tilde{A}$.

The membership function $\mu_{\tilde{A}}(X)$ is defined as

$$\mu_{\tilde{A}}(x)=\begin{cases} \frac{x-l}{m-l} & l \le x \le m \\ \frac{x-u}{m-u} & m \le x \le u \\ 0 & oterwise \end{cases} \quad (1)$$

Where l, m, and u are also considered as the lower bound, the mean bound, and the upper bound, respectively. The triangular fuzzy number $\tilde{N}$ is often represented as (l,m,u). According to Table 1, criteria compare with each other. After pairwise comparisons, are finished at a level, a fuzzy reciprocal judgment matrix $\tilde{A}$ can be established as

$$\tilde{A} = \{\tilde{a}_{ij}\} = \begin{bmatrix} \tilde{a}_{11} & \tilde{a}_{12} & \cdots & \tilde{a}_{1n} \\ \tilde{a}_{21} & \tilde{a}_{22} & \cdots & \tilde{a}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{a}_{n1} & \tilde{a}_{n2} & \cdots & \tilde{a}_{nn} \end{bmatrix} \quad (2)$$

Table 1. Linguistic variables for important of each criteria

linguistic variables	triangular fuzzy numbers
very low	(0.00,0.00,0.00)
low	(0.10,0.20,0.30)
medium low	(0.20,0.35,0.50)
medium	(0.40,0.50,0.60)
medium high	(0.50,0.65,0.80)
high	(0.70,0.80,0.90)
very high	(0.80,1.00,1.00)

Where n is the number of the related elements at this level, and $a_{ij} = 1/a_{ij}$. Basic arithmetic operations on triangular fuzzy numbers $A_1 = (l_1, m_1, u_1)$, where $l_1 \le m_1 \le u_1$, and $A_2 = (l_2, m_2, u_2)$, where $l_2 \le m_2 \le u_2$, can be shown as follows:

Addition:

$$A_1 \oplus A_2 = (l_1 + l_2, m_1 + m_2, u_1 + u_2) \quad (3)$$

Subtraction:

$$A_1 \ominus A_2 = (l_1 - u_2, m_1 - m_2, u_1 - l_2) \quad (4)$$

Multiplication: if K is a scalar

$$K \otimes A_1 = \begin{cases} (kl_1, km_1, ku_1), & k > 0 \\ (ku_1, km_1, kl_1), & k < 0 \end{cases}$$

$$A_1 \otimes A_2 \approx (l_1 l_2, m_1 m_2, u_1 u_2), \text{ if } l_1 \geq 0, l_2 \geq 0 \quad (5)$$

Division: $A_1 \oslash A_2 \approx \left(\frac{l_1}{u_2}, \frac{m_1}{m_2}, \frac{u_1}{l_2}\right)$,

if $l_1 \geq 0$, $l_2 \geq 0$ (6)

Although multiplication and division operations on triangular fuzzy numbers do not necessarily yield a triangular fuzzy number, triangular fuzzy number approximations can be used for many practical applications (Kaufmann and Gupta, 1988). Triangular fuzzy numbers are appropriate for quantifying the vague information about most decision problems including Facility location selection. The primary reason for using triangular fuzzy numbers can be stated as their intuitive and computational-efficient representation (Karsak, 2002). A linguistic variable is defined as a variable whose values are not numbers, but words or sentences in natural or artificial language. The concept of a linguistic variable appears as a useful means for providing approximate characterization of phenomena that are too complex or ill-defined to be described in conventional quantitative terms (Zadeh, 1975).

3. RESEARCH METHODOLOGY

In this paper, the weights of each criterion are calculated using FPM. After that, VIKOR is utilized to rank the alternatives. Finally, we select the best FMS based on these results.

3.1. Fuzzy Prioritization Method (FPM)

Fuzzy prioritization method is described by Wang et al (2006) as follow: suppose that a fuzzy judgment matrix is constructed as Eq. (2) in a prioritization problem, where n elements are taken into account. Among this judgment matrix A, the triangular fuzzy number a_{ij} is expressed as (l_{ij}, m_{ij}, u_{ij}), i and j=1,2,…,n, where l_{ij}, m_{ij}, and u_{ij} are the lower bound, the mean bound, and the upper bound of this fuzzy triangular set, respectively. Furthermore, we assume that $l_{ij} < m_{ij} < u_{ij}$ when $i \neq j$. If i=j, then $a_{ij} = a_{ji} = (1, 1, 1)$. Therefore, an exact priority vector $w = (w_1, w_2, \ldots, w_n)^T$ derived from A must satisfy the fuzzy inequalities:

$$l_{ij} \lesssim \frac{w_i}{w_j} \lesssim m_{ij} \quad (7)$$

Where $w_i > 0$, $w_j > 0$, $i \neq j$, and the symbol $\lesssim$ means "fuzzy less or equal to".To measure the degree of satisfaction for different crisp ratios w_i/w_j with regard to the double side inequality (7), a function can be defined as:

$$\mu_{ij}\left(\frac{w_i}{w_j}\right) = \begin{cases} \dfrac{m_{ij} - (w_i/w_j)}{m_{ij} - l_{ij}} & 0 < \dfrac{w_i}{w_j} \le m_{ij} \\ \dfrac{(w_i/w_j) - m_{ij}}{u_{ij} - m_{ij}} & , \dfrac{w_i}{w_j} > m_{ij} \end{cases} \quad (8)$$

Where $i \neq j$. Being different from the membership function (1) of triangular fuzzy numbers, the function value of $\mu_{ij}(w_i/w_j)$ may be larger than one, and is linearly decreasing over the interval $(0, m_{ij}]$ and linearly increasing over the interval $[m_{ij}, \infty)$, as shown in Fig. 2. The less value of $\mu_{ij}(w_i/w_j)$ indicates that the exact ratio w_i/w_j is more acceptable.

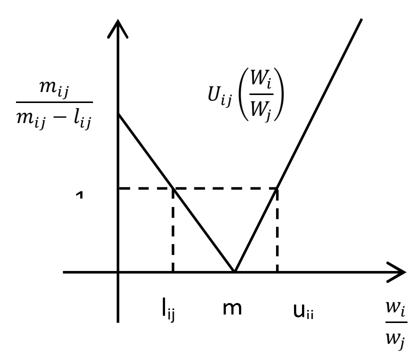

Fig 2. Function for measuring the satisfaction degree of w_i/w_j

To find the solution of the priority vector $(w_1, w_2, \ldots, w_n)^T$, the idea is that all exact ratios w_i/w_j should satisfy n(n-1) fuzzy comparison judgments

(l_{ij},m_{ij},u_{ij}) as possible as they can, i and j=1,2,…,n, i≠j. Therefore, in this study, the crisp priorities assessment is formulated as a constrained optimization problem:

Min J $(w_1, w_2, \ldots, w_n)$

$$= \min \sum_{i=1}^{n} \sum_{j=1}^{n} \left[m_{ij} \left(\frac{w_i}{w_j}\right) \right]$$

$$= \min \sum_{i=1}^{n} \sum_{j=1}^{n} \left[\begin{array}{l} \delta\left(m_{ij} - \frac{w_i}{w_j}\right)\left(\frac{m_{ij}-(w_i/w_j)}{m_{ij}-l_{ij}}\right)^P \\ +\delta\left(\frac{w_i}{w_j} - m_{ij}\right)\left(\frac{(w_i/w_j)-m_{ij}}{u_{ij}-m_{ij}}\right)^P \end{array} \right]$$

Subject to

$\sum_{k=1}^{n} w_k = 1, w_k > 0$, k=1,2,…,n.

Where i≠j ,P ϵ N , and

$$\delta(x) = \begin{cases} 0 & , x < 0 \\ 1 & , x \geq 0 \end{cases} \quad (9)$$

The power index P is fixed, and chosen by decision makers in a specific MCDM problem. A larger P is suggested, e.g. 10, as illustrated briefly in the next section. The function J $(w_1,w_2,\ldots,w_n)$ is non-differentiable. In some cases, decision-makers are unable or unwilling to give all pairwise comparison judgments of n elements. However, provided that the known fuzzy set of pairwise comparisons involves n elements, such as $F=\{a_{ij}\} =\{a_{12}, a_{13}, \ldots, a_{n1}\}$ or $\{a_{21}, a_{31}, \ldots, a_{n1}\}$, the solution of priority vector $(w_1,w_2,\ldots,w_n)^T$ will be still able to be derived based on the optimization problem above. In order to measure the consistency degree of the fuzzy comparison judgment matrix A as Eq. (2), an index γ can be defined after the optimal crisp priority vector $(w_1^*, w_2^*, \ldots, w_n^*)^T$ is obtained:

$$\gamma = \exp\left\{-\max\left\{\mu_{ij}\left(\frac{w_i^*}{w_j^*}\right) \middle| i, j = 1,2,\ldots,n, i \neq j\right\}\right\} \quad (10)$$

Where $\mu_{ij}(w_i^*/w_j^*)$ is the function of (8). The value of γ satisfies $0 < \gamma \leq 1$ always. If it is larger than $e^{-1}=0.3679$, all exact ratios satisfy the crisp inequalities $i_{ij} \leq w_i^*/w_j^* \leq m_{ij}$, i and j=1,2,…,n, i≠j, and the corresponding fuzzy judgment matrix has good consistency. γ=1 indicates that the fuzzy judgment matrix is completely consistent. In conclusion, the fuzzy judgment matrix with a larger γ value is more consistent. For solving this optimization problem that has non-linear constraints, we used the genetic algorithm.

3.2. The VIKOR Method

The VIKOR method is a compromise MADM method, developed by Opricovic .S and Tzeng (Opricovic, 1998; Opricovic, S. and Tzeng, G. H., 2002) started from the form of Lp-metric:

$$L_{pi} = \left\{\sum_{j=1}^{n} \left[w_j (f_j^* - f_{ij})/(f_j^* - f_j^-)\right]^p\right\}^{1/p} \quad 1 \leq p \leq +\infty \,; i = 1,2,\ldots I.$$

The VIKOR method can provide a maximum ‘‘group utility’’ for the “majority’’ and a minimum of an individual regret for the ‘‘opponent’’ (Opricovic, 1998; Opricovic, S; Tzeng, G. H., 2002; Serafim Opricovic & Gwo-Hshiung Tzeng, 2004).

3.2.1. Working Steps of VIKOR Method

1) Calculate the normalized value

Assuming that there are m alternatives, and n attributes. The various I alternatives are denoted as x_i. For alternative x_j, the rating of the jth aspect is denoted as x_{ij}, i.e. x_{ij} is the value of jth attribute. For the process of normalized value, when x_{ij} is the original value of the ith option and the jth dimension, the formula is as follows:

$$f_{ij} = x_{ij} \Big/ \sqrt{\sum_{j=1}^{n} x_{ij}^2}\,, i = 1,2,\ldots,m\,; j = 1,2,\ldots,n \quad (11)$$

2) Determine the best and worst values

For all the attribute functions the best value was f_j^* and the worst value was f_j^-, that is, for attribute J=1-n, we get formulas (12) and (13)

$$f_j^* = \max \ f_{ij}\,, i = 1,2,\ldots,m \quad (12)$$

$$f_j^- = \min \ f_{ij}\,, i = 1,2,\ldots,m \quad (13)$$

Where f_j^* the positive ideal solution for the jth criteria is, f_j^- is the negative ideal solution for the jth criteria. If one associates all f_j^*, one will have the optimal combination, which gets the highest scores, the same as f_j^-.

3) Determine the weights of attributes

The weights of attribute should be calculated to express their relative importance.

4) Compute the distance of alternatives to ideal solution

This step is to calculate the distance from each alternative to the positive ideal solution and then get the sum to obtain the final value according to formula (14) and (15).

$$S_i = \sum_{j=1}^{n} w_j (f_j^* - f_{ij})/(f_j^* - f_j^-) \quad (14)$$

$$R_i = \max_j \left[w_j (f_j^* - f_{ij})/(f_j^* - f_j^-) \right] \quad (15)$$

Where S_i represents the distance rate of the ith alternative to the positive ideal solution (best combination), R_i represents the distance rate of the ith alternative to the negative ideal solution (worst combination). The excellence ranking will be based on S_i values and the worst rankings will be based on R_i values. In other words, S_i, R_i indicate L_{1i} and L_{*i} of L_p-metric respectively.

5) Calculate the VIKOR values Q_i for i=1,2, … ,m, which are defined as

$$Q_i = v\left[\frac{S_i - S^*}{S^- - S^*}\right] + (1-v)\left[\frac{R_i - R^*}{R^- - R^*}\right] \quad (16)$$

Where $S^- = \max_i S_i$, $S^* = \min_i S_i$, $R^- = \max_i R_i$, $R^* = \min_i, R_i$, and v is the weight of the strategy of "the majority of criteria'' (or ''the maximum group utility''). $[(S - S^*)/(S^- - S^*]$ represents the distance rate from the positive ideal solution of the ith alternative's achievements In other words, the majority agrees to use the rate of the ith.$[(R - R^*)/(R^- - R^*]$ represents the distance rate from the negative ideal solution of the ith alternative; this means the majority disagree with the rate of the ith alternative. Thus, when the v is larger (> 0.5), the index of Q_i will tend to majority agreement; when v is less (< 0.5), the index Q_i will indicate majority negative attitude; in general, v = 0.5, i.e. compromise attitude of evaluation experts.

6) Rank the alternatives by Q_i values

According to the Q_i values calculated by step (4), we can rank the alternatives and to make-decision.

4. A NUMERICAL APPLICATION OF PROPOSED APPROACH

The criteria for this example are taken from Shamsuzzaman et al (2003). These criteria are including: Flexibility (C_1), Cost (C_2), Risk (C_3), Production rate (C_4), and Throughput time (C_6). In addition, there are six alternatives include A_1, A_2, A_3, A_4, A_5 and A_6.In this paper, the weights of criteria are calculated by using FPM, and these calculated weight values are used as VIKOR inputs. Then, after VIKOR calculations, evaluation of the alternatives and selection of Flexible Manufacturing System is realized.

Fuzzy Prioritization Method:

After forming the decision hierarchy for flexible manufacturing system selection problem, the criteria to be used in evaluation process are assigned weights by using FPM method. Geometric means of these values are found to obtain the pairwise compassion matrix on which there is a consensus (Table 2).

Table 2. Fuzzy comparison matrix

D	C1			C2			C3			C4			C5		
	L	m	u	L	m	u	L	m	u	L	m	u	L	m	u
C1	1.00	1.00	1.00	1.00	2.33	3.00	3.00	5.00	7.00	0.20	2.47	7.00	0.33	3.44	7.00
C2	0.33	0.56	1.00	1.00	1.00	1.00	1.00	3.25	5.00	0.14	1.83	5.00	0.20	2.07	3.00
C3	0.14	0.23	0.33	0.20	0.51	1.00	1.00	1.00	1.00	0.20	1.18	3.00	0.14	0.23	0.33
C4	0.14	3.38	5.00	0.20	3.40	7.00	0.33	0.85	5.00	1.00	1.00	1.00	0.33	2.11	3.00
C5	0.14	1.16	3.00	0.33	1.89	5.00	3.00	1.00	7.00	0.33	1.22	3.00	1.00	1.00	1.00

After that we formulate the fuzzy comparison matrix as a constrained optimization problem and we solve this optimization problem using Genetic algorithm. In order to employ Genetic algorithm, we use the MATLAB toolbox. Some settings that are used: Population Size equal to 100, the number of direct transfer to the next generation (Elite count) equal to 2, crossover fraction equal to 0.8 and the stopping conditions are described as follow: transfer from 100 generation and a lack of improvement in 50 generation. The results obtained from solving optimization problem using of Genetic algorithm are presented in Table 3.

Table 3.The weight of criteria

W_1	W_2	W_3	W_4	W_5
0.280883	0.175129	0.06327	0.240667	0.240052

Then, weighted normalized matrix is formed by multiplying each value with their weights. After that we formed the Total weighted values of criteria as shown in Table 4.

Table 4. Total weighted values of main criteria

Ai - Cj	C_1	C_2	C_3	C_4	C_5
A_1	0.26	0.18	0.01	0.26	0.11
A_2	0.26	0.12	0.02	0.24	0.13
A_3	0.28	0.01	0.05	0.04	0.19
A_4	0.00	0.08	0.04	0.12	0.04
A_5	0.17	0.02	0.05	0.00	0.18
A_6	0.18	0.09	0.02	0.03	0.20

The ranking of alternatives are shown in Table 5. According to result, if the best one is needed to be selected, then the alternative A_4 must be chosen.

Table 5. Rankings of alternatives according to Qi values

	$E_i=\Sigma e_i$	$F_i=\mathrm{Max}\,(e_i)$	Q_i	Ranking
A_1	0.796574	0.278979	0.929987	5
A_2	0.845344	0.288269	1	6
A_3	0.51279	0.274223	0.675601	4
A_4	0.253205	0.127156	0	1
A_5	0.408987	0.16989	0.264161	2
A_6	0.543172	0.192014	0.446127	3
Min	0.253205	0.127156		
Max	0.845344	0.288269		

5. CONCLUSIONS

Selection of a flexible manufacturing system (FMS) is a challenging task because of the insufficient experience and data about this still-evolving technology. Further, the large investment involved makes the selection process critical. This paper illustrates an application of FPM along with VIKOR in selecting FMS. Fuzzy set theory is incorporated to overcome the vagueness in the preferences. Two steps FPM and VIKOR methodology is structured here that FPM uses VIKOR result weights as input weights. According to this methodology, A_4 are selected as the best FMS.

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