On Soft Multi Matrix Theory

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Abstract: In this paper, we define soft multi matrices as matrix representations of soft multi sets. We also define soft multi matrices operations, discuss their basic properties and show that these soft multi matrices operations are equivalent to their corresponding soft multi sets operations.
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1. Introduction

Multi sets introduced by Bruijn N.G,(1983) as sets where an element can occur more than once are useful structure arising in many areas of mathematics and computer science such as data queries.

Again, the concept of soft set initiated by D Molodtsov, (1999) for modeling uncertainty, present in real life, is roughly defined as a parameterized family of subsets of the universe. Several applications of soft sets in areas like in decision making, medical diagnosis, texture classification, data analysis and forecasting have been shown by many authors (P. K. Maji, A.R. Roy and R. Biswas, 2002; A.Khaval and B. Ahmed,2011; Mushrif, M., 2006 etc.)

Maji et al.,(2003) and(Ali et al.,2009) defined several operations and their basic properties on soft sets. Cagman and Enginoglu,(2010) defined soft matrices and their operations. Later many other authors (P.K. Maji, R.Biswas and A.Roy, 2001; F. Feng, C.Li,B. Dayvaz and M.I.Ali,2010;P.K.Maji, R. Biswas and A.Roy,2001 etc.) combined soft sets with other sets to generate hybrid structures such as fuzzy soft set, rough soft set, intuitionistic fuzzy soft set and vague soft set.

In recent times, some researchers introduced a new hybrid set called soft multi set as a generalization of Molodtsov's soft set by combining soft sets and multisets.

Alkhazaleh et al.(2011) initiated this concept of soft multiset and discussed its basic operations such as complement, union, intersection, among others.

Pinaki Majumdar(2012) redefined the notion of soft multi set using soft count function and introduced some operations on them. He also applied soft multi sets in student's evaluation process.

As a special case of Pinaki's definition of soft multi set,(Babitha and John 2013) initiated the novel concept of soft multi set as a mapping from parameter set to a subset of the power set of multi subsets of the universe and established the relationship between soft multi sets and multi-valued information systems(Babitha and John 2013) finally gave an application of soft multi set in decision making problem.

Tridiv and Dusmanta(2012) redefined the notions of complement of a soft multi set introduced by Alkhazaleh et al.(2011) and showed that the laws of exclusion, contradiction, involution and De Morgan's are valid for soft multi sets.

In this paper, an attempt is made to extend the concepts and results on soft matrices to soft multi matrices. To carry out this, we first define a soft multi matrix as a matrix representing a soft multi set. We also define soft multi matrices operations and discuss their basic properties. We finally show that the operations on soft multi sets are equivalent to their corresponding operations on soft multi matrices representing them.

2. Preliminaries

We first recall some basic notions related to soft sets multisets and soft multisets with illustrative examples.

Definition 2.1[12] (Soft set)

Let $\cup$ be an initial universal set and E be a set of parameters. Let $P(\cup)$ denote the power set of $\cup$ and $A \subseteq E$. A pair (E,A) is called a soft set over $\cup$ if and only if F is a mapping given by $F: A \rightarrow P(\cup)$.

Example 2.1

Let $\cup = \{s_1, s_2, s_3, s_4\}$ be a universal set consisting of four students and $E = \{e_1, e_2, e_3, e_4\}$ be the set of parameters under consideration.

Let $A = \{e_1, e_3, e_4\}$ where $F(e_1)=\{s_1, s_3\}$, $F(e_3)=\{s_1, s_2, s_4\}$ and $F(e_4)=\{s_4\}$. Then the soft (F, A) over $\cup$ is given by $(F,A)= \{F(e_1)=\{s_1, s_3\}, F(e_3)=\{s_1,$

s_2, s_4} and F(e_4)={s_4}} or {(e_1,{s_1, s_3}), (e_3,{s_1, s_2, s_4}) and (e_4,{s_4})} as a set of ordered pair.

Definition 2.2(Soft matrix)

Let (F, A) be a soft set over U, where U={u_1, u_2, …, u_m}, E={e_1, e_2, …, e_n} and A⊆E. Then the matrix [a_{ij}] representing (F, A) is called the soft matrix over U and is defined as

$$\left[a_{ij}\right]_{m\times n}=\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & A_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn}\end{bmatrix}$$

where $a_{ij} = \begin{cases} 1, \text{if } u_i \in F(e_j) \\ 0, \text{otherwise}\end{cases}$

Example 2.2

Let (F,A) be the soft set in Example 2.1. Then the soft matrix [a_{ij}] over U is given by

$$\left[a_{ij}\right]=\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1\end{bmatrix}$$

Definition 2.3[6] (Multiset)

Let X be a set of elements. A multiset M drawn from X is represented by a C_M:X →IN where IN represents the set of non-negative integers. For each x∈X, $C_{M(x)}$ is the multiplication of x in M. ie, the number of occurrences of x∈M. In other words, a multiset M is a collection of elements in which elements are allowed to repeat. The word "Multisets" often shortened to m.set.

If x_i appears k_1 times, x_2 appears k_2 times, ….. X_n appears k_n times in an mset M, then M is expressed as

$$M = \left\{\frac{k_1}{x_1}, \frac{k_2}{x_2}, \frac{\ldots k_n}{\ldots x_n}\right\}$$

Definition 2.4[6] (Sub mset)

Let M1 and M2 be two msets drawn from a set X. M1 is a submset of M2 denoted by M1 ⊆ M2, if $C_{M1}(x) \le C_{M2}(x)$ for all x∈X.

Definition 2.5[2] (whole submset)

A submset M_1 of M_2 is called a whole submset of M_2 if $C_{M1}(x) = C_{M2}(x)$ for all x∈M_1.

Definition 2.6[2] (Soft mset)

Let U be a universal mset and E be a set of parameters. Let PW(U) denote power whole mset of M ie the set of all whole submsets of U. Then an ordered pair (F,A) is called a soft multi set where F is a mapping given by F: A→PW(U), where A⊆ E.

Example 2.3 Let U = $\left\{\frac{k_1}{u_1}, \frac{k_2}{u_2}, \frac{k_3}{u_3}, \frac{k_4}{u_4}, \frac{k_5}{u_5}\right\}$ be a universal mset and E={e_1, e_2, e_3, e_4} be the set of parameters with respect to U.

Let A = { e_1, e_2, e_4} ⊂ E, where F(e_1)= $\left\{\frac{k_1}{u_1}, \frac{k_2}{u_2}, \frac{k_5}{u_5}\right\}$

$F(e_2) = \left\{\frac{k_1}{u_1}, \frac{k_3}{u_3}\right\}$ and F(e_4)= U.

Then the soft mset (F,A) over U is given by

(F,A) = $\left\{ F(e_1) = \left\{\frac{k_1}{u_1}, \frac{k_2}{u_2}, \frac{k_5}{u_5}\right\}, F(e_2) = \left\{\frac{k_1}{u_1}, \frac{k_3}{u_3}\right\}, F(e_4) = U \right\}$.

Definition 2.7[2]

Let (F,A) and (G,B) be two soft msets over a common universal mset U, we say that

a) (F,A) is a **soft submset** of (G,B) denoted by (F,A) $\tilde{\subseteq}$ (G,B) if (i) A⊆ B and (ii) F(e) is a submset of G(e) for all e∈A.

b) (F,A) and (G,B) are **soft equal msets** if (F,A) $\tilde{\subseteq}$ (G,B) and (G,B) $\tilde{\subseteq}$ (F,A).

Definition 2.8[2] (Complement)

The complement of a soft mset (F,A) denoted by $(F,A)^c$ is defined by $(F,A)^c = (F^c, ⅂A)$ where $F^c(⅂a) = U - F(a)$ for all ⅂a ∈ ⅂A.

Example 2.4

Consider the soft mset (F,A) in example 2.3

$(F,A)^c = \left\{ F(⅂e_1) = \left\{\frac{k_3}{u_3}, \frac{k_4}{u_5}\right\}, \left\{ F(⅂e_2) = \left\{\frac{k_2}{u_2}, \frac{u_4}{u_4}, \frac{u_5}{u_5}\right\}\right.\right.$

Definition 2.9[2] (Null soft mset)

A soft mset (F,A) over a universe U is said to be a null soft mset, denoted by Φ, if for all a∊A. F(a)= Φ.

Definition 2.10[2] (Absolute soft mset)

A soft mset (F,A) over a universe U is said to be absolute soft mset, denoted by Ã if for all a∈A. F(a)= U.

Definition 2.11[2] (Soft mset Operations)

Let (F,A) and (G,B) be soft msets over a common universe.

i) The **union** of (F,A) and (G,B) denoted by (F,A) ∪ (G,B) is a soft mset (H,C) where C=A∪B and ∀ e ∈ C

H(e)

$$= \begin{cases} F(e)\ ,\ \ if\ e \in A-B \\ G(e)\ ,\ \ if\ e \in B-A \\ F(e)\ \cup\ G(e)\ (\text{the union of msets } F(e) \text{ and } G(b)),\ e \in A\cap B \end{cases}$$

ii) The **intersection** of (F,A) and (G,B) denoted (F,A) $\tilde{\cap}$ (G,B) is a soft mset (H,C) where C=A∩B and ∀ e ∈ C

H(e) = F(e) ∩ G(e) (the intersection of msets F(e) and G(e).

iii) The **AND-operation** of (F,A) and (G,B) denoted by (F,A) AND (G,B) or (F,A) ∧(G,B) is a soft mset (H,C) where C = A×B and ∀ (a,b) ∈ C

H(a,b) = F(e) ∩ G(e) (the intersection of msets F(e) and G(e).

iv) The **OR-operation** of (F,A) and (G,B), denoted by (F,A) OR (G,B) or (F,A) ∨ (G,B) is a soft mset (H,C) where C=A×B and ∀ (a,b) ∈ C

H(a,b) = F(a) ∪ G(b) (the union of msets F(a) and G(b))

Proposition 2.1

Let (F,A) and (G,B) be two soft msets over a common universe U. Then the following hold;

i) $$\left.\begin{array}{l}(F,A)\cup(F,A) = (F,A)\\ (F,A)\cap(F,A) = (F,A)\end{array}\right\}\ldots\ldots.$$ Idempotent Laws

ii) $$\left.\begin{array}{l}(F,A)\ \cup\ \Phi\ = (F,A)\\ (F,A)\ \cap\ \tilde{A}\ = (F,A)\end{array}\right\}\ldots\ldots.$$ Identity Laws

iii) $$\left.\begin{array}{l}(F,A)\ \tilde{\cup}\ \tilde{A}\ = \tilde{A}\\ (F,A)\ \tilde{\cap}\ \Phi\ = \Phi\end{array}\right\}\ldots\ldots\ldots$$ Domination Laws

iv) $$\left.\begin{array}{l}(F,A)\ \tilde{\cup}\ (F,A)^c\ = \tilde{A}\\ (F,A)\ \tilde{\cap}\ (F,A)^c\ = \Phi\end{array}\right\}\ldots\ldots..$$ Complementation Laws

v) $\Phi^c = \tilde{A}$, $\tilde{A}^c = \Phi$………… Complementation Laws

vi) $((F,A)^c)^c = (F,A)$…………….. Double Complementation(Involution) law

vii) $$\left.\begin{array}{l}(F,A)\tilde{\cup}\ (G,B) = (G,B)\tilde{\cup}(F,A)\\ (F,A)\tilde{\cap}\ (G,B) = (G,B)\tilde{\cap}\ (F,A)\end{array}\right\}\ldots\ldots$$ Commutative Laws

viii) $$\left.\begin{array}{l}\left[(F,A)\tilde{\cup}\ (G,B)\right]^c = (F,A)^c\ \tilde{\cap}\ (G,B)^c\\ \left[(F,A)\tilde{\cap}\ (G,B)\right]^c = (F,A)^c\ \tilde{\cup}\ (G,B)^c\end{array}\right\}\ldots..$$ De Morgan's Laws

3. Soft Multi Matrices

In this section, we give the definition of msets. We also define operations on them with illustrative examples and discuss their basic properties.

Definition 3.1 (Soft multi matrix)

Let (F,A) be a soft multi set over U, where U = $\left\{\frac{k_1}{u_1}, \frac{k_2}{u_2}, \ldots\ldots, \frac{k_m}{u_m}\right\}$ and A={e_1, e_2, ……, e_m}. Then the matrix $[a_{ij}]$ representing (F,A) is called the soft multi matrix over U and is defined as

$$\{a_{ij}\} = \begin{bmatrix} a_{11} & a_{12\ldots} & a_{1n} \\ a_{21} & a_{22\ldots} & a_{2n} \\ a_{m1} & a_{m2\ldots} & a_{mn} \end{bmatrix} \quad \text{where } a_{ij} = \begin{cases} 1, \text{if } u_i \in F(e_j) \\ 0, \text{otherwise} \end{cases}$$

Example 3.1

Consider the soft mset (F,A) in Example 2.3. The matrix $[a_{ij}]$, representing (F,A) is given by

$$[a_{ij}] = \begin{bmatrix} k_1 & k_2 & 0 & k_1 \\ k_2 & 0 & 0 & k_2 \\ 0 & k_3 & 0 & k_3 \\ 0 & 0 & 0 & k_4 \\ k_5 & 0 & 0 & k_5 \end{bmatrix}$$

Definition 3.2 (Types of soft multi matrices)

Let $SMM(\cup)_{m\times n}$ denote the set of all m×n soft multi matrices over ∪.

Let $[a_{ij}]$ and $[b_i] \in SMM(\cup)$. Then

i) $[a_{ij}]$ is called a **soft multi sub** matrix of $[b_{ij}]$ denoted $[a_{ij}] \tilde{\subseteq} [b_{ij}]$ if $a_{ij} \leq b_{ij}$ for all $_i$ and $_j$.

ii) $[a_{ij}]$ and $[b_{ij}]$ are said to be **soft equal multi** matrices denoted $[a_{ij}] = [b_{ij}]$, if $a_{ij} = b_{ij}$ for all $_i$ and $_j$.

iii) $[a_{ij}]$ is called a **zero soft multi** matrix denoted [0] if $a_{ij} = 0$ for all $_i$ and $_j$.

iv) $[a_{ij}]$ is called a **universal soft multi** matrix denoted [k], if $a_{ij} = k_i$ for all $_j$.

Definition 3.3 (Operations on soft multi matrices)

Let $[a_{ij}]$ and $[b_{ij}] \in SMM(\cup)$. Then

i) The **union** of $[a_{ij}]$ and $[b_{ij}]$ denoted $[a_{ij}] \tilde{\cup} [b_{ij}]$ is the soft multi matrix $[c_{ij}] \in SMM(\cup)$ such that $c_{ij} = \max\{a_{ij}, b_{ij}\}$, for all $_i$ and $_j$.

ii) The **intersection** of $[a_{ij}]$ and $[b_{ij}]$ denoted $[a_{ij}] \tilde{\cap} [b_{ij}]$ is the soft multi matrix $[c_{ij}] \in SMM_{m\times n}$ such that $c_{ij} = \min\{a_{ij}, b_{ij}\}$, for all $_i$ and $_j$.

iii) The **complement** of $[a_{ij}]$ denoted $[a_{ij}]^0$ is the soft multi matrix defined by

$[a_{ij}]^0 = [k] - [a_{ij}]$ for all $_i$ and $_j$.

iv) The **AND-operation** of $[a_{ij}]$ and $[b_{ij}]$ denoted $[a_{ij}] \wedge [b_{ij}]$ is the soft multi matrix $[c_{ip}]_{m\times n^2}$ where

$c_{i1} = \min\{a_{i1}, b_{i1}\}$, $c_{i2} = \min\{a_{i1}, b_{i2}\}\ldots$ $c_{in^2} = \min\{a_{in}, b_{in}\}$ i= 1, 2, 3,…. m

v) The **OR-operation** $[a_{ij}]$ and $[b_{ij}]$ denoted $[a_{ij}] \vee [b_{ij}]$ is the soft multi matrix $[c_{ip}]_{m\times n^2}$ where

$c_{i1} = \max\{a_{i1}, b_{i1}\}$, $c_{i2} = \min\{a_{i1}, b_{i2}\}\ldots$ $c_{in^2} = \max\{a_{in}, b_{in}\}$ i= 1, 2, 3,…. m

Example 3.2

Let $\cup = \left\{\frac{k_1}{u_1}, \frac{k_2}{u_2}, \frac{k_3}{u_3}, \frac{k_4}{u_4}\right\}$ be a universal mset and E = {e_1, e_2, e_3, e_4} be the set of parameters and A={e_1, e_2, e_3} ⊂ E. suppose (F,A) is a soft mset over ∪ given by

$$(F,A) = \left\{ \left(e_1, \left\{\frac{k_1}{u_1}, \frac{k_2}{u_2}\right\}\right), (e_2, \cup), \left(e_3, \left\{\frac{k_1}{u_1}, \frac{k_3}{u_3}, \frac{k_4}{u_4}\right\}\right) \right\}$$

Then the soft multi matrix $[a_{ij}]$ representing (F,A) is given by

$$[a_{ij}] = \begin{bmatrix} k_1 & k_1 & k_1 & 0 \\ k_2 & k_2 & 0 & 0 \\ 0 & k_3 & k_3 & 0 \\ 0 & k_4 & k_4 & 0 \end{bmatrix}$$

Suppose B = {e_1, e_2} and (G,B) is a soft mset given by $(G,B) = \left\{\left(e_1, \left\{\frac{k_1}{u_1}, \frac{k_2}{u_2}\right\}\right), \left(e_2, \left\{\frac{k_1}{u_1}, \frac{k_3}{u_3}, \frac{k_4}{u_4}\right\}\right)\right\}$. Then the soft multi matrix $[b_{ij}]$ representing (G,B) is given by

$$[b_{ij}] = \begin{bmatrix} k_1 & k_1 & 0 & 0 \\ k_2 & 0 & 0 & 0 \\ 0 & k_3 & 0 & 0 \\ 0 & k_4 & 0 & 0 \end{bmatrix}$$

Thus we have

i) $[b_{ij}] \tilde{\subseteq} [a_{ij}]$ since $b_{ij} \leq a_{ij}$ for all i andj

ii) $$[a_{ij}] \tilde{\cup} [b_{ij}] = \begin{bmatrix} k_1 & k_1 & k_1 & 0 \\ k_2 & k_2 & 0 & 0 \\ 0 & k_3 & k_3 & 0 \\ 0 & k_4 & k_4 & 0 \end{bmatrix}$$

$$[a_{ij}]\ \tilde{\cap}\ [b_{ij}] = \begin{bmatrix} k_1 & k_1 & 0 & 0 \\ k_2 & 0 & 0 & 0 \\ 0 & k_3 & 0 & 0 \\ 0 & k_4 & 0 & 0 \end{bmatrix}$$

iii) if (H,E) is a soft mset over ∪ such that H(e) = ∪ for all e∈E, then the soft multi matrix of (H,E) is a universal soft multi matrix [K] given by

$$[\mathrm{K}] = \begin{bmatrix} k_1 & k_1 & k_1 & k_1 \\ k_2 & k_2 & k_2 & k_2 \\ k_3 & k_3 & k_3 & k_3 \\ k_4 & k_4 & k_4 & k_4 \end{bmatrix}$$

iv) if (M,A) is a soft mset over ∪ such that M(e)=$\boldsymbol{\phi}$, then the soft multi matrix of (M,A) is a zero or null soft multi matrix [0] given by

$$[0] = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

v) $[a_{ij}]^0 = [K] - [a_{ij}] =$

$$\begin{bmatrix} 0 & 0 & 0 & k_1 \\ 0 & 0 & k_2 & k_2 \\ k_3 & 0 & 0 & k_3 \\ k_4 & 0 & 0 & k_4 \end{bmatrix}$$

vi) $[a_{ij}] \wedge [b_{ij}] =$

$$\begin{bmatrix} k_1 & k_1 & 0 & 0 & k_1 & k_1 & 0 & 0 & k_1 & k_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ k_2 & 0 & 0 & 0 & k_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & k_3 & 0 & 0 & 0 & k_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & k_4 & 0 & 0 & 0 & k_4 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

vii) $$\left[a_{ij}\right] \vee \left[b_{ij}\right] = \begin{bmatrix} k_1 & k_1 & k_1 & k_1 & k_1 & k_1 & k_1 & k_1 & k_1 & k_1 & k_1 & k_1 & k_1 & k_1 & 0 & 0 \\ k_2 & k_2 & k_2 & k_2 & k_2 & k_2 & k_2 & k_2 & k_2 & 0 & 0 & 0 & k_2 & 0 & 0 & 0 \\ 0 & k_3 & 0 & 0 & k_3 & k_3 & k_3 & k_3 & k_3 & k_3 & k_3 & k_3 & 0 & k_3 & 0 & 0 \\ 0 & k_4 & 0 & 0 & k_4 & k_4 & k_4 & k_4 & k_4 & k_4 & k_4 & k_4 & 0 & k_4 & 0 & 0 \end{bmatrix}$$

Proposition 3.2

Let $\left[a_{ij}\right]$, $\left[b_{ij}\right]$, $\left[c_{ij}\right]$ ∈ SMM(∪)$_{m\times n}$. Then the following hold

i) $\left[a_{ij}\right] \tilde{\cup} \left[a_{ij}\right] = \left[a_{ij}\right]$; $\left[a_{ij}\right] \tilde{\cap} \left[a_{ij}\right] = \left[a_{ij}\right]$............ Idempotent laws

ii) $\left[a_{ij}\right] \tilde{\cup} [0] = \left[a_{ij}\right]$; $\left[a_{ij}\right] \tilde{\cap} [K] = \left[a_{ij}\right]$............ Identity laws

iii) $\left[a_{ij}\right] \tilde{\cup} [K] = [K]$; $\left[a_{ij}\right] \tilde{\cap} [0] = [0]$............ Domination laws

iv) $\left[a_{ij}\right] \subseteq \left[b_{ij}\right]$ *and* $\left[b_{ij}\right] \subseteq \left[c_{ij}\right] = \left[a_{ij}\right] \subseteq \left[c_{ij}\right]$............ Inclusion law

v) $\left(\left[a_{ij}\right]^0\right)^0 = \left[a_{ij}\right]$........... Double complementation or involution law

vi) $[K]^0 = [0]$; $[0]^0 = [K]$ .... Complementation laws

vii) $\left[a_{ij}\right] \tilde{\cup} \left[a_{ij}\right]^0 = [K]$; $\left[a_{ij}\right] \tilde{\cap} \left[a_{ij}\right]^0 = [0]$............. Complementation laws

viii) $[a_{ij}] \tilde{\cup} [b_{ij}] = [b_{ij}] \cup [a_{ij}]; [a_{ij}] \tilde{\cap} [b_{ij}] = [b_{ij}] \tilde{\cap} [a_{ij}]$ .... Commutative laws

ix) $\left.\begin{array}{l} [a_{ij}] \tilde{\cup} [b_{ij}] \tilde{\cup} [c_{ij}] = ([a_{ij}] \tilde{\cup} [b_{ij}]) \tilde{\cup} [c_{ij}] \\ [a_{ij}] \tilde{\cap} [b_{ij}] \tilde{\cap} [c_{ij}] = ([a_{ij}] \tilde{\cap} [b_{ij}] \tilde{\cap} [c_{ij}]) \end{array}\right\}$ ...... Associative laws

x) $\left.\begin{array}{l} ([a_{ij}] \tilde{\cup} [b_{ij}])^0 = [a_{ij}]^0 \tilde{\cap} [b_{ij}]^0 \\ ([a_{ij}] \tilde{\cap} [b_{ij}])^0 = [a_{ij}]^0 \tilde{\cup} [b_{ij}]^0 \end{array}\right\}$ .... De Morgan's laws

xi) $\left.\begin{array}{l} [a_{ij}] \tilde{\cup} [b_{ij}] \tilde{\cap} [c_{ij}] = ([a_{ij}] \tilde{\cup} [b_{ij}] \tilde{\cap} ([a_{ij}] \tilde{\cup} [c_{ij}]) \\ [a_{ij}] \tilde{\cap} [b_{ij}] \tilde{\cup} [c_{ij}] = ([a_{ij}] \tilde{\cap} [b_{ij}] \tilde{\cup} ([a_{ij}] \tilde{\cap} [c_{ij}]) \end{array}\right\}$ .Distributive laws

xii) $\left.\begin{array}{l} ([a_{ij}] \vee [b_{ij}])^0 = [a_{ij}]^0 \wedge [b_{ij}]^0 \\ ([a_{ij}] \wedge [b_{ij}])^0 = [a_{ij}]^0 \vee [b_{ij}]^0 \end{array}\right\}$ .... De Morgan's type of laws

Proof:

The proofs follow from definitions. Let us for example prove ix(b), x(a), xi(b) and also verify xii(a) with an example.

ix(b) For each i and j

$$[a_{ij}] \tilde{\cap} ([b_{ij}] \tilde{\cap} [c_{ij}]) = [\{min\{a_{ij}, min(b_{ij}, c_{ij})\}]$$

$$= [min\{min(a_{ij}, b_{ij}), c_{ij}\}]$$

$$= ([a_{ij}] \tilde{\cap} [b_{ij}]) \tilde{\cap} [c_{ij}].$$

x(a) $([a_{ij}] \tilde{\cup} [b_{ij}])^0 = [\max\{a_{ij}, b_{ij}\}]^0$

$$= [K - \max\{a_{ij}, b_{ij}\}]$$

$$= [\min\{K - a_{ij}, K - b_{ij}\}]$$

$$= [a_{ij}]^0 \tilde{\cap} [b_{ij}]^0.$$

xi(b) $[a_{ij}] \tilde{\cap} ([b_{ij}] \tilde{\cup} [c_{ij}])$

$$= [\{\min\{a_{ij}, \max(b_{ij}, c_{ij})\}]$$

$$= [\max\{\min\{a_{ij}, b_{ij}\}, \min\{a_{ij}, c_{ij}\}]$$

$$= ([a_{ij}] \tilde{\cap} [b_{ij}]) \tilde{\cup} ([a_{ij}] \tilde{\cap} [c_{ij}])$$

Example 3.3 (Verifying Xii(a))

Let $[a_{ij}]$ and $[b_{ij}] \in SMM(U)_{4\times4}$ given by

$$\left[a_{ij}\right]=\begin{bmatrix} 0 & k_1 & 0 & k_1 \\ 0 & k_2 & k_2 & k_2 \\ 0 & k_3 & k_3 & 0 \\ 0 & k_4 & 0 & 0 \end{bmatrix} \; and \; \left[b_{ij}\right]=\begin{bmatrix} 0 & 0 & k_1 & k_1 \\ k_2 & 0 & 0 & k_2 \\ 0 & 0 & k_3 & k_3 \\ 0 & 0 & k_4 & k_4 \end{bmatrix}$$

Then

$$([a_{ij}] \wedge [b_{ij}]) = \begin{bmatrix} 0&0&0&0&0&0&k_1&k_1&0&0&0&0&0&0&k_1&k_1 \\ 0&0&0&0&k_2&0&0&k_2&k_2&0&0&k_2&k_2&0&0&k_2 \\ 0&0&0&0&0&0&k_3&k_3&0&0&k_3&k_3&0&0&0&0 \\ 0&0&0&0&0&0&k_4&k_4&0&0&0&0&0&0&0&0 \end{bmatrix}$$

$$\left(\left[a_{ij}\right] \wedge \left[b_{ij}\right]\right)^0 = \begin{bmatrix} k_1&k_1&k_1&k_1&k_1&k_1&0&0&k_1&k_1&k_1&k_1&k_1&k_1&0&0 \\ k_2&k_2&k_2&k_2&0&k_2&k_2&0&0&k_2&k_2&0&0&k_2&k_2&0 \\ k_3&k_3&k_3&k_3&k_3&k_3&0&0&k_3&k_3&0&0&k_3&k_3&k_3&k_3 \\ k_4&k_4&k_4&k_4&k_4&k_4&0&0&k_4&k_4&k_4&k_4&k_4&k_4&k_4&k_4 \end{bmatrix}$$

$$\left[a_{ij}\right] \vee \left[b_{ij}\right]^0 = \begin{bmatrix} k_1&0&k_1&0 \\ k_2&0&0&0 \\ k_3&0&0&k_3 \\ k_4&0&k_4&k_4 \end{bmatrix} \vee \begin{bmatrix} k_1&k_1&0&0 \\ 0&k_2&k_2&0 \\ k_3&k_3&0&0 \\ k_4&k_4&0&0 \end{bmatrix}$$

$$= \begin{bmatrix} k_1&k_1&k_1&k_1&k_1&k_1&0&0&k_1&k_1&k_1&k_1&k_1&k_1&0&0 \\ k_2&k_2&k_2&k_2&0&k_2&k_2&0&0&k_2&k_2&0&0&k_2&k_2&0 \\ k_3&k_3&k_3&k_3&k_3&k_3&0&0&k_3&k_3&0&0&k_3&k_3&k_3&k_3 \\ k_4&k_4&k_4&k_4&k_4&k_4&0&0&k_4&k_4&k_4&k_4&k_4&k_4&k_4&k_4 \end{bmatrix}$$

Thus $([a_{ij}] \vee [b_{ij}])^0 = [a_{ij}]^0 \wedge [b_{ij}]^0$.

Similarly xii(b) can be verified.

Comparing Operations on Soft Multi sets and Soft Multi matrix

It is shown in this section that operations on soft multi-sets are equivalent to operations onthe corresponding soft multi matrices representing them.

Consider Example (3.2) where,

$U = \left\{\frac{k_1}{u_1}, \frac{k_2}{u_2}, \frac{k_3}{u_3}, \frac{k_4}{u_4}\right\}$, A $=\{e_1, e_2, e_3\}$, B $= \{e_1, e_2\}$

(F,A) $= \{(e_1, \left\{\frac{k_1}{u_1}, \frac{k_2}{u_2}\right\}), (e_2, \left\{\frac{k_1}{u_1}, \frac{k_3}{u_3}, \frac{k_4}{u_4}\right\})\}$. Then

i) (F,A) $\tilde{\cup}$ (G,B) = (H,C) $= \{(e_1, \left\{\frac{k_1}{u_1}, \frac{k_2}{u_2}\right\}), (e_2, U), (e_3, \left\{\frac{k_1}{u_1}, \frac{k_3}{u_3}, \frac{k_4}{u_4}\right\})\}$. Whose soft multi matrix $[c_{ij}]$ is given by

$$\left[c_{ij}\right] = \begin{bmatrix} k_1 & k_1 & k_1 \\ k_2 & k_2 & 0 \\ 0 & k_3 & k_3 \\ 0 & k_4 & k_4 \end{bmatrix} = [a_{ij}] \,\tilde{\cup}\, [b_{ij}]$$

ii) $(F,A) \,\tilde{\cap}\, (G,B) = [H,D] = \{ (e_1, \left\{\frac{k_1}{u_1}, \frac{k_2}{u_2}\right\}), (e_2, \left\{\frac{k_1}{u_1}, \frac{k_3}{u_3}, \frac{k_4}{u_4}\right\}) \}$ whose soft multi matrix $[d_{ij}]$ is given by

$$[d_{ij}] = \begin{bmatrix} k_1 & k_1 & 0 \\ k_2 & 0 & 0 \\ 0 & k_3 & 0 \\ 0 & k_4 & 0 \end{bmatrix} = [a_{ij}] \,\tilde{\cap}\, [b_{ij}]$$

iii) $(F,A)^c = (F^c, \neg A) = \{(e_1, \left\{\frac{k_3}{u_3}, \frac{k_4}{u_4}\right\}), (e_3, \left\{\frac{k_2}{u_2}\right\}) \}$ whose soft multi matrix is given by

$$[e_{ij}] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & k_2 \\ k_3 & 0 & 0 \\ k_4 & 0 & 0 \end{bmatrix} = [a_{ij}]^0$$

iv) $(F,A) \wedge (G,B) = \Big\{ \left((e_1, e_1), \left\{\frac{k_1}{u_1}, \frac{k_2}{u_2}\right\}\right), \left((e_1, e_2), \left\{\frac{k_1}{u_1}\right\}\right), \left((e_2, e_1), \left\{\frac{k_1}{u_1}, \frac{k_2}{u_2}\right\}\right),$ $\left((e_2, e_2), \left\{\frac{k_1}{u_1}, \frac{k_3}{u_3}, \frac{k_4}{u_4}\right\}\right), \left((e_3, e_1), \left\{\frac{k_1}{u_1}\right\}\right), \left((e_3, e_2), \left\{\frac{k_1}{u_1}, \frac{k_3}{u_3}, \frac{k_4}{u_4}\right\}\right) \Big\}$

whose soft multi matrix $[f_{ip}]$ is given by

$$[f_{ip}]_{4\times 9} = \begin{bmatrix} k_1 & k_1 & 0 & k_1 & k_1 & 0 & k_1 & k_1 & 0 \\ k_2 & 0 & 0 & k_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & k_3 & 0 & 0 & k_3 & 0 \\ 0 & 0 & 0 & 0 & k_4 & 0 & 0 & k_4 & 0 \end{bmatrix} = [a_{ij}] \wedge [b_{ij}]$$

v) $(F,A) \vee (G,B) = \{((e_1, e_1), \left\{\frac{k_1}{u_1}, \frac{k_2}{u_2}\right\}), ((e_1, e_2), U),$

$((e_1, e_3), \left\{\frac{k_1}{u_1}, \frac{k_2}{u_2}\right\}), ((e_2, e_1), U),$

$((e_2, e_2), U), ((e_2, e_3), U),$

$((e_3, e_1), U), ((e_3, e_2), \left\{\frac{k_1}{u_1}, \frac{k_3}{u_3}, \frac{k_4}{u_4}\right\}),$

$$((e_3, e_3), \left\{\frac{k_1}{u_1}, \frac{k_3}{u_3}, \frac{k_4}{u_4}\right\})\}$$

whose soft multi matrix $[g_{ip}]$ is given by

$$[g_{ip}]_{4\times9} = \begin{bmatrix} k_1 & k_1 & k_1 & k_1 & k_1 & k_1 & k_1 & k_1 & k_1 \\ k_2 & k_2 & k_2 & k_2 & k_2 & k_2 & k_2 & 0 & 0 \\ 0 & k_3 & 0 & k_3 & k_3 & k_3 & k_3 & k_3 & k_3 \\ 0 & k_4 & 0 & k_4 & k_4 & k_4 & k_4 & k_4 & k_4 \end{bmatrix} = [a_{ij}] \vee [b_{ij}]_{\dots}$$

Conclusion

Soft multi matrices are defined as representations of soft multisets as matrices. Operations such as union, intersection, AND-operation, OR-operation among others are defined on soft multi matrices and their basic properties are discussed. It is shown that these operations are equivalent to their corresponding operations on soft multi sets they are representing. This work can be extended to multi soft set theory.

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