

## Double-layer Optical Fiber Coating Using Viscoelastic Phan Thien Tanner Fluid

Zeeshan<sup>1,2</sup>, Rehan Ali Shah<sup>3</sup>, Saeed Islam<sup>1</sup>, A. M. Siddique<sup>4</sup>

Department of Mathematics, Abdul Wali Khan University, Mardan, KPK, Pakistan

Sarhad University of Science and Information Technology, Peshawar, KPK, 92000, Pakistan

Department of Basic Sciences and Islamiat, University of Engineering and Technology, Peshawar, KPK, Pakistan

Department of Mathematics, Pennsylvania State University, York Campus, 1031 Edgecombe Avenue, York, PA

17403, [USAzeeshansuit@gmail.com](mailto:USAzeeshansuit@gmail.com), [mmrehan79@yahoo.com](mailto:mmrehan79@yahoo.com)

**Abstract:** In mass manufacturing of optical fibers, the wet-on-wet polymer resin coating is an efficient process for applying double layer coating on the glass fiber. This paper presents an analytical study on the behavior of an incompressible viscoelastic PTT fluid in the double layer coating liquid flow inside a secondary coating die of the optical fiber coating applicator. Based the approximations of fully developed laminar flow and the coating liquid flow of two immiscible resin layers is modeled for the simplified geometry of capillary annulus, where the surface of glass fiber moves at high fiber drawing speed. Exact analytical expression for the axial velocity, shear stress, volume flux and temperature distribution are obtained for both the layers separately. Thickness of coated fiber optics is also obtained.

[Zeeshan, Rehan Ali Shah, Saeed Islam, A. M. Siddique. **Double-layer Optical Fiber Coating Using Viscoelastic Phan Thien Tanner Fluid**. *N Y Sci J* 2013;6(10):66-73]. (ISSN: 1554-0200). <http://www.sciencepub.net/newyork>. 12

**Keywords:** PTT fluid, Optical fiber, Theoretical analysis, Wet-on-Wet coating, Two-layer coating flow, pressure gradient is zero

### 1: Introduction

Since the modern concept of fiber optics was developed in the 1960s, remarkable advance have been achieved in the optical fiber manufacturing technology. The consequences led to high volume production of optical fibers at a low cost and the extensive utilities of optical fibers in many areas of communication applications. An optical fiber consists of high purity Silica glass fiber, which carries the information in the form light wave signal, and the protective polymer coatings on the glass fiber. As depicted in Fig. 1, the manufacturing of optical fibers is a series of automated inline process such as the drawing of glass fiber from a softened Silica preform in draw furnace, the coating of freshly drawn glass fiber in helium injected Coating System, and the double layer coating of polymers on glass fibers. Then, the optical fiber manufacturing becomes complete as the liquid fiber coatings are cured by Ultra violet (UV) Lamps. The coatings are necessary to provide mechanical protection and to prevent the ingress of moisture into microscopic flaws on fiber surface. The optical fibers today in general are characterized by a double-layer coating structure: an inner layer (called primary layer) made of soft coating material and an outer layer (called secondary layer) made of hard coating material. The role of the Primary layer is to minimize attenuation due to micro bending, while the secondary layer protects the primary coating against mechanical damage. The widespread industrial success of optical fibers as a

practical alternative to copper wiring could be attributed to these UV-curable Coatings. There are two types of double layer polymer resin coating for the optical fibers which are being continuously drawn at high speed: wet-on-dry and wet-on-wet techniques. In wet-on-dry coating process, glass fiber passes through two separable coating applicators with UV curing immediately after having each coating layer. In contrast, in wet-on-wet process, double layer liquid coatings are applied in one coating applicator and both coating layers are cured simultaneously by two different wavelength ranges of UV light system. Due to its efficiency, the wet-on-wet coating technique becomes popular recently in fiber optics industry.

Early stages of fiber coating used the open cup applicator without any pressurization. However, as the drawing speed of optical fiber increased, this coating system revealed several critical shortcoming such as air bubble entrainment, meniscus collapse of coating liquid and difficulties in controlling the coating thickness [1, 3].

There have been serious research endeavors in theoretical analysis of fiber coating process in the applicators. Kimura [4] and Sakaguchi et al. [5] developed a one-dimensional viscous flow analysis of fiber coating in a pressurized coating die. Recently, Cheng et al. [6] also studied a similar problem in unpressurized and pressurized coating applicators and showed good agreement between the prediction and the experimental results of coating

thickness. However, all these work based on the single-layer coating on a glass fiber.

There have been a series of investigations on multilayer liquid coating. Tiu et al. [7, 8] conducted the extensive studies on the wire and the cable coatings including the multilayer coating. Kyoungjin et al. [9] use double-layer coating flow using power law model. Papanastasiou et al. [9] examined the planner multilayer coatings using the lubrication approximation. However, specifically for the optical fiber manufacturing, there have been few studies on the double-layer coating. Here we consider wet-on-wet coating technique for optical fiber in two layers using Non-Newtonian viscoelastic PTT fluid of different viscosity and take the pressure gradient equal to zero. The effects of different parameters on the velocity, shear stress, volume flow rate and on the thickness of coated fiber optics are also discussed.

**2: Problem statement**

The Wet-On-Wet (WOW)-type double-coating system is illustrated in Figure 1. The bare glass fiber of radius  $R_1$  kept at temperature  $\Theta_0$  is fed, with a constant velocity  $V$ , into primary coating applicator which is filled with an incompressible viscoelastic PTT fluid of constant density. Subsequently, the glass fiber with the uncured primary coating enters the secondary coating applicators kept at temperature  $\Theta_i$ . Through the secondary costing die of radius  $R_2$  and length  $L$ , the fiber leaves the coating system with two coated layers in liquid phase, as shown in the Figure. 2. Then, the coated resins are cured by the UV lamp system. For mathematical modeling the Co-ordinate system is selected at the centre of the die. In which z-axis is taken at the centre of the die and r-axis is perpendicular to it.

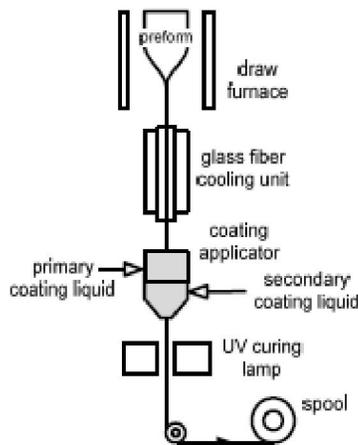


Figure 1: Double layer optical fiber coating in wet-on-wet coating process

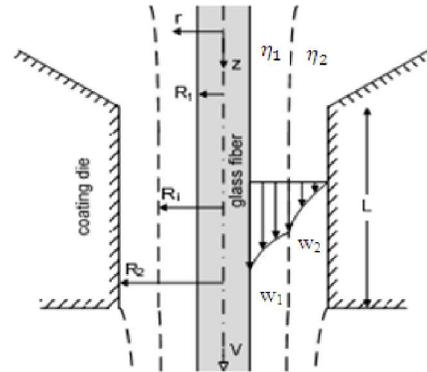


Figure 2: PTT flow model of double-layer coating in the secondary coating die

**3: Formulation of the problem**

Assuming that the flow is steady, laminar, axisymmetric and neglecting the entrance and exit effects. The fluid velocity and temperature are consider as,

$$\mathbf{u} = (0, 0, w(r)), \mathbf{S} = \mathbf{S}(r), \text{ and } \Theta = \Theta(r). \quad (1.1)$$

The model adopted here to illustrate the viscoelastic behavior of the PTT model which may be expressed as

$$f(tr\mathbf{S})\mathbf{S} + \lambda\dot{\mathbf{S}} = \eta\mathbf{A}_1. \quad (1.2)$$

In which  $\eta$  is the constant viscosity coefficient of the fluid,  $\lambda$  the relaxation time,  $tr\mathbf{S}$  is the trace of the extra stress tensor  $\mathbf{S}$  and  $\mathbf{A}_1$  is the deformation rate tensor given by

$$\mathbf{A}_1 = \mathbf{L}^T + \mathbf{L}, \quad (1.3)$$

where superscript  $T$  stand for the transpose of matrix.

The upper contra-variant convected derivative  $\dot{\mathbf{S}}$  in Eq. (1.2) is defined as

$$\dot{\mathbf{S}} = \frac{D\mathbf{S}}{Dt} - [(\nabla\mathbf{u})^T\mathbf{S} + \mathbf{S}(\nabla\mathbf{u})]. \quad (1.4)$$

The function  $f$  is given by

$$f(tr\mathbf{S}) = 1 + \frac{\epsilon\lambda}{\eta}(tr\mathbf{S}). \quad (1.5)$$

In Eq. (1.5),  $f(tr\mathbf{S})$  is the stress function in which  $\epsilon$  is related to the elongation behavior of the fluid. For  $\epsilon=0$ , the model reduces to the well-known Maxwell model and for  $\lambda=0$ , the model reduce to Newtonian one.

The fundamental equations governing the flow of an incompressible fluid with thermal effects are the continuity equation, equation of motion and the energy equation which are given below, respectively.

$$div\mathbf{u} = 0, \quad (1.6)$$

$$\rho \frac{D\mathbf{u}}{Dt} = div\mathbf{T}, \quad (1.7)$$

$$\rho c_p \frac{D\theta}{Dt} = k\nabla^2\theta + \Phi, \quad (1.8)$$

Where  $\frac{D}{Dt}$  denotes the substantive acceleration, consist of the local derivative  $\frac{\partial}{\partial t}$  and the convective derivative  $\mathbf{u} \cdot \nabla$ , i.e.,  $\frac{D}{Dt} = \frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla$ .

The Cauchy stress tensor  $\mathbf{T}$  is defined as

$$\mathbf{T} = -p\mathbf{I} + \mathbf{S}. \tag{1.9}$$

Using Eq. (1.1), the continuity Eq. (1.6) is identically satisfied and from Eq. (1.2)-(1.5) and (1.7), we arrive at

$$\frac{\partial p}{\partial r} = 0, \tag{1.10}$$

$$\frac{\partial p}{\partial \theta} = 0, \tag{1.11}$$

$$\frac{\partial p}{\partial z} = \frac{1}{r} \frac{d}{dr} (r\mathbf{S}_{rz}), \tag{1.12}$$

$$k \left( \frac{d^2}{dr^2} + \frac{1}{r} \frac{d}{dr} \right) \Theta + \mathbf{S}_{rz} \frac{dw}{dr} = 0, \tag{1.13}$$

$$f(tr\mathbf{S})\mathbf{S}_{zz} = 2\lambda\mathbf{S}_{rz} \frac{dw}{dr}, \tag{1.14}$$

$$f(tr\mathbf{S})\mathbf{S}_{rz} = \eta \frac{dw}{dr}, \tag{1.15}$$

$$\Phi = \mathbf{S}_{rz} \frac{dw}{dr}. \tag{1.16}$$

From Eq. (1.10) and (1.11), it is concluded that  $p$  is a function of  $z$  only. Eq. (1.12) represents that the flow due to pressure gradient. After leaving the die, there is only drag flow. Hence we consider

$$\frac{d}{dr} (r\mathbf{S}_{rz}) = 0. \tag{1.17}$$

Integrating Eq. (1.17) with respect to  $r$ , we get

$$\mathbf{S}_{rz} = \frac{c_1}{r}. \tag{1.18}$$

Where  $c_1$  is an arbitrary constant of integration.

Using Eq. (1.18) in Eq. (1.15), we get

$$f(tr\mathbf{S}) = \frac{\eta \frac{dw}{dr}}{\frac{c_1}{r}}. \tag{1.19}$$

Combining Eq. (1.14), (1.15) and (1.16), we obtain explicit expression for the normal stress component  $\mathbf{S}_{zz}$  as

$$\mathbf{S}_{zz} = 2 \frac{\lambda}{\eta} \left( \frac{c_1}{r} \right)^2. \tag{1.20}$$

According to equation Eq. (1.19) and definition of  $f(tr\mathbf{S})$  given in Eq. (1.5), we have

$$\eta \frac{dw}{dr} = \left( 1 + \varepsilon \frac{\lambda}{\eta} \mathbf{S}_{zz} \right) \left( \frac{c_1}{r} \right). \tag{1.21}$$

Inserting Eq. (1.20) in Eq. (1.21), we obtain an analytical expression for the axial velocity gradient

$$\frac{dw_1}{dr} = \frac{1}{\eta_1} \left( \frac{c_1}{r} \right) + 2\varepsilon_1 \frac{\lambda_1^2}{\eta_1^3} \left( \frac{c_1}{r} \right)^3, \tag{1.22}$$

$$\frac{dw_2}{dr} = \frac{1}{\eta_2} \left( \frac{c_2}{r} \right) + 2\varepsilon_2 \frac{\lambda_2^2}{\eta_2^3} \left( \frac{c_2}{r} \right)^3. \tag{1.23}$$

Where  $r$  is the radial coordinate,  $w$  the axial velocity component, and  $\eta$  the viscosity of the coating resin. The subscript 1 and 2 represent the primary and secondary coating resins, respectfully.

At the surface of bare glass fiber and die wall, the boundary conditions are obviously given as

$$w_1 = V \quad \text{at } r = R_1, \tag{1.24}$$

$$w_2 = 0 \quad \text{at } r = R_2. \tag{1.25}$$

At the interface between two coating resins, the continuity should be satisfied both in the velocity and shear stress, i.e.

$$w_1 = w_2, \tag{1.26}$$

and

$$\eta_1 \mathbf{S}_{rz_1} = \eta_2 \mathbf{S}_{rz_2} \quad \text{at } r = R_i, \tag{1.27}$$

where  $R_i$  is the radial location at the liquid-liquid interface between two coating resins.

Introduce the following dimensionless parameters

$$r^* = \frac{r}{R_1}, \quad w_1^* = \frac{w_1}{V}, \quad \Theta^* = \frac{\Theta - \Theta_1}{\Theta_2 - \Theta_1}, \quad c_1^* = \frac{c_1}{\eta_1 V},$$

$$\Lambda_1 = \frac{2\varepsilon_1 \lambda_1^2}{\eta_1^2 R_1^2 V}, \quad \Lambda_3 = \frac{vc_1 R_1^2}{k(\Theta_2 - \Theta_1)}, \quad \Omega^* = \frac{r}{R_i}, \tag{1.28}$$

$$r^* = \frac{r}{R_2}, \quad w_2^* = \frac{w_2}{V}, \quad c_2^* = \frac{c_2}{\eta_2 V},$$

$$\Lambda_2 = \frac{2\varepsilon_2 \lambda_2^2}{\eta_2^2 R_2^2 V}, \quad \Lambda_4 = \frac{vc_1 R_2^2}{k(\Theta_2 - \Theta_1)}, \quad \frac{R_2}{R_1} = \delta > 1, \tag{1.29}$$

Using these new variables Eq. (1.22) and (1.23) after dropping the asterisks takes the following form

$$\frac{dw_1}{dr} = \frac{c_1}{r} + \frac{\Lambda_1}{r^3} c_1^3, \tag{1.30}$$

$$\frac{dw_2}{dr} = \frac{c_2}{r} + \frac{\Lambda_2}{r^3} c_2^3. \tag{1.31}$$

The boundary conditions given by Eq. (1.24) – (1.27) becomes

$$w_1 = V \quad \text{at } r = 1, \tag{1.32}$$

$$w_2 = 0 \quad \text{at } r = \delta, \tag{1.33}$$

and

$$w_1 = w_2, \tag{1.34}$$

$$\eta_1 \mathbf{S}_{rz_1} = \eta_2 \mathbf{S}_{rz_2} \quad \text{at } r = \Omega, \tag{1.35}$$

#### 4: Solution of the problem

To obtain the solution for velocity field, we integrate Eq. (1.30) and (1.31) with respect to  $r$  and after considerable simplification, we find that

$$w_1 = c_1 \ln r - \frac{\Lambda_1}{2r^2} c_1^3 + c_3, \tag{1.36}$$

and

$$w_2 = c_2 \ln r - \frac{\Lambda_2}{2r^2} c_2^3 + c_4, \tag{1.37}$$

where  $c_3$  and  $c_4$  are another constant of integration to be determined. The expression given in Eq. (1.36) and (1.37) represents the general solution for the velocity field in the primary and secondary coating resins, respectively.

Now we proceed to find the constant involved in velocity field. For this, we insert the boundary conditions given in Eq. (1.31)-(1.34), we get

$$c_3 = 1 + \frac{\Lambda_1}{2} c_1^3, \tag{1.38}$$

$$c_4 = \frac{\Lambda_2}{2\delta^2} c_2^3 - c_2 \ln \delta, \tag{1.39}$$

$$A_1 c_1 + A_2 c_1^3 - A_3 c_2 - A_4 c_2^3 + 1 = 0, \tag{1.40}$$

$$B_1 c_1 + B_2 c_1^3 - B_3 c_2 - B_4 c_2^3 = 0, \tag{1.41}$$

where the values of the constant  $A_1, A_2, A_3,$

$A_4, B_1, B_2$  and  $B_4$  are

$$A_1 = \ln R_i,$$

$$A_2 = \frac{\Lambda_1}{2} - \frac{\Lambda_1}{2\Omega^2},$$

$$A_3 = \ln\Omega - \ln\delta,$$

$$A_4 = \frac{\Lambda_2}{2\delta^2} - \frac{\Lambda_2}{2\Omega^2},$$

$$B_1 = \frac{1}{\Omega} = B_3,$$

$$B_2 = \frac{2\varepsilon_1\lambda_1^2}{\eta_1^2\Omega^3},$$

$$B_4 = \frac{2\varepsilon_2\lambda_2^2}{\eta_2^2\Omega^3}.$$

Now Eq. (1.40) and (1.41) are highly non-linear system of algebraic equations. Therefore solution we use special cases.

**4. (a) Special Case-1**

Let us suppose that

$$A_1 = B_1 \text{ and } A_2 = B_2,$$

From Eq. (1.39) and (1.40), we end up with a cubic equation

$$Gc_2^3 + Mc_2 + 1 = 0, \tag{1.42}$$

with coefficients

$$G = B_3 - A_4, M = B_1 - A_3,$$

The real root to the cubic equation Eq. (1.42) can be obtained explicitly by the formula for the third order algebraic equation as

$$c_2 = -\frac{\left(\frac{2}{3}\right)^{1/2}M}{\left((-9G^2 + \sqrt{3}\sqrt{27G^4 - 4G^2M^3})\right)^{1/3}} + \frac{\left((-9G^2 + \sqrt{3}\sqrt{27G^4 - 4G^2M^3})\right)^{1/3}}{(2)^{1/3}(3)^{2/3}G}. \tag{1.43}$$

Substitute Eq. (1.43) into Eq. (1.40) or (41), we get the value of  $c_1$ .

The volume flow rate for the primary and secondary coating resins, are as

$$Q_1 = c_1 \left(\frac{\Omega^2}{2} \ln\delta - \frac{\Omega^2}{4} + \frac{1}{4}\right) - \frac{\Lambda_1}{2} \ln\Omega c_1^3 + \left(\frac{\Omega^2}{2} - \frac{1}{2}\right) c_3, \tag{1.44}$$

$$Q_2 = c_2 \left(\frac{1}{2}(\delta^2 \ln\delta - \Omega^2 \ln\Omega) - \frac{1}{4}(\delta^2 - \Omega^2)\right) - \frac{\Lambda_2 c_2^3}{2} (\ln\delta - \ln\Omega) + \frac{c_4}{4} (\delta^2 - \Omega^2). \tag{1.45}$$

The thickness of the primary and secondary coating layers,  $h_1$  and  $h_2$  is explicitly determined by the supplied volumetric flow rate of both coating resins  $Q_1$  and  $Q_2$  [9].

$$h_1 = R_1 \left( \left(1 + \frac{Q_1}{\pi v R_1^2}\right)^{1/2} - 1 \right), \tag{1.46}$$

$$h_2 = R_1 \left( \left( \left(1 + \frac{h_1}{R_1}\right)^2 + \frac{Q_2}{\pi v R_1^2} \right)^{1/2} + \left(1 + \frac{h_1}{R_1}\right) \right). \tag{1.47}$$

An important physical property is the shear stress on the bare glass fiber, which is associated with the drawing tension of the glass to cause fiber breakage. The shear rate at the glass fiber surface is derived from Eq. (1.18) as

$$\gamma_w = S_{rz}|_{r=1} = \frac{c_1}{R_1}. \tag{1.48}$$

Consider a special case in which the viscosities of the two coating resins are identical i.e ( $\eta_1 = \eta_2$ ), then, the above analysis goes back to the single-coating fluid analysis and the total volume flow rate of coating resin ( $Q_1 + Q_2$ ) becomes

$$Q = c_1 \left(\frac{\Omega^2}{2} \ln\delta - \frac{\Omega^2}{4} + \frac{1}{4}\right) - \frac{\Lambda_1}{2} \ln\Omega c_1^3 + c_2 \left(\frac{1}{2}(\delta^2 \ln\delta - \Omega^2 \ln\Omega) - \frac{1}{4}(\delta^2 - \Omega^2)\right) - \frac{\Lambda_2 c_2^3}{2} (\ln\delta - \ln\Omega) + \frac{c_4}{4} (\delta^2 - \Omega^2) + \left(\frac{\Omega^2}{2} - \frac{1}{2}\right) c_3 \tag{1.49}$$

**4. (b) Special Case-2**

Now we suppose that

$$A_3 = B_3 \text{ and } A_4 = B_4,$$

From Eq. (1.40) and (1.41), we end up with a cubic equation

$$Ic_1^3 + Hc_1 + 1 = 0, \tag{1.50}$$

with coefficient

$$H = A_1 - B_1, I = A_2 - B_2,$$

The real root to the cubic equation Eq. (1.50) can be obtained explicitly by the formula for the third order algebraic equation as

$$c_1 = -\frac{\left(\frac{2}{3}\right)^{1/2}H}{\left((-9I^2 + \sqrt{3}\sqrt{27I^4 - 4I^2H^3})\right)^{1/3}} + \frac{\left((-9I^2 + \sqrt{3}\sqrt{27I^4 - 4I^2H^3})\right)^{1/3}}{(2)^{1/3}(3)^{2/3}I}. \tag{1.51}$$

Substitute Eq. (1.51) into Eq. (1.40) or (41), we get the value of  $c_2$ .

The volume flow rate for the primary and secondary coating resins, are as

$$Q_1 = c_1 \left(\frac{\Omega^2}{2} \ln\Omega - \frac{\Omega^2}{4} + \frac{1}{4}\right) - \frac{\Lambda_1}{2} \ln\Omega c_1^3 + \left(\frac{\Omega^2}{2} - \frac{1}{2}\right) c_3. \tag{1.52}$$

$$Q_2 = c_2 \left(\frac{1}{2}(\delta^2 \ln\delta - \Omega^2 \ln\Omega) - \frac{1}{4}(\delta^2 - \Omega^2)\right) - \frac{\Lambda_2 c_2^3}{2} (\ln\delta - \ln\Omega) + \frac{c_4}{4} (\delta^2 - \Omega^2) \tag{1.53}$$

The thickness of the primary and secondary coating layers,  $h_1$  and  $h_2$  is explicitly determined by the supplied volumetric flow rate of both coating resins  $Q_1$  and  $Q_2$ .

$$h_1 = R_1 \left( \left(1 + \frac{Q_1}{\pi v R_1^2}\right)^{1/2} - 1 \right). \tag{1.54}$$

$$h_2 = R_1 \left( \left( \left(1 + \frac{h_1}{R_1}\right)^2 + \frac{Q_2}{\pi v R_1^2} \right)^{1/2} + \left(1 + \frac{h_1}{R_1}\right) \right). \tag{1.55}$$

The shear rate at the glass fiber surface is

$$\gamma_w = S_{rz}|_{r=1} = \frac{c_1}{R_1}. \tag{1.56}$$

Consider a special case in which the viscosities of the two coating resins are identical i.e ( $\eta_1 = \eta_2$ ), then, the above analysis goes back to the single-coating fluid analysis and the total volume flow rate of coating resin ( $Q_1 + Q_2$ ) becomes

$$Q = c_1 \left( \frac{\Omega^2}{2} \ln \delta - \frac{\Omega^2}{4} + \frac{1}{4} \right) - \frac{\Lambda_1}{2} \ln \Omega c_1^3 + c_2 \left( \frac{1}{2} (\delta^2 \ln \delta - \Omega^2 \ln \Omega) - \frac{1}{4} (\delta^2 - \Omega^2) \right) - \frac{\Lambda_2 c_2^3}{2} (\ln \delta - \ln \Omega) + \frac{c_4}{4} (\delta^2 - \Omega^2) + \left( \frac{\Omega^2}{2} - \frac{1}{2} \right) c_3. \quad (1.57)$$

**5: Temperature distribution**

From Eq. (1.13), we have

$$k \left( \frac{d^2}{dr^2} + \frac{1}{r} \frac{d}{dr} \right) \Theta_{0,i} + S_{rz} \frac{dw_{1,2}}{dr} = 0, \quad (1.58)$$

where the subscript 1 and 2 represents the primary and secondary coating resins, respectfully.

The corresponding boundary conditions are

$$\Theta_0 = \Theta_1 \text{ at } r = R_1, \quad (1.59)$$

$$\Theta_i = \Theta_2 \text{ at } r = R_2, \quad (1.60)$$

and

$$\Theta_0 = \Theta_i \quad (1.61)$$

$$k_1 \frac{d\Theta_0}{dr} = k_2 \frac{d\Theta_i}{dr} \text{ at } r = R_i, \quad (1.62)$$

Introduce the dimensionless parameters given in Eq. (1.28) and (1.29), Eq. (1.58) after dropping the asterisks takes the following form

$$\left( \frac{d^2}{dr^2} + \frac{1}{r} \frac{d}{dr} \right) \Theta_0 + \Lambda_3 \frac{dw_1}{dr} = 0, \quad (1.63)$$

$$\left( \frac{d^2}{dr^2} + \frac{1}{r} \frac{d}{dr} \right) \Theta_i + \Lambda_4 \frac{dw_2}{dr} = 0. \quad (1.64)$$

The boundary conditions given by Eq. (1.59)-(1.62) becomes

$$\Theta_0 = 0 \text{ at } r = 1, \quad (1.65)$$

$$\Theta_i = 0 \text{ at } r = \delta, \quad (1.66)$$

And

$$\Theta_0 = \Theta_i, \quad (1.67)$$

$$k_1 \frac{d\Theta_0}{dr} = k_2 \frac{d\Theta_i}{dr} \text{ at } r = \Omega. \quad (1.68)$$

Now substituting the expression for the velocity profile in the energy equation Eq. (1.63) and (1.64) and solving the resulting equation corresponding to boundary conditions given by Eq. (1.65)-(1.68), we obtain the temperature distribution for the primary and secondary coating resins, respectfully, as

$$\Theta_0 = D_1 \ln r + D_2 - \Lambda_3 \left( c_1 \ln r + \frac{\Lambda_1}{4r^2} c_1^3 \right), \quad (1.69)$$

$$\Theta_i = D_3 \ln r + D_4 - \Lambda_4 \left( c_2 \ln r + \frac{\Lambda_2}{4r^2} c_2^3 \right), \quad (1.70)$$

where the values of the constant  $D_1$ ,  $D_2$ ,  $D_3$  and  $D_4$  are

$$D_1 = \beta \left( c_1 \left( \frac{\Lambda_3}{r} \ln \Omega - \frac{\Lambda_3}{r} \ln \Omega \ln r \right) + c_1^3 \left( \frac{\Lambda_1}{r} \ln \Omega - \frac{1}{r} \left( \frac{\Lambda_1 \Lambda_3}{4} \ln \Omega - \frac{\Lambda_1 \Lambda_3}{4\Omega^2} \right) \right) + \frac{c_2^3}{r} \left( \frac{\Lambda_2}{4\delta^2} + \frac{\Lambda_2 \Lambda_4}{4\Omega^2} \right) + \frac{c_2}{r} (\Lambda_4 \ln \delta + \Lambda_4 \ln \Omega) - \frac{1}{r} (\ln \frac{\delta}{\Omega} + \beta \ln \Omega), \quad (1.71)$$

$$D_2 = 1 + \frac{\Lambda_1 \Lambda_3}{4} c_1^3, \quad (1.72)$$

$$D_3 = \left( c_1 \left( \frac{\Lambda_3}{r} \ln \Omega - \frac{\Lambda_3}{r} \ln \Omega \ln r \right) + c_1^3 \left( \frac{\Lambda_1}{r} \ln \Omega - \frac{1}{r} \left( \frac{\Lambda_1 \Lambda_3}{4} \ln \Omega - \frac{\Lambda_1 \Lambda_3}{4\Omega^2} \right) \right) + \frac{c_2^3}{r} \left( \frac{\Lambda_2}{4\delta^2} + \frac{\Lambda_2 \Lambda_4}{4\Omega^2} \right) + \frac{c_2}{r} (\Lambda_4 \ln \delta + \Lambda_4 \ln \Omega) - \frac{1}{r} (\ln \frac{\delta}{\Omega} + \beta \ln \Omega) \right) / 1r (\ln \delta \Omega + \beta \ln \Omega) + \Lambda 4 \ln \Omega - 1r) / 1r (\ln \delta \Omega + \beta \ln \Omega) + \Lambda 3 c_1 \ln r - \Lambda 1 2 r 2 c_1 3 - \Lambda 4 c_2 \ln r - \Lambda 2 2 r 2 c_2 3, \quad (1.71)$$

$$\frac{c_2}{r} (\Lambda_4 \ln \delta + \Lambda_4 \ln \Omega) - \frac{1}{r} \left( \ln \frac{\delta}{\Omega} + \beta \ln \Omega \right), \quad (1.73)$$

$$D_4 = -\ln \delta \left( c_1 \left( \frac{\Lambda_3}{r} \ln \Omega - \frac{\Lambda_3}{r} \ln \Omega \ln r \right) + c_1^3 \left( \frac{\Lambda_1}{r} \ln \Omega - \frac{1}{r} \left( \frac{\Lambda_1 \Lambda_3}{4} \ln \Omega - \frac{\Lambda_1 \Lambda_3}{4\Omega^2} \right) \right) + \frac{c_2^3}{r} \left( \frac{\Lambda_2}{4\delta^2} + \frac{\Lambda_2 \Lambda_4}{4\Omega^2} \right) + \frac{c_2}{r} (\Lambda_4 \ln \delta + \Lambda_4 \ln \Omega) - \frac{1}{r} (\ln \frac{\delta}{\Omega} + \beta \ln \Omega) \right) + \Lambda 4 c_2 \ln \delta + \Lambda 2 2 \delta 2 c 2 3, \quad (1.74)$$

and

$$\beta = \frac{k_2}{k_1}.$$

**6: Results and Discussion**

The expression for the velocity of fluid and temperature distribution is given by Eq. (1.36)-(1.37) and Eq. (1.69)-(1.70) for the primary and secondary coating resins, respectively. Also the volume flow rate, shear stress and thickness of coated fiber optic are derived. Figure.3 and 4 presents the velocity profile as a function of  $r$  for several values of viscosity parameter  $\eta_1$  and speed of fiber drawing  $v$  in the primary coating resins. In Figure.3, we varied the viscosity parameter  $\eta_1$  i.e.,  $\eta_1 = 5, 9, 14$  and fixed  $\Lambda_1 = 2, R_1 = 4, \Omega = 5, \delta = 9, \Lambda_2 = 4, \eta_2 = 4, \varepsilon_2 = 5, \lambda_2 = 0.3, \varepsilon_1 = 4, \lambda_1 = 4$ . It is obvious from Figure.3 that velocity decreases with increase in viscosity parameter  $\eta_1$ . Also when the speed of fiber drawing increase, the velocity of fluid is increasing, as shown in Figure.4. The effect of viscosity parameter  $\eta_2$  and the speed of fiber drawing  $v$  on the velocity profile in the secondary coating resin is expressed in Figure 5 and Figure 6. From Figure.5, it is clear that the velocity decrease with increase in viscosity and increase with increasing the fiber drawing speed as shown in Figure.6.

The expression in Eq. (1.55) representing the thickness of coated fiber optic in the secondary coating resin, is plotted in Figure.7 and 8. In Figure.7, we varied  $h_1$  (Thickness of coated fiber optic in primary coating resin) i.e.,  $h_1 = 0.04, 0.06, 0.08$  and fixed  $\Lambda_1 = 4, \Lambda_2 = 5, \varepsilon_2 = 4, \lambda_2 = 4, \eta_2 = 7, v = 0.4, \Omega = 2, \delta = 6$ .

It is clear from Figure.7 that the thickness in the secondary coating die increases with increasing  $h_1$ . Also in Figure.8 the thickness increase with increasing  $\delta$ . Fig. 9 and 10 are plotted for the variation of shear stress for different values of  $\delta$  and  $\eta_2$ , respectively. It is to be noted that both the stresses decrease with increase in  $\delta$  and  $\eta_2$ . In Figures.11 and 12, we plotted dimensionless temperature profiles  $\Theta_0$  and  $\Theta_i$  versus  $\delta$  with selected sets of parameters. The subscript 1 and 2 represents the primary and secondary coating resin. It can be observed that the temperature increase with

increasing the viscosity parameters  $\Lambda_1$  and  $\Lambda_2$  as shown in Figure.11 and 12, respectively.

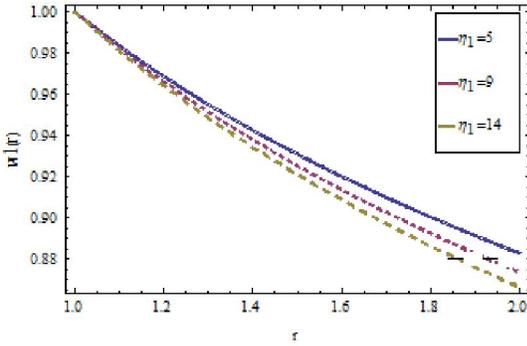


Figure 3: Dimensionless velocity profiles for different values of  $\eta_1$  at fixed values of  $\Lambda_1 = 2, R_1 = 4, \Omega = 5, \delta = 9, \Lambda_2 = 4, \eta_2 = 4, \varepsilon_2 = 5, \lambda_2 = 0.3, \varepsilon_1 = 4, \lambda_1 = 4, v = 2$ .

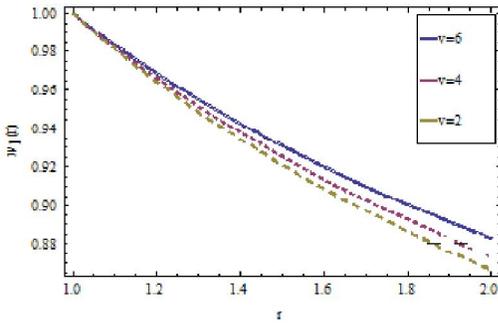


Figure 4: Dimensionless velocity profiles for different values of  $v$  at fixed values of  $\Lambda_1 = 2, R_1 = 4, \Omega = 5, \delta = 9, \Lambda_2 = 4, \eta_2 = 4, \varepsilon_2 = 5, \lambda_2 = 0.3, \eta_1 = 3, \varepsilon_1 = 4, \lambda_1 = 4$ .

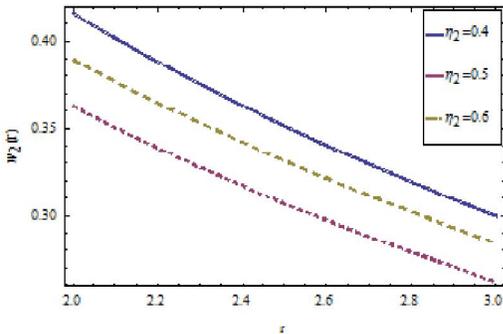


Figure 5: Dimensionless velocity profiles for different values of  $\eta_2$  at fixed values of  $\Lambda_1 = 2, R_1 = 4, \Omega = 5, \delta = 9, \Lambda_2 = 4, \varepsilon_2 = 5, \lambda_2 = 0.3, \varepsilon_1 = 4, \lambda_1 = 4, v = 2$ .

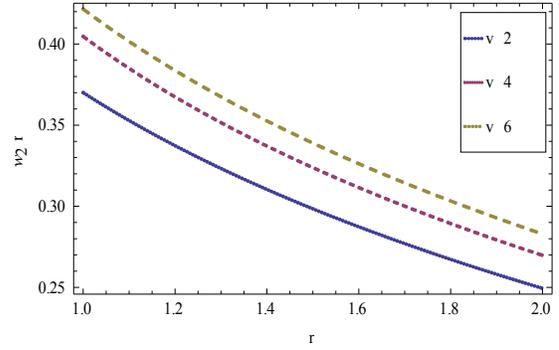


Figure 6: Dimensionless velocity profiles for different values of  $v$  at fixed values of  $\Lambda_1 = 2, R_1 = 4, \Omega = 5, \delta = 9, \Lambda_2 = 4, \eta_2 = 4, \varepsilon_2 = 5, \lambda_2 = 0.3, \varepsilon_1 = 4, \lambda_1 = 4$ .

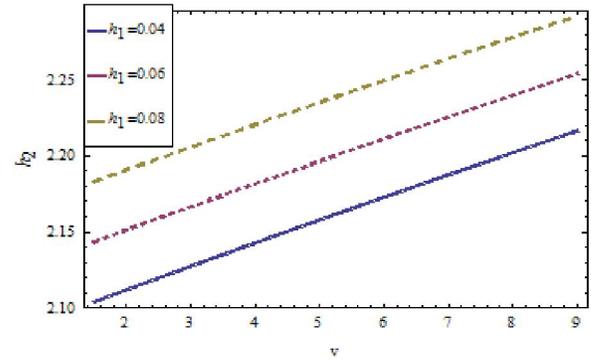


Figure 7: Thickness of coated fiber optics versus ratio of radii for different values of  $h_1$  at fixed values of  $\Lambda_1 = 4, \Lambda_2 = 5, \varepsilon_2 = 4, \lambda_2 = 4, \eta_2 = 7, v = 0.4, \Omega = 2, \delta = 6$ .

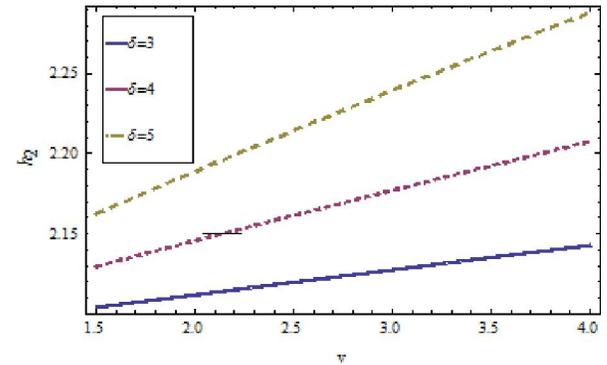


Figure 8: Thickness of the coated fiber optics versus ratio of the radii for different values of  $\delta$  at fixed values of  $\Lambda_1 = 4, \Lambda_2 = 5, \varepsilon_2 = 4, \lambda_2 = 4, \eta_2 = 7, v = 0.4, \Omega = 2, h_1 = 0.06$ .

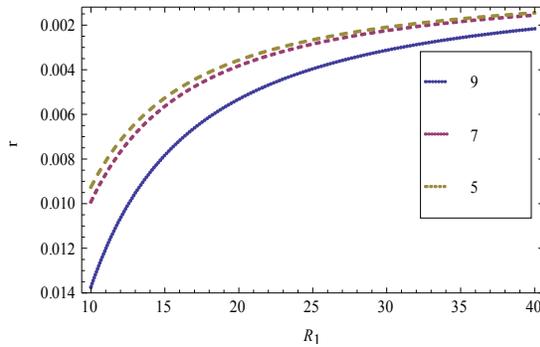


Figure 9: Dimensionless shear stress profiles for different values of  $\delta$  at fixed values of  $\Lambda_1 = 4$ ,  $\Lambda_2 = 5$ ,  $\epsilon_2 = 4$ ,  $\lambda_2 = 4$ ,  $\eta_2 = 7$ .

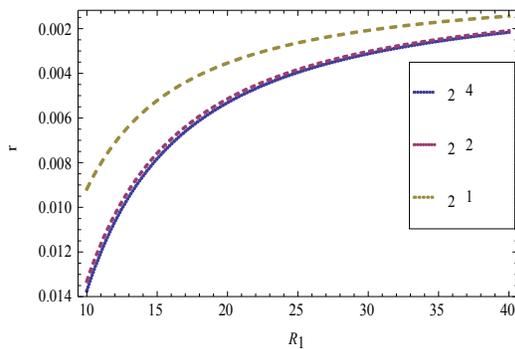


Figure 10: Dimensionless shear stress profiles for different values of  $\eta_2$  at fixed values of  $\Lambda_1 = 4$ ,  $\Lambda_2 = 5$ ,  $\epsilon_2 = 4$ ,  $\lambda_2 = 4$ ,  $\delta = 7$ .

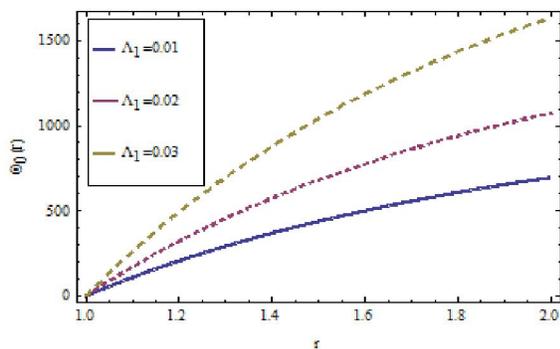


Figure 11: Dimensionless temperature distribution for different values of  $\Lambda_1$  at fixed values of  $R_1 = 0.5$ ;  $R_i = 0.7$ ;  $\delta = 1.8$ ;  $\Lambda_2 = 0.01$ ;  $\epsilon_2 = 5$ ;  $\lambda_2 = 1$ ;  $\eta_2 = 4$ ;  $\Lambda_4 = 0.6$ ;  $\Lambda_3 = 5$ ;  $\beta = 3$ .

**7: Conclusion**

This paper presents an analytical modeling on the wet-on-wet coating process on glass fiber in a capillary coating die. Analytical solutions are derived for the axisymmetric flow of nonlinear viscoelastic

PTT fluid in optical glass fiber coating analysis. Expressions are presented for the radial variation of the axial velocity and temperature distribution. Exact solutions for volume flux, thickness of coated optical glass fiber and shear stress on glass fiber are obtained. The Effect of viscosity parameter  $\eta_1$  and  $\eta_2$ , the speed of fiber drawing  $V$ , ratio of the radii  $\delta$  and thickness of primary coating resin  $h_1$  are discussed. It was found that the velocity of the fluid decrease with increasing of  $\eta_1$  and  $\eta_2$  and increase with increasing of  $V$ . The thickness of coated fiber optic in the secondary coating resin increase as the thickness increases in the primary coating resin. The thickness is also increase with increasing  $\delta$ . It is also found that the shear stress increase with decreasing  $\delta$  and  $\eta_2$ . The fluid temperature depends upon  $\Lambda_1$  and  $\Lambda_2$  and increase very quickly with increasing these parameters. Also, it respectively, reduces to Maxwell and linear viscous model by setting  $\epsilon$  and  $\lambda$  equal to zero.

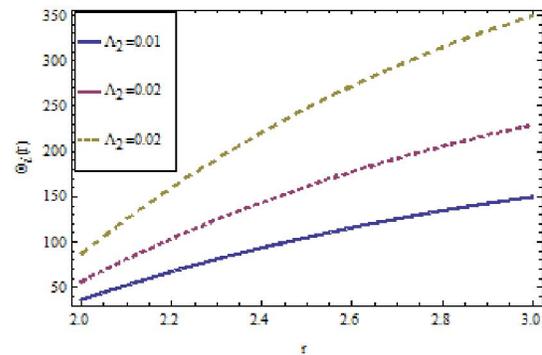


Figure 12: Dimensionless temperature distribution for different values of  $\Lambda_2$  at fixed values of  $\Lambda_1 = 3$ ,  $R_1 = 0.5$ ,  $R_i = 0.7$ ,  $\delta = 1.8$ ,  $\epsilon_2 = 5$ ,  $\lambda_2 = 1$ ,  $\eta_2 = 4$ ,  $\Lambda_4 = 0.6$ ,  $\Lambda_3 = 5$ ,  $\beta = 3$ .

**References**

1. Aloisio, CJ, Lenahan, TA, Taylor, CR. Coating Optical Fibers with Dual Dies Separated by a Pressurized Chamber. *Fiber Int. Opt.* 1984; **5** 81-97.
2. Jochevm, CMG, van der Ligt, JWC, cooling and Bubble-Free Coating of Optical Fiber at a High Drawing Rate. *J. Lightwave Technol.* 1986; **LT-4** 739.
3. Ravinutala, S. Polymeripoulos. Entrance Meniscus in a Pressurized Optical Fiber Coating Applicator. *Exp. Them. Fluid Sci.* 2002; **26** 573-580.
4. Sakagushi, Kimura. High-Speed Drawing of Optical Fiber with Pressurized Coating. *J. LightwaveTechnol.* 1985; **LT-3** 669-673.

5. Panoliaskos, A, Hallett, WLH, Garis. Prediction of Optical Fiber Coating Thickness. *Appl. Opt.* 1985 **24** 2309-2312.
6. Yang, A, Tao, XM, Cheng, XY. Prediction of Fiber Coating Via Liquid-Phase Process. *J. Master. Process. Technol.* 2008;**202** 365-373.
7. Tiu, Podolsak, AK, Mitsoulis. Process and Simulation of Wire Coating. IN: Cheremisinoff, NP (ed.) *Handbook of Polymer Science and Technology, Vol. 3: Application and Processing Operation.* Marcel Dekker Inc, New York. 1989; 609-647.
8. Mitsoulis, E. Fluid Flow and Heat Transfer in Wire Coating: A Review. *Adv. Polym. Technol.* 1986; **6** 467-487.
9. Kyoungjin Kim and Ho. Sang Kwak. Analytical Study of Non-Newtonian Double Layer Coating Liquid Flows in Optical Fiber Manufacturing. *App. Mech. Mat. Vo.* 2012; 224 260-263.
10. Anturkar, NR, Papanastasiou, TC, Wilkes, JO. Lubrication Theory for n-layer Thin-Film Flow with Application to Multilayer Extrusion and Coating. *Chem. Eng. Sci.* 1990 **45** 3271-3282.

9/28/2013