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Applications Of Laplace Transform To Differential Equations With Discontinuous Functions

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Abstract: Most of the problems in different areas of science, engineering and technology are solved by the Laplace transformation method. In this paper, we will apply Laplace transformation method to differential equations with discontinuous functions. Laplace transformation makes it easier to solve the problems and makes differential equations with discontinuous functions simple to solve. This paper presents a new technological approach to solve differential equations with discontinuous functions.

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Key words: Laplace transformation, Differential equations with discontinuous functions.
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Introduction

Laplace transformation is very useful tool in various areas of engineering and science. It helps us to find the solution of initial value problems involving homogeneous and non- homogeneous equations. It minimizes the problem of solving differential equations to an algebraic problem which becomes much easier to solve. It is very powerful technique, because it replaces operations of calculus by operation of algebra [1, 2, 3, 4, 5, 6,]. In this paper, we apply Laplace transformation method to differential equations with discontinuous functions.

Definition

Let f (t) is a function of t which is well defined for all t ≥ 0. The Laplace transformation [1, 2, 7, 8,] of f (t), denoted by f (p) or L {F (t)}, is defined as

L {F (t)} $=\int_0^\infty e^{-pt}F(t)dt$, provided that the integral exists, i.e. convergent. The Laplace transformation of some derivatives [3, 4, 5, 6,] is

$L\{F'(t)\} = pL\{F(t)\} - F(0)$,

$L\{F''(t)\} = p^2 f(p) - F(0) - F'(0)$,

And so on.

Problem I: Using Laplace Transformations to solve

$$+10y' + 9y = g(t)$$
$$y(0) = y'(0) = 0$$

Where,

$$g(t) = \begin{cases} \cos\left(t - \frac{2\pi}{3}\right),\ t > \frac{2\pi}{3} \\ 0, otherwise \end{cases}$$

Solution:

$$y'' + 10y' + 9y = g(t)$$

Taking Laplace transformation [1, 2, 7,] on both sides

$$L\{y''\} + 10\,L\{y'\} + 9L(y) = g(t)$$

Or

$$+$$

$$10[p\,\bar{y}(p) - p(0)] + 9\bar{y}(p) = \frac{p\,e^{-\frac{2\pi p}{3}}}{p^2+1}$$

Now,

$$\bar{y}(p) = \frac{p\,e^{-\frac{2\pi p}{3}}}{(p^2+1)(p+9)(p+1)}$$

Or

$$y = L^{-1}\left[\frac{p\,e^{-\frac{2\pi p}{3}}}{(p^2+1)(p+9)(p+1)}\right]$$

$$= -\frac{1}{16}L^{-1}\left[\frac{1}{(p+1)}\right] + \frac{9}{656}L^{-1}\left[\frac{1}{(p+9)}\right] + L^{-1}\left[\frac{\frac{2}{41}p - \frac{5}{82}}{(p^2+1)}\right]$$

$$= -\frac{1}{16}e^{-t} + \frac{9}{656}e^{-9t} + \frac{2}{41}cost - \frac{5}{82}sint$$

Hence

$$L^{-1}\left[\frac{p\,e^{-\frac{2\pi p}{3}}}{(p^2+1)(p+9)(p+1)}\right] =$$

$$-\frac{1}{16}e^{-\left(t-\frac{2\pi}{3}\right)} + \frac{9}{656}e^{-9\left(t-\frac{2\pi}{3}\right)} + \frac{2}{41}\cos\left(t - \frac{2\pi}{3}\right) - \frac{5}{82}\sin\left(t - \frac{2\pi p}{3}\right)$$

Problem II: Using Laplace Transformations to solve

$$Solve\ y'' + 4y = 4u(t-2)$$
$$y(0) = y'(0) = 0$$
$$and\ u(t-2)\ is\ unit\ step\ function$$
Solution:
$$y'' + 4y = 4u(t-2)$$

Taking Laplace transformation [3, 4, 8,] on both sides

$$L\{y''\} + 4L(y) = 4Lu(t-2)$$

Or

$$\bar{y}(p) - py(0) - y'(0) + 4\bar{y}(p) = 4\frac{e^{-2p}}{p}$$

Or

$$+4)\bar{y}(p) = 4\frac{e^{-2p}}{p}$$

Or

$$\bar{y}(p) = \frac{4e^{-2p}}{p(p^2+4)}$$

Or

$$\bar{y}(p) = \left[\frac{1}{p} - \frac{p}{p^2+4}\right]e^{-2p}$$

Hence

$$y = L^{-1}\left[\frac{1}{p} - \frac{p}{p^2+4}\right]e^{-2p}$$

Or

$$y = L^{-1}\left[\frac{1}{p}\right]e^{-2p} - L^{-1}\left[\frac{p}{p^2+4}\right]e^{-2p}$$

Or

$$y = u(t-2) - u(t-2)\cos 2(t-2)$$

Or

$$y = u(t-2)[1 - \cos 2(t-2)]$$

Problem III: Charge on capacitor and initial current is zero to solve for current in LC Circuit if L=1henry, C=1 fared

$$V(t) = f(x) = \begin{cases} 10t, 0 < t < 0 \\ 0, otherwise \end{cases}$$

The differential equation of LC Circuit is given by

$$L\ddot{Q} + \frac{Q}{C} = E$$
$$L = 1, C = 1\ and\ E = V(t) = 10t.$$

Solution: We have

$$+q = \int_0^{\infty} e^{-pt}\,V(t)dt$$

Or

$$+q = \int_0^1 e^{-pt}\,10t\,dt$$

Taking Laplace transformation [6, 9, 10]on both sides

$$L\{Q''\} + L(Q) = L\int_0^1 e^{-pt}\,10t\,dt$$

Or

$$\bar{Q}(p) - pQ(0) - Q'(0) + \bar{Q}(p) = 10\left[-\frac{e^{-p}}{p} - \frac{e^{-p}}{p^2} + \frac{1}{p^2}\right]$$

Or

$$(p^2+1)\bar{Q}(p) = 10\left[-\frac{e^{-p}}{p} - \frac{e^{-p}}{p^2} + \frac{1}{p^2}\right]$$

Or

$$\bar{Q}(p) = 10\left[\frac{e^{-p}}{(p^2+1)p} \quad \frac{e^{-p}}{(p^2+1)p^2} + \frac{1}{(p^2+1)p^2}\right]$$

Or

$$Q = L^{-1}\left\{10\left[-\frac{e^{-p}}{(p^2+1)p} - \frac{e^{-p}}{(p^2+1)p^2} + \frac{1}{(p^2+1)p^2}\right]\right\}$$

Or

$$Q = -10[\{1 - \cos(t-1) + (t-1) - \sin(t-1)\}u(t-1) - (t - \sin t)$$

Conclusion:

This paper has presented a new technological approach to solve differential equations with discontinuous functions. It may be finish that this technique is very foremost and accomplished in finding the solution of differential equations with discontinuous functions.

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