Absolute Parametric Instability in a Nonuniform Plane Plasma waveguide

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Abstract: The paper reports an analysis of the effect of spatial plasma nonuniformity on absolute parametric instability (API) of electrostatic waves in a magnetized plane waveguides subjected to an intense high frequency (HF) electric field. It is shown that allowance for the spatial nonuniformity leads to 1) localization of unstable waves in a finite region of a plasma volume, 2) increases in the threshold value of the pump wave amplitude above which parametric amplification occurs and 3) decreases in the value of the growth rate of unstable waves.
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1. Introduction

The parametric interaction of an external HF electric field with an electrostatic surface wave in anisotropic nonuniform plasma has been previously investigated using a special method based on the separation of variables (Demchenko and Omelchenko, 1976). The method makes it possible to separate the problem into two parts. The "dynamical" (temporal) part describes the parametric excitation of waves and corresponding equations within the renormalization of natural (eigen) frequencies coincides with equations for parametrically unstable waves in uniform plasma (Aliev and Silin, (1965) and Silin, (1965)). Natural frequencies of surface waves and spatial distribution of the self-consistent electric field amplitude are determined from the solution of a boundary-value problem ("spatial" part) taking into account specific spatial distribution of plasma density. The proposed approach ("separation method") is significantly simpler than the method previously used in the theory of parametric resonance in a nonuniform plasma (e.g., ref. Kaw, Kruer, Liu and Nishikawa, (1976) and references therein). Therefore, it is of special interest to apply the separation method to solve different problems involving parametric excitation of electrostatic waves in bounded nonuniform plasma.

It is known that (e.g. Perkins and Flick, (1971)) the spatial nonuniformity of plasma density may lead to localization of a parametrically unstable in a finite region of a plasma volume. This suggests that instability has assumed an absolute character. From an experimental point of view, it is quite important to know whether a given parametric instability is absolute or convective. This is so essential because the nature of instability determines the mechanism of their saturation. The convective instability reaches saturation at a comparatively low level, due to convection of energy of the unstable waves away from the resonance region. The absolute instability saturates at a higher level of energy under the action of various nonlinear effects. From this point of view, an absolute parametric instability (API) play a crucial role in the process of the energy transfer from the electromagnetic radiation to the plasma and may have important consequences for experiments on RF plasma heating in tokamaks and for laser fusion (Rosenbluth, (1972), Piliya, (1973), Pesme, Javal and Pellat, (1973), Silin and Starodub, (1974), White, Kaw, Pesme, Rosenbluth, Javal, Huff and Varma, (1974), and Mourou, Tajima and Bulonov, (2006)).

In ref. (Demchenko and Omelchenko, 1976) the problem of parametric excitation of natural modes of semi-infinite plasma (surface waves) was analyzed as an initial value problem. In other words, surface waves are excited due to an initial perturbation at the boundary and a dispersion equation determines the complex frequency ω as a function of the real wave number k. It is of practical interest to use the separation method described in (Demchenko and Omelchenko, 1976) for the solution of an eigen-values problems when the wave number k is found as a function of the real frequency ω. This means that one has to treat a forced oscillations excited in a plasma by an external source (generator) with a fixed frequency ω_g (for more details see e.g., Demchenko and Zayed, 1972), where the initial value problem for the problem of surface wave transformation at the plasma resonance in a transition layer).

Demchenko et al. (1998) have reported an analysis of the effect of spatial plasma nonuniformity on parametric instability of electrostatic waves in a magnetized cylindrical waveguides subjected to an intense HF electric field.

A method is expounded in this paper which permits reducing the problem of absolute parametric instability excited by a monochromatic pumping field of arbitrary amplitude in nonuniform magneto active plasma to the problem of parametric excitation of spatial oscillations in uniform isotropic plasma. Below, we will discuss the parametric excitation of low-frequency waves whose dispersion is completely determined by a high-frequency field, in a strong magnetic field i.e., here, we shall apply the method of ref. (Demchenko and Omelchenko, 1976) to investigate the API in a $1-D$ nonuniform plasma waveguide subjected to an intense HF electric field as an eigen-value problem.

Using the separation method, we investigate the API in bounded nonuniform relativistic plasma under the effect of pump field and static magnetic field. The pump field $\vec{E}_P = \vec{E}_0 \sin(\omega_0 t)$ and the static magnetic field $\vec{B}_0$ are both directed along the $z-$ axis. Assuming the intensity of the magnetic field to be strong enough $\left(\omega_{c_\alpha} >> \omega_{P_\alpha}\right)$, the motion of plasma particles are considered to be confined along $z-$ axis only. We are going to study API in $1-D$ nonuniform bounded plasma.

2. Separation method in the problem of API in a $1-D$ nonuniform bounded plasma

Let us suppose that plane waveguide is filled by nonuniform plasma $(n_{\alpha_0} = n_{\alpha_0}(x); \alpha = e,i)$. A uniform strong static magnetic field $\vec{B}_0(\omega_{c_\alpha} >> \omega_{P_\alpha})$ and a HF electric field $\vec{E}_P = \vec{E}_0 \sin(\omega_0 t)$ are directed along the z axis. We choose the electric field of an ordinary wave as an HF pump field. The equilibrium particles velocity $\vec{u}_\alpha(0,0,u_\alpha)$ is determined by the following expression:

$$\vec{u}_\alpha = -\frac{e_\alpha \vec{E}_0}{m_\alpha \omega_0} \cos(\omega_0 t) \tag{1}$$

Representing the perturbations of velocity $\delta\vec{V}_\alpha(0,0,\delta V_\alpha)$, density and electrical potential Φ in the form $(\delta\vec{V}_\alpha, \delta n_\alpha, \Phi) \sim \exp(ikz)$.

The initial system of equations consists of the two fluid equations in combination of the Poisson equation:

$$\frac{\partial \vec{V}_\alpha}{\partial t} + (\vec{V}_\alpha \cdot \nabla)(\vec{V}_\alpha) = \frac{e_\alpha}{m_\alpha}(\vec{E}_P + \frac{1}{c}[\vec{V}_\alpha \times \vec{B}_0] - \nabla\Phi) \tag{2}$$

$$\frac{\partial n_\alpha}{\partial t} + div(n_\alpha \vec{V}_\alpha) = 0, \tag{3}$$

$$\Delta\Phi = -4\pi \sum_\alpha e_\alpha n_\alpha \tag{4}$$

Where, n_α and $\vec{V}_\alpha$ are the density and velocity of particles of species α, and Φ is the potential self-consistent electric field.

Suppose that

$$k_z \equiv k; \ \frac{\partial \delta V_\alpha}{\partial t} \sim \omega \delta V_\alpha \sim \omega_{P_\alpha} \delta V_\alpha, \ k u_\alpha \sim \omega_{P_\alpha}.$$

It means that particles are "frozen" and could not move across the magnetic field $\delta V_{\alpha_x} = \delta V_{\alpha_y} = 0$. From linearized equation (2) we find

$$ikn_{\alpha_0} \frac{\partial \delta V_\alpha}{\partial t} = k^2 \frac{e_\alpha}{m_\alpha} n_{\alpha_0} \Phi, \tag{5}$$

The continuity equation (3) reduces to

$$-[\frac{\partial \delta n_\alpha}{\partial t} + iku_\alpha \delta n_\alpha] = ikn_{\alpha_0} \delta V_\alpha, \tag{6}$$

Introducing new variable

$$\nu_\alpha = e_\alpha \delta n_\alpha e^{iA_\alpha}, \ A_\alpha = -a_\alpha \sin(\omega_0 t), \ a_\alpha \equiv \frac{e_\alpha k E_0}{m_\alpha \omega_0^2} \approx a_e,$$

set of equations (5) and (6) can be rewritten as following

$$\frac{\partial^2 \nu_\alpha}{\partial t^2} = -\frac{e_\alpha^2}{m_\alpha} e^{-iA_\alpha} \hat{L}_2 \Phi, \tag{7}$$

Where; $\hat{L}_2\Phi = -k^2 n_{\alpha_0}(x)$.

The Poisson's equation takes the form

$$\frac{\partial^2\Phi}{\partial x^2} - k^2\Phi = -4\sum_{\alpha} \nu_\alpha e^{iA_\alpha(t)} \tag{8}$$

Assuming $\nu_\alpha(x,t) = \nu_{\alpha_1}(t)\nu_{\alpha_2}(x),\ \Phi(x,t) = \Phi_1(t)\Phi_2(x)$ and separating variables in equations (7) and (8), we have

$$\frac{d^2\nu_{\alpha_1}}{\partial t^2} = -\frac{m_e p_\alpha^2}{e^2} e^{iA_\alpha} \sum_{\beta=e,i} \nu_{\beta_1} \frac{e_\beta^2}{m_\beta} e^{-iA_\beta}, \tag{9}$$

$$\frac{\partial^2\Phi_2}{\partial x^2} - k^2\varepsilon(x,p)\Phi_2 = 0 \tag{10}$$

Where; $\varepsilon(x,p) = 1 - \omega_{P_e}^2/p^2$ and p is the separation constant. Set of equations (9) describe "temporal" (dynamical) part of the problem. Comparing the derived equations with the system describing volumetric oscillations in a uniform plasma (Pesme, Javal and Pellat, (1973), Silin and Starodub, (1974)), we find that the presence of plasma nonuniformity results in a renormalization of the natural plasma frequencies $\omega_{P_e}^2 \to p^2, \omega_{P_i}^2 \to (m_e/m_i)p^2$. This fact enables us to use the method developed in (Silin, 1965) to solve the system of equations with periodical coefficients (9). Equation (10) corresponds to the "spatial" (stationary) part of the problem. If the profile of plasma density and boundary conditions are specified, solution of equation (10) gives us the needed value of constant p. The distinguishing feature of the equation (10) is that the amplitude of HF electric field is not part of it.

3. Solution of the spatial equation (10)

We will consider API in nonuniform plasma in which the density distribution is determined by the relation $n = n_0(1 - x^2/L^2)$ [14] where L is the characteristic scale of nonuniformity. In this case equation (10) takes the form

$$\frac{\partial^2\Phi_2}{\partial x^2} - k^2\Phi_2 + \frac{k^2}{p^2}\left(\frac{4\pi e^2 n_0}{m}\right)\left(1 - \frac{x^2}{L^2}\right)\Phi_2 = 0 \tag{11}$$

Equation (10) yields

$$\frac{\partial^2\Phi_2}{\partial x^2} + \left(A - Bx^2\right)\Phi_2 = 0 \tag{12}$$

Where;

$$A = -k^2\varepsilon_0,\ B = \frac{\omega_{P_0}^2 k^2}{p^2L^2},\quad \varepsilon_0 = 1 - \frac{4\pi e^2 n_0}{m_e p^2}.$$

The solution of equation (12), which describes trapped oscillations, is possible for $A < 0$ $(\varepsilon_0 < 0)$ in the region $-\sqrt{\frac{|A|}{B}} < x < \sqrt{\frac{|A|}{B}}$. Then equation (12), takes the form

$$\frac{\partial^2\Phi_2}{\partial\xi^2} + \left(\frac{A}{\sqrt{B}} - \xi^2\right)\Phi_2 = 0,\ \frac{|A|}{\sqrt{B}} = \frac{k|\varepsilon_0|pL}{\omega_{P_0}} \tag{13}$$

Where; $\xi = (k\omega_{P_0}/Lp)^{1/2}x$. Making the substitution $\Phi_2 = \psi(\xi)\exp(-\xi^2/2)$ and introducing the notation

$$2n+1 = \frac{|A|}{\sqrt{B}} = \frac{k|\varepsilon_0|pL}{\omega_{P_0}} \tag{14}$$

We obtain the equation

$$\psi''_{\xi\xi} - 2\xi\psi'_\xi + 2n\psi = 0 \tag{15}$$

for the function $\psi(\xi)$. The solutions of this equation are Hermite polynomials (Richards, 2009):

$$\psi \sim H_n(\xi) = (-1)^n e^{\xi^2}\frac{d^n e^{-\xi^2}}{d\xi^n} \tag{16}$$

satisfying the localizability condition (the width of the region of localizability of the oscillations is assumed to be significantly less than the width of the plasma layer) only for integral positive values of the number n (including zero). This fact permits considering equation (14) as an analog of the quantization rule, which serves to determine the possible values of the quantity p. Thus, the solution of equation (13) takes the form

$$\Phi_2 = c_n e^{-\xi^2/2}H_n(\xi) \tag{17}$$

From equation (14), we get

$$p^2 + \left(\left[\frac{2n+1}{kL}\right]\omega_{P_0}\right)p - \omega_{P_0}^2 = 0 \tag{18}$$

Thus from equation (18), we get

$$p = \left(\frac{\omega_{P_0}}{2}\right)\left\{\left(\left[\frac{2n+1}{kL}\right]^2 + 4\right)^{1/2} - \left(\frac{2n+1}{kL}\right)\right\} \tag{19}$$

At $L \to \infty, \quad p \to \omega_{P_0}$ (plasma waves in uniform plasma). Equation (18), takes the form

$$\left.\begin{aligned} & p^2 = \omega_{P_0}^2 (1-\delta), \\ & \delta = \frac{1}{2}\cdot\frac{2n+1}{kL}\left\{\left[4+\left(\frac{2n+1}{kL}\right)^2\right]^{1/2} - \frac{2n+1}{kL}\right\} \end{aligned}\right\} \quad (20)$$

This equation is the same one in a uniform plasma case (Demchenko and Omelchenko, 1976); i.e., the nonuniform plasma has no effect on the space part of the problem.

4. Solution of the "Temporal" (Time-Dependent) equations

Following the procedure, developed in (Aliev and Silin, (1965) and Silin, (1965)), from equations (9) we can derive dispersion equation of low-frequency oscillations $(\omega \approx (m_e / m_i) p)$.

Under the parametric resonance condition ($n\omega_0 \approx s, \quad n-$ integer), we get

$$\omega^4 - \frac{\Delta_n^2 p^2}{4}\omega^2 - \frac{m_e}{2m_i} p^4 \Delta_n J_n^2(a) = 0 \quad (21)$$

where: $\Delta_n = (p / n\omega_0)^2 - 1,$, and we suppose here that the resonance "mismatch" Δ_n satisfies the inequalities $(m_e / m_i) << |\Delta_n| << 1$. From equation (21) we find the frequencies of parametrically excited plasma oscillations

$$\omega^2 = \frac{\Delta_n^2 p^2}{8}\left(1 \pm \left[1 + \frac{32}{\Delta_n^3} J_n^2(a)\frac{m_e}{m_i}\right]^{1/2}\right) \quad (22)$$

where: $J_n(a)$ is the Bessel function. Expression (22) yields an unstable solution in two cases:

a) Periodic instability $(\Delta_n < 0)$

In this case $\gamma_{per} = \operatorname{Im}\omega > 0$, i.e., small perturbations in plasma grow exponentially in time, if the following condition is satisfied

$$0 > \Delta_n > -2\left(4J_n^2(a)\frac{m_e}{m_i}\right)^{1/3} \quad (23)$$

The growth rate of instability is determined by the expression

$$\gamma_{per} = p\frac{|\Delta_n|}{4}\left[-1 + \left(\frac{32J_n^2(a)}{|\Delta_n|^3}\frac{m_e}{m_i}\right)^{1/2}\right]^{1/2} \quad (24)$$

The maximum value of the growth rate γ_{per} reached at

$$(n\omega_0)_{\max} = p\left[1 + \left(\frac{1}{4}J_n^2(a)\frac{m_e}{m_i}\right)^{1/3}\right] \quad (25)$$

Substituting (25) into (24) we find

$$\gamma_{per}^{\max} = p\left[\frac{\sqrt{27}}{32}J_n^2(a)\frac{m_e}{m_i}\right]^{1/3} \quad (26)$$

b) A periodic instability $(\Delta_n > 0)$

In this case expressions (22) describe the growth of oscillations when the minus sign is taken. We have then the following expression for the growth rate

$$\gamma_{aper} = \frac{\Delta_n}{2\sqrt{2}} p\left[\left(1 + \frac{32J_n^2(a)}{\Delta_n^{\ 3}}\frac{m_e}{m_i}\right)^{1/2} - 1\right]^{1/2} \quad (27)$$

The maximum of the growth rate

$$\gamma_{aper}^{\max} = p\left[\frac{1}{2}J_n^2(a)\frac{m_e}{m_i}\right]^{1/3} \quad (28)$$

is attained under the condition

$$(n\omega_0)_{aper}^{\max} = p\left[1 - \left(\frac{1}{2}J_n^2(a)\frac{m_e}{m_i}\right)^{1/3}\right] \quad (29)$$

The main feature of equations (24) – (26) is in the existence of a separation constant p which enables us to account for the plasma nonuniformity.

At $\delta << 1$ expressions (26) and (28) become

$$\gamma_{per}^{\max} \cong (1-\frac{\delta}{2})\gamma_{per}^{\max U}, \quad \gamma_{aper}^{\max} \cong (1-\frac{\delta}{2})\gamma_{aper}^{\max U} \quad (30)$$

Where $\gamma_{per}^{\max U}$ and $\gamma_{aper}^{\max U}$ are the values of the growth rates of periodical and a periodical API at vanishing density gradients.

From equation (24) we conclude that, the threshold value of the HF field amplitude in case of periodic instability is determined by the relation

$$32J_n^2(a_{thr})\frac{m_e}{m_i} = |\Delta_n|^3,\ |\Delta_n| = -\frac{p^2}{(n\omega_0)^2} + 1 \qquad (31)$$

At small amplitudes of the pumping wave $a << 1,\ n = 1$, from equation (31) we get

$$\left.\begin{aligned} a_{thr}^2 &= \frac{m_i}{8m_e}\left[|\Delta_0| + \delta\frac{\omega_{P_0}^2}{(n\omega_0)^2}\right]^3, \\ |\Delta_0| &= \frac{\omega_{P_0}^2}{(n\omega_0)^2} - 1 \end{aligned}\right\} \qquad (32)$$

It follows from expressions (30) and (32) that nonuniformity of the plasma density results in a decrease of the growth rate of absolutely unstable oscillations and an increase in the threshold value of the pump wave amplitude in comparison with the case of a uniform plasma waveguide.

It should be noted that our approach is significantly simpler than the method ordinarily employed in theory of a parametric excitation of waves in nonuniform plasma. Therefore, it is of practical interest to apply the method to solve different problems in parametric resonance in nonuniform plasma taking into account finite plasma temperature and nonuniformities of the HF electric field and static magnetic field.

5. Results and Conclusions

We study in this paper the effect of $1-D$ plasma nonuniformity on absolute parametric instability (API) of electrostatic waves in magnetized pump plasma is in plane geometry by using the separation method,

It follows from equations (20), (26), (28), (31) and (32) that taking account of nonuniformity of plasma density results in decrease of the maximum values of the oscillation build up increments and an increase in the threshold value of the electric field amplitude of the pumping wave in comparison with the case of uniform plasma. These results are consistent with the results of Refs. Perkins and Flick, (1971) and Demchenko et al (1976).

Equation (20) is the same equation in uniform plasma case Perkins and Flick, (1971) and Demchenko et al (1976); i.e., the nonuniformity plasma has no effect on the space part of the problem. The main feature of equation (9) enables us to account for the plasma nonuniformity.

From expressions (26) and (29), we conclude that the growth rate of periodic API decreases in nonuniform plasma more than in uniform plasma which is considered by .Demchenko et al (1976).

It should be noted that our approached is significantly simpler than the method ordinary employed in theory of a parametric excitation of waves in nonuniform plasma. Therefore it is of practical interest to apply the method to solve different problems in parametric resonance in nonuniform plasma taking into account relativistic electron plasma and nonuniformities of the HF electric field and static magnetic field.

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