# Thermodynamic modeling of an irreversible dual cycle: effect of mean piston speed

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**Abstract:** In this paper, the theory of finite time thermodynamics is used to determine the effect of mean piston speed on performance of an irreversible dual cycle. In the model, the non-linear relation between the specific heat ratio of the working fluid and its temperature, the friction loss computed according to the mean velocity of the piston, the internal irreversibility described by using the compression and expansion efficiencies and the heat transfer loss are considered. The relations between the power output and the compression ratio, between the power output and the thermal efficiency are derived by detailed numerical examples. The results shows that if compression ratio is less than certain value, the power output increases with increasing mean engine speed, while if compression ratio exceeds certain value, the power output first increases and then starts to decrease with increasing mean engine speed. With further increase in compression ratio, the increase of mean piston speed results in decreasing the power output. This paper provides an additional criterion for use in the evaluation of the performance and the suitability of a dual engine. [Report and Opinion. 2009;1(5):25-30]. (ISSN: 1553-9873).

**Key words**: dual cycle; internal irreversibility; performance optimization

## 1. Introduction

Traditional thermodynamics is a theory about equilibrium states and about limits on process variables for transformations from one equilibrium state to another. In order to obtain more realistic limits to the performance of real processes, thermodynamics is extended to finite-time thermodynamics to deal with processes which have explicit time or rate dependencies (Andresen et al., 1984; Bejan, 1996; Aragon-Gonzalez et al., 2000). Thus, much work has been performed for the performance analysis and optimization of finite time processes and finite size devices (Aragon-Gonzalez et al., 2006; Chen et al., 2007; Aragon-Gonzalez et al., 2008; Ge et al., 2008a). Wu and Blank (1992) and Blank and Wu (1994) carried out the effect of combustion on the work or power-optimized Otto, Diesel and dual cycles. Chen et al. (1996) and Chen et al. (1998) derived the relations between net work output and efficiency of the Diesel cycles. The relation between net work output and the efficiency as well as the maximum net-work output and the corresponding efficiency for internal-combustion dual cycles are derived in this paper. Sahin et al. (2002a, 2002b) optimized the performance of a new combined power cycle based on power density analysis of the dual cycle and made a comparative performance analysis of an endoreversible dual cycle under a maximum ecological function and maximum power conditions. Parlak et al. (2004) optimized the performance of an irreversible dual cycle: the predicted behavior was corroborated by experimental results. Chen et al. (2004) determined the characteristics of power and efficiency for dual cycle with heat transfers and friction losses. It is found that there are optimal values of the cut-off ratio at which the power output and efficiency attain their maxima. Ust et al. (2005) performed an ecological performance analysis for an irreversible dual cycle by employing the new thermo-ecological criterion as the objective function. They compared the effects of cut-off ratio on performance of the cycle. Al-Sarkhi et al. (2006) investigated the effects of friction, temperature-dependent specific heat of the working fluid and cut-off ratio on the performances of the Diesel-cycle. Parlak and Sahin (2006) defined the internal irreversibility by using entropy production, and analyzed the effect of the internal irreversibility on the performance of irreversible dual cycle. Chen et al. (2006) and Ghatak and Chakraborty (2007) analyzed the effect of variable specific heats and heat transfer loss on the performance of the dual cycle when variable specific heats of working fluid are linear functions of its temperature. Zhao et al. (2007) defined the internal irreversibility by using compression and expansion efficiencies and analyzed the performance of dual cycle. Ge et al. (2008a; 2008b; 2009) analyzed the performance of an air standard Otto, Diesel and dual cycles. In the irreversible cycle model, the non-linear relation between the specific heat of the working fluid and its temperature, the friction loss computed according to the mean velocity of the piston, the internal irreversibility described by using the compression and expansion efficiencies, and the heat transfer loss are considered.

As can be seen in the relevant literature, the investigation of the effect of mean piston speed on performance of dual cycle does not appear to have been published. Therefore, the objective of this study is to examine the effect of mean piston speedon performance of air standard dual cycle.

## 1. Thermodynamic analysis

The temperature-entropy diagram of an irreversible dual heat engine is shown in Fig. 1, where $T_1$, $T_{2s}$, $T_2$, $T_3$, $T_4$, $T_{4s}$ and $T_5$ are the temperatures of the working substance in state points 1, 2s, 2, 3, 4, 4s and 5. Process $1 \to 2s$ is a reversible adiabatic compression, while process $1 \to 2$ is an irreversible adiabatic process that takes into account the internal irreversibility in the real compression process. The heat additions are an isochoric process $2 \to 3$ and an isobaric process $3 \to 4$. The process $4 \to 5s$ is a reversible adiabatic expansion, while $4 \to 5$ is an irreversible adiabatic process that takes into account the internal irreversibility in the real expansion process. The heat-removing process is the reversible constant volume $5 \to 1$.

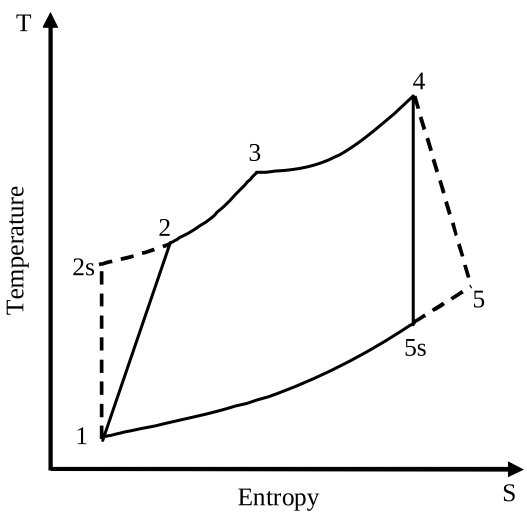


Figure 1. $T-S$ diagram of a dual cycle

According to Refs. (Abu-Nada 2005; Chen et al. 2009), for the temperature range of 300-3500 K, the specific heat with constant pressure can be written as:

$$c_p = 2.506\times10^{-11}T^2 + 1.454\times10^{-7}T^{1.5} - 4.246\times10^{-7}T + 3.162\times10^{-5}T^{0.5} + 1.3303 - 1.512\times10^{4}T^{-1.5} + 3.063\times10^{5}T^{-2} - 2.212\times10^{7}T^{-3} \quad (1)$$

where $T$ is the absolute temperature. The unit of $c_p$ is $kJ\,kg^{-1}K^{-1}$.

The specific heat with constant volume can be written as:

$$c_v = c_p - R_{air} = 2.506\times10^{-11}T^2 + 1.454\times10^{-7}T^{1.5} - 4.246\times10^{-7}T + 3.162\times10^{-5}T^{0.5} + 1.0433 - 1.512\times10^{4}T^{-1.5} + 3.063\times10^{5}T^{-2} - 2.212\times10^{7}T^{-3} \quad (2)$$

The heat added per second in the isobaric ($2 \to 3$) and isobaric ($3 \to 4$) heat addition processes may be written as

$$\begin{aligned} Q_{in} &= M_{Sp}\left[\int_{T_2}^{T_3} c_v dT + \int_{T_3}^{T_4} c_p dT\right] = M_{Sp}\Big[8.353\times10^{-12}T^3 + 5.816\times10^{-8}T^{2.5} - 2.123\times10^{-7}T^2 + 2.108\times10^{-5}T^{1.5} + \\ &+1.0433T + 3.024\times10^{4}T^{-0.5} - 3.063\times10^{5}T^{-1} + 1.106\times10^{7}T^{-2}\Big]_{T_2}^{T_3} + M_{Sp}\Big[8.353\times10^{-12}T^3 + \\ &5.816\times10^{-8}T^{2.5} - 2.123\times10^{-7}T^2 + 2.108\times10^{-5}T^{1.5} + 1.3303T + 3.024\times10^{4}T^{-0.5} \\ &-3.063\times10^{5}T^{-1} + 1.106\times10^{7}T^{-2}\Big]_{T_3}^{T_4} \end{aligned} \quad (3)$$

where $M_{sp}$ is the molar number of the working fluid which is function of mean engine speed.

The heat rejected per second in the isochoric heat rejection process ($5 \to 1$) may be written as:

$$\begin{aligned} Q_{out} &= M_{Sp}\int_{T_1}^{T_5} c_v dT = \\ &M_{Sp}\big(8.353\times10^{-12}T^3 + 5.816\times10^{-8}T^{2.5} - 2.123\times10^{-7}T^2 + 2.108\times10^{-5}T^{1.5} + 1.0433T + \\ &3.024\times10^{4}T^{-0.5} - 3.063\times10^{5}T^{-1} + 1.106\times10^{7}T^{-2}\big) \end{aligned} \quad (4)$$

The compression and expansion efficiencies can be defined as (Ge et al., 2008a; Ge et al., 2008b):

$$\eta_c = (T_{2s} - T_1)/(T_2 - T_1) \quad (5)$$

and

$$\eta_e = (T_5 - T_4)/(T_{5s} - T_4) \quad (6)$$

These two efficiencies can be used to describe the internal irreversibility of the processes.

Since specific heat with constant volume and specific heat with constant pressure are dependent on temperature, the adiabatic exponent will vary with temperature as well. Therefore, the equation often used in a reversible adiabatic process with constant specific heat ratio cannot be used in a reversible adiabatic process with variable specific heat ratio. However, according to Refs (Ge et al. 2007; Chen et al., 2008), the equation for a reversible adiabatic process with variable specific heat ratio can be written as follows:

$$TV^{\gamma-1} = (T+dT)(V+dV)^{\gamma-1} \quad (7)$$

From Eq. (6), one gets

$$C_v \ln\frac{T_j}{T_i} = R_{air}\ln\frac{V_i}{V_j} \quad (8)$$

where the temperature in the equation of $c_v$ is $T = (T_j - T_i)/\ln(T_j/T_i)$.

The compression ratio, $r_c$, and pressure ratio, $\alpha$, are defined as

$$r_c = V_1/V_2 \quad (9)$$

and

$$\alpha = T_3/T_2 \quad (10)$$

Therefore, the equations for processes ($1 \to 2s$) and ($4 \to 5s$) are shown, respectively, by the following:

$$C_v \ln\frac{T_{2s}}{T_1} = R_{air}\ln r_c \quad (11)$$

and

$$C_v \ln\frac{T_4}{T_{5s}} = R_{air}\ln\frac{T_2}{T_4} + R_{air}\ln(r_c\alpha) \quad (12)$$

The energy transferred to the working fluid during combustion is given by the following linear relation (Chen et al., 2008; Ebrahimi, 2009b)

$$Q_{leak} = M_{Sp}B(T_2 + T_4 - 2T_0) \quad (13)$$

where $B$ are a constant related to heat transfer.

Taking into account the friction loss of the piston and assuming a dissipation term represented by a friction force that is a linear function of the piston velocity gives (Chen et al., 2006; Ge et al., 2007; Ebrahimi, 2009a)

$$f_\mu = -\mu S_p = -\mu\frac{dx}{dt} \quad (14)$$

where $\mu$ is the coefficient of friction, which takes into account the global losses, $x$ is the piston's displacement and $S_p$ is the piston's velocity. Therefore, the lost power due to friction is

$$P_\mu = \frac{dW_\mu}{dt} = -\mu\left(\frac{dx}{dt}\right)^2 = -\mu(S_p)^2 \quad (15)$$

Thus, the lost power is

$$P_\mu = -\mu(\bar{S}_p)^2 \quad (16)$$

where $\bar{S}_p$ is the mean velocity of the piston.

Thus, the power output of the dual cycle engine can be written as

$$P_{out} = Q_{in} - Q_{out} - P_\mu \quad (17)$$

The efficiency of the dual cycle engine is expressed by

$$\eta_{th} = \frac{Q_{in} - Q_{out} - P_\mu}{Q_{in} + Q_{leak}} = \frac{P_{out}}{Q_{in} + Q_{leak}} \quad (18)$$

When $r_c$, $\alpha$, $T_1$, $T_4$, $\eta_c$ and $\eta_e$ are given, $T_{2s}$ can be obtained from Eq. (11), then, substituting $T_{2s}$ into Eq. (5) yields $T_2$. $T_3$ can be obtained from Eq. (10), $T_{5s}$ can be obtained from Eq. (12) and the last, $T_5$ can be found by substituting $T_{5s}$ into Eq. (6). Substituting $T_1$, $T_2$, $T_3$, $T_4$ and $T_5$ into Eqs. (17) and (18), respectively, the power output and thermal efficiency of the dual cycle engine can be obtained. Therefore, the relations between the power output, the thermal efficiency and the compression ratio can be derived.

## 3. Results and discussion

The following constants and parameters have been used in this exercise: $T_4 = 2200\,K$, $T_1 = 350\,K$, $L = 70\ mm$, $\eta_c = 0.97$, $\eta_e = 0.97$, $\mu = 12.9\,Nsm^{-1}$, $B = 0.2\ kJ.kg^{-1}K^{-1}$, $\bar{S}_p = 7 \to 23\ rev/s$, $r_c = 1.5 \to 100$, $\alpha = 1.5$ $T_0 = 345\,K$ and $M_{Sp} = 5.4204E-4 \times \bar{S}_P\ kg.s^{-1}$ (Heywood, 1988; Chen et al. 2007; Ghatak and Chakraborty, 2007; Ge et al., 2009; Ebrahimi, 2009a). Using the above constants and range of parameters, the power output versus compression ratio characteristic and the power output versus efficiency characteristic with varying the mean piston speed can be plotted. Numerical examples are shown as follows.

Figures 2 and 3 show the effects of the variable mean piston speed on the cycle performance with heat resistance, internal irreversibility and friction losses (the dashed lines in the figures denote where the cycle cannot work normally). From these figures, it can be found that the mean piston speed plays important roles on the power output. It is clearly seen that the effect of mean piston speed on the power output is related to compression ratio. They reflect the performance characteristics of a real irreversible dual cycle engine. It should be noted that the heat added and the heat rejected by the working fluid increase with increasing mean piston speed (see Eqs. (3) and (4)).

Figure 2 indicates the effects of the mean piston speed on the power output of the cycle for different values of the compression ratio. It can be seen that the power output versus compression ratio characteristic is approximately parabolic like curves. In other word, the power output increases with increasing compression ratio, reach their maximum values and then decreases with further increase in compression ratio. The maximum power output increases with increasing mean piston speed up to about 15 rev/s where it reaches its peak value then starts to decline as the mean

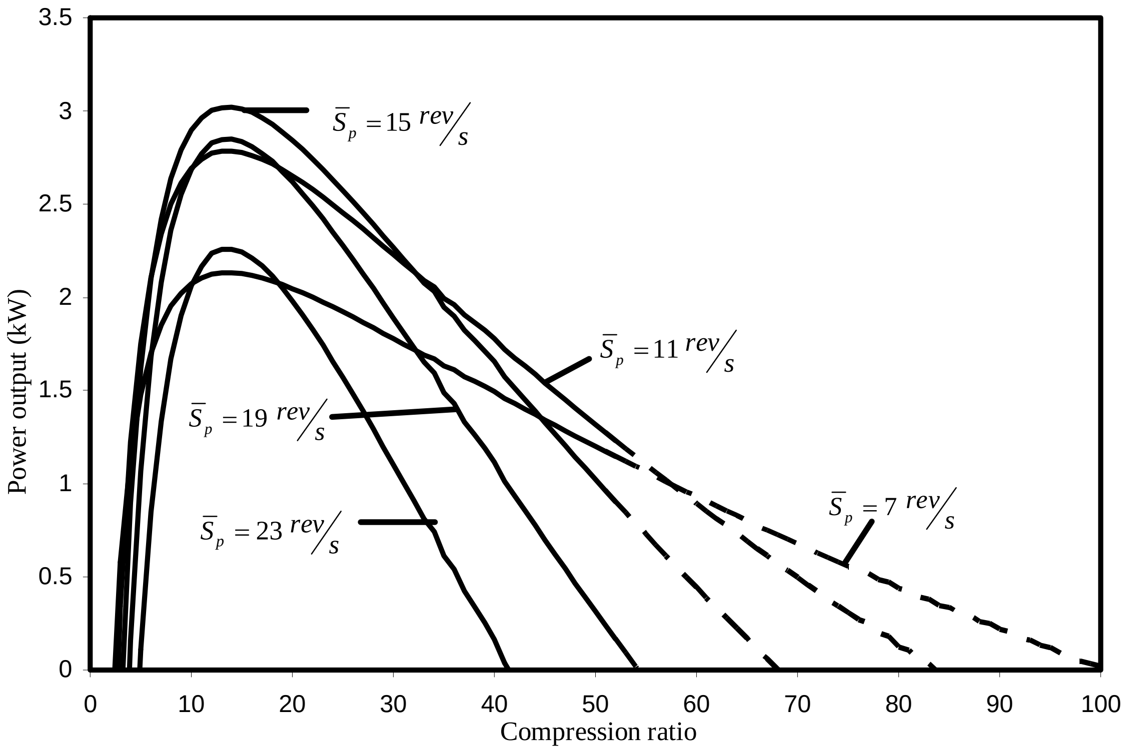


Figure 2. Effect of mean piston speed on the variation of the efficiency with compression ratio

piston speed increases. This is consistent with the experimental results in the internal combustion engine (Mercier, 2006).

The optimal compression ratio corresponding to maximum power output point remains constant with increase of mean engine speed. The results shows that if compression ratio is less than certain value, the power output increases with increasing mean engine speed, while if compression ratio exceeds certain value, the power output first increases and then starts to decrease with increasing mean engine speed. With further increase in compression ratio, the increase of mean piston speed results in decreasing the power output. Numerical calculation shows that for any same compression ratio, the smallest power output is for $\bar{S}_p = 23\ rev/s$ when $r_c \le 10$ or $r_c > 19$ and is for $\bar{S}_p = 7\ rev/s$ when $10 < r_c \le 19$ and also the largest power output is for $\bar{S}_p = 7\ rev/s$ when $r_c \le 3.8$ or $r_c > 57$, is for $\bar{S}_p = 11\ rev/s$ when $3.8 < r_c \le 6$ or $32 \le r_c \le 57$ and is for $\bar{S}_p = 15\ rev/s$ when $6 \le r_c < 32$.

The influence of the mean piston speed on the power output versus thermal efficiency is displayed in figure 3. As can be seen from this figure, the power output versus thermal efficiency is loop shaped one. It can be seen that the power output at maximum thermal efficiency improves with increasing mean piston speed from 7 to around $\bar{S}_p = 15\ rev/s$. With further increase in mean engine speed, the power output at maximum thermal efficiency decreases. It can also be seen that the thermal efficiency at maximum power decreases with increase of mean piston speed from 7 to $\bar{S}_p = 23\ rev/s$.

According to above analysis, it can be found that the effects of the mean piston speed on the cycle performance are obvious, and they should be considered in practice cycle analysis in order to make the cycle model be more close to practice.

## 4. Conclusion

In this paper, an irreversible air standard dual cycle model which is closer to practice is established. The relations between net power output, efficiency, compression ratio, and the mean piston speed are derived. The maximum power output and the corresponding efficiency and the maximum efficiency and the corresponding power output are also calculated. The detailed effect analyses are shown by numerical examples. The analysis helps us to understand the strong effect of mean piston speed on the performance

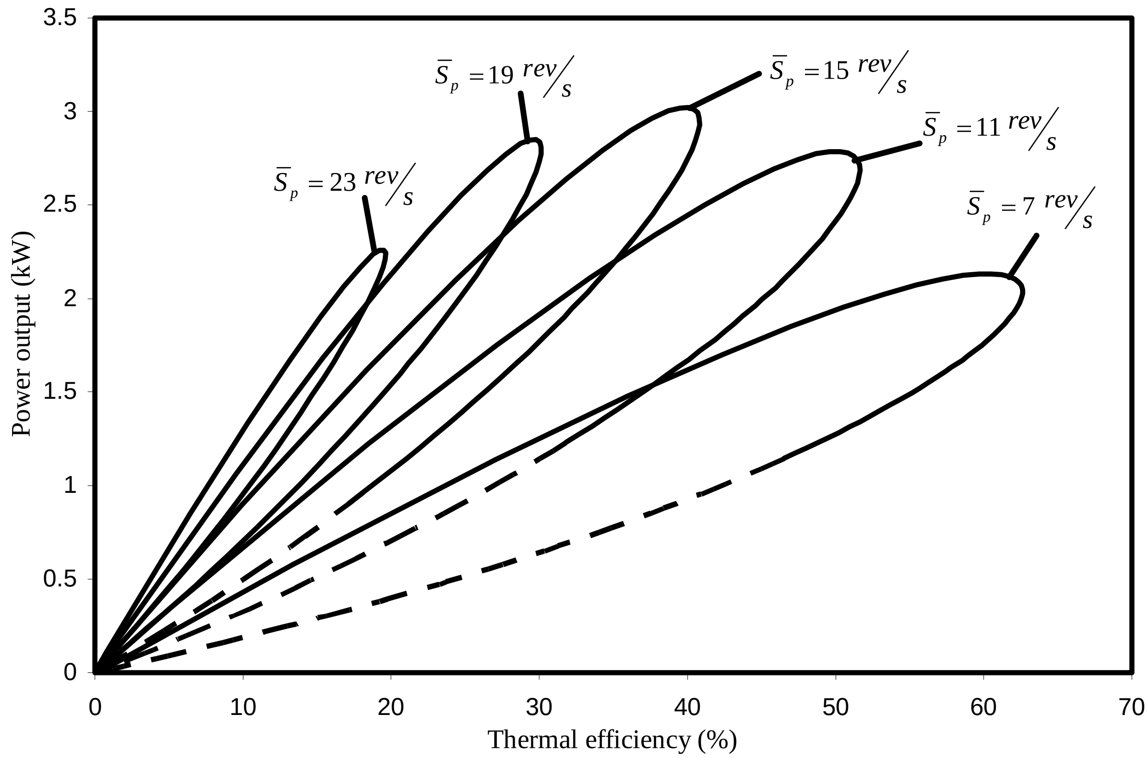


Figure 3. Effect of mean piston speed on the variation of the efficiency with compression ratio

of the dual cycle. This paper provides an additional criterion for use in the evaluation of the performance and the suitability of a dual engine.

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