

## Radius of photon orbit of rotating Black hole

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**Abstract** : A photon sphere is a spherical region of space where gravity is strong enough that photons are forced to travel in orbits . As photons travel near the event horizon of a black hole they can escape being pulled in by the gravity of a black hole by traveling at a nearly vertical direction known as an exit cone. A photon on the boundary of this cone will not completely escape the gravity of the black hole. Instead it orbits the black hole. These orbits are not stable. Radius of photon orbit around the rotating black hole is defined as a function of intense gravitational field intensity of rotating blackhole ,Spin parameter of rotating blackhole and Angular momentum of rotating blackhole. The above equation was developed based on the newton gravitational concepts,basic concepts of blackhole and Einstein's famous mass energy equivalence relationship. [Report and Opinion 2010:2(1):78-81] (ISSN: 1553-9873).

**Keywords:** Mass, force, spin parameter,energy

### Introduction

A rotating black hole is a black hole that possesses angular momentum. Rotating black holes are formed in the gravitational collapse of a massive spinning star or from the collapse of a collection of stars or gas with a total non-zero angular momentum. As most stars rotate it is expected that most black holes in nature are rotating black holes. Rotating black hole can produce large amounts of energy at the expense of its rotational energy. The above equation tells that how intense gravitational field of this black hole has influence on the radius of photon orbit around the rotating black hole. The equation  $r = [GJ/a\mathbf{c}]^{1/2}$  also states that how radius of photon orbit are related to angular momentum of rotating blackhole, spin parameter of rotating blackhole, strong intense gravitational field intensity of rotating blackhole .The intense gravitational field is strong enough to cause the trapped photons to move in photon orbit around the rotating blackhole .

### Derivation

According to Einstein's mass energy equivalence relationship:

Mass of rotating blackhole is the measure of it's energy content .

Total energy associated with the rotating Blackhole is given by  $E=Mc^2$

where M=Mass of rotating blackhole ,c = Speed of light in vaccum( $3*10^8$ m/s).

Rotating blackhole also possess spin parameter given by the relation  $a=J/Mc$

where M=Mass of rotating blackhole, J=Angular momentum of this blackhole .

By rearranging of equation  $a=J/Mc$  we get  $Mc=J/a$ .

Then the equation  $E=Mc^2$  i.e  $E=(Mc)c$  becomes  $E= Jc/a$  ,where a= spin parameter of rotating blackhole.

Photon sphere is a spherical region of space where gravity is strong enough that photons are forced to travel in orbits.

Consider photon of relativistic mass 'm' is moving in the photon orbit around this black hole.

Then the gravitational force of rotating Black hole experienced by the photon is given by  $F = GMm/r^2$  where  $G$ =Universal gravitational constant,  $M$ =Mass of rotating blackhole,  $m$ = relativistic mass of photon ,  $r$ = radius of photon orbit ( distance between rotating Black hole and photon ) .

Total energy associated with the rotating Blackhole is given by  $E = Mc^2$

then the equation  $F = GMm/r^2$  becomes  $F = GEm/r^2 c^2$  .

As the total energy of rotating black hole is also given by  $E = Jc/a$  then the equation  $F = GEm/r^2 c^2$  becomes  $F = GJcm/a r^2 c^2$ . Thus  $F = GJm/a r^2 c$  is obtained.

Intense gravitational field also surrounds this black hole ,then gravitational force of rotating Black hole experienced by the photon moving in photon orbit can also be given by  $F = mI$  where  $I$ =strong gravitational field intensity of this blackhole,  $F$ = gravitational force of rotating Black hole experienced by the photon of mass 'm' moving in photon orbit .

By equating the equations  $F = mI$  and  $F = GJm/a r^2 c$

we get the equation  $r^2 = GJ/aIc$  i.e  $r = [GJ/aIc]^{1/2}$

where  $r$  = radius of photon orbit of rotating black hole  $G$ = Universal gravitational constant,  $J$ =Angular momentum of rotating blackhole,  $a$ = spin parameter of rotating blackhole,  $I$ = strong gravitational field intensity of this black hole,  $c$ =speed of light in vaccum ( $3 \times 10^8$ m/s).

**Note:** The above equation  $r^2 = GJ/aIc$  can be applied to charged rotating blackhole then the Electric potential of rotating charged blackhole can be given by  $\phi_E = QR/(R^2 + (J/Mc)^2)$

where  $R$ =horizon radius,  $Q$ =charge on this blackhole ,  $a$ = spin parameter of this black hole( $a = J/Mc$ ) then the equation

$\phi_E = QR/(R^2 + a^2)$  is obtained.

$a^2 = (QR/\phi_E - R^2)$  i.e  $a = (QR/\phi_E - R^2)^{1/2}$ .

The equation  $r^2 = GJ/aIc$  becomes  $r^2 = GJ/(QR/\phi_E - R^2)^{1/2} Ic$  .

## Result

- 1) Radius of photon orbit of rotating blackhole is given by the relation  $r = [GJ/aIc]^{1/2}$  .  
where  $r$  = Radius of photon orbit of rotating black hole  $G$ = Universal gravitational constant,  $J$ =Angular momentum of rotating blackhole,  $a$ = Spin parameter of rotating blackhole,  $I$ =Intense gravitational field intensity of this black hole,  $c$  = Speed of light in vaccum ( $3 \times 10^8$ m/s).
- 2) Radius of photon orbit of charged rotating blackhole is given by the relation  $r^2 = GJ/(QR/\phi_E - R^2)^{1/2} Ic$  .  
where  $\phi_E$ =Electric potential of charged rotating black hole ,  $R$ =Horizon radius,  $Q$ =Charge on this blackhole ,  
 $a$ = Spin parameter of this black hole( $a = J/Mc$ ) .

## Discussions

As intense gravitational field is more, then space between rotating blackhole and photon orbit is less i.e space is dragged towards the rotating black hole. Radius of photon orbit varies directly with the angular momentum of

rotating blackhole i.e greater the angular velocity (angular momentum) of the rotation of a blackhole, greater is the distance between the rotating blackhole and photon orbit. Radius of photon orbit varies inversely with the spin parameter of rotating blackhole ( $a=J/Mc$ ). The universal constants like  $G$  (Universal gravitational constant),  $c$  (Speed of light in vacuum/air) are included in the above equations to relate radius of photon orbit, spin parameter, angular momentum, intense gravitational field of rotating blackhole, electric potential of rotating black hole, charge on black hole and horizon radius of rotating black hole. As a black hole rotates it drags space with it.

The photon sphere that is closer to the black hole is moving in the same direction as the rotation, whereas the photon sphere further away is moving against it. The black hole has an axis of rotation this only holds true if approaching the black hole in the direction of the equator.

**Conclusion :** The equation  $r = [GJ/aIc]^{1/2}$  also states that how radius of photon orbit, angular momentum of rotating blackhole, spin parameter of rotating blackhole, strong intense gravitational field intensity of rotating blackhole are related to each other.

$r \propto J$  radius of photon orbit increases with angular momentum of rotating blackhole.

$r \propto 1/a$  radius of photon orbit decreases with spin parameter of rotating blackhole.

$r \propto 1/I$  radius of photon orbit decreases with strong intense gravitational field intensity of rotating blackhole.

The equation  $r^2 = GJ/(QR/\Phi_E - R^2)^{1/2}Ic$  states that how radius of photon orbit around charged rotating blackhole is related with angular momentum, strong gravitational field intensity, electric potential, charge on black hole and horizon radius of charged rotating blackhole.

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